EML3041 Computational Methods

Fall 2023

Week Seven: September 25 – September 29

Session 02 Questions

Answer the free-response questions starting each one on a fresh sheet of paper. *Solve the problem as if you were submitting them for a test.* Identify each part separately.

An AMD chip uses 4 clock cycles to conduct one multiplication, 4 clock cycles to conduct one addition, 4 clock cycles to conduct one subtraction, and 16 clock cycles to conduct one division. The computer uses a 2.5 GHz processor. The computational times (*CT*) are given for various parts of Naïve Gauss elimination and LU decomposition methods of solving simultaneous linear equations as below.

$T =$ Clock cycle time

n= Number of rows or columns of a square matrix

$CT|_{DE} = T\left(\frac{8n^3}{3} + 4n^2 - \frac{20n}{3}\right)$,

$CT|_{FS} = T(4n^2 - 4n)$,

$CT|_{BS} = T(4n^2 + 12n)$

$CT|_{FE} = T\left(\frac{8n^3}{3} + 8n^2 - \frac{32n}{3}\right)$

The computational time to find the inverse of a matrix by LU decomposition method as derived in class is given by

$$CT|_{inverse\ by\ LU\ decomposition\ method} = T\left(\frac{32n^3}{3} + 12n^2 - \frac{20n}{3}\right)$$

Answer the following questions. Please do not look at the final answer before attempting a question.

a) What is the clock cycle time? Give units.

b) How much time in seconds does it take to conduct one division?

c) How much time would it take to find the value of $2.03 \times 3.06 + 5.07 - 3.03/4.05 + 1.03/2.915$

d) Find the formula for the time it takes to multiply two matrices $[A]_{n\times n}[C]_{n\times 1}$.

e) Estimate the computational time to find the solution of one set of 85 simultaneous linear equations using Naive Gaussian elimination.

f) Estimate the computational time to find the solution of one set of 85 simultaneous linear equations using LU decomposition method.

g) Estimate the computational time to find the solution of 1000 sets of 85 simultaneous linear equations using Naive Gaussian elimination *where the coefficient matrix is the same for all sets of equations*.

h) Estimate the computational time to find the solution of 1000 sets of 85 simultaneous linear equations using LU decomposition method *where the coefficient matrix is the same for all sets of equations*.

i) To find the solution of 1000 sets of 85 simultaneous linear equations *where the coefficient matrix stays the same*, a student claims that finding first the inverse of the coefficient matrix by LU decomposition and then using matrix multiplication for each set of equations would be more efficient than what one is doing in parts (g) and (h). Show if this claim is true.

Answers

(a) $T = 4 \times 10^{-10}$ seconds
(b) 6.4×10^{-9} seconds
(c) 1.92×10^{-8} seconds
(d) $T(8n^2 - 4n)$
(e) 6.89792×10^{-4} seconds
(f) 6.89792×10^{-4} seconds
(g) 0.689792 seconds
(h) 0.0240584 seconds
(i) Answer not given intentionally