EML3041 Computational Methods

Fall 2023

Week 8: October 9- October 13

Answer the free-response questions on fresh sheets of paper. Solve the problem as if you were submitting them for a test. Identify each part separately. You will be asked to submit the solution to Q1 at the end of the class.

1. The following incomplete y vs. x data is given

x	1	2	4	6	7
У	5	11	??	??	32

The data is fit by quadratic spline interpolants given by

$$f(x) = \begin{cases} ax - 1, 1 \le x \le 2\\ -2x^2 + 14x - 9, 2 \le x \le 4\\ bx^2 + cx + d, 4 \le x \le 6\\ 25x^2 - 303x + 928, 6 \le x \le 7 \end{cases}$$

where *a*, *b*, *c*, and *d*, are constants.

a) Estimate f(2) without looking at the table data.

- b) Estimate f(4).
- c) Estimate f'(3).
- d) Estimate f'(4). Which quadratic did you use? e) Estimate $\int_{1.6}^{3.7} f(x) dx$

f) Estimate b. You can use the internet (google search for "wolfram widget 3 equation solver" or "wolfram widget 2 equation solver" depending on how many equations you need to solve) to solve sets of equations.

Answer

a) 11 b) 15 c) 2 d) -2e) 28.0147 f) -0.25

2) A robot follows a path generated by a quadratic interpolant through three consecutive data points (2, 4), (3, 9) and (4, 16) from x = 2 to x = 4. The quadratic interpolant is given as a courtesy to you as $y = x^2$. The length of a curve is given by $L = \int_a^b \sqrt{1 + (dy/dx)^2} dx$

a) Find the approximate length of the interpolant path from x = 2 to x = 4. You can approximate a general definite integral by Simpson's $1/3^{rd}$ rule approximation as given below.

 $\int_{a}^{b} f(x)dx \cong \frac{(b-a)}{6} \left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b) \right]$

- b) Find the approximate length of the interpolant path from x = 2 to x = 4. You can use three straight lines to approximate the curve, each of whose *x*-coordinate is equidistantly spaced.
- c) What is the exact length of the length of the interpolant path from x = 2 to x = 4. Hint:

$$\int \sqrt{1 + ax^2} \, dx = \frac{\sinh^{-1}(|a|x)}{2|a|} + \frac{x\sqrt{a^2x^2 + 1}}{2} + C$$
$$\int x^2 \, dx = \frac{x^3}{3} + C$$

Answer:

a) 12.17

b) 12.17

c) 12.17