EML3041 Computational Methods
Fall 2023
Week 12: November 6-November 10

Answer each question in sequence on a fresh sheet of paper. Solve the problem as if you were submitting them for a test. Identify each part separately if a question has parts. Submit problem #1 at the end of class.

1) The one-point Gauss quadrature rule of integration is derived by starting with

$\int_a^b f(x)dx \approx c_1 f(x_1)$, where $a \le x_1 \le b$.

The values of c_1 and x_1 are found by assuming that the above formula is exact for the polynomial integrand of the form $a_0 + a_1 x$ polynomial. We obtain the one-point Gauss quadrature rule as

$$\int_a^b f(x)dx \approx (b-a)f\left(\frac{a+b}{2}\right) \qquad (1)$$

a) What would be the highest order polynomial integrand for which equation (1) be exact?
b) What is the exact value of the integral $\int_2^7 (3x^2 + 5x)dx$?
c) Estimate the value of the integral $\int_2^7 (3x^2 + 5x)dx$ by using the one-point Gauss quadrature rule.
d) Is the answer in part (c) exact? If not, what is the true error?
e) Did you expect it to give the exact value for part (c), and why or why not?

Now answer these nxt parts of the question. An engineer named Adele does not like Gauss for reasons known to us all. She develops an approximate formula for integration as

$$\int_a^b f(x)dx \approx c_1 f(x_1), \text{ where } a \le x_1 \le b$$

The values of c_1 and x_1 were found by Doja Cat by assuming that the above formula is exact for the polynomials of the form $a_0 x + a_1 x^2$. She calls the rule – one-point Doja Cat quadrature rule.
f) Find the values of c_1 and x_1.
g) Verify if the one-point Adele quadrature formula gives the exact value for the integral

$$\int_2^7 (3x^2 + 5x)dx$$

h) Verify if the one-point Adele quadrature formula gives the exact value for the integral of

$$\int_2^7 2dx$$

i) Based on what you have seen so far, was this a good idea for Doja Cat to have chosen $a_0x + a_1x^2$ polynomial for deriving her one-point rule? Why or why not?

Answer
a) Answer not given intentionally
b) 447.50
c) 416.25
d) Answer to the first part not given intentionally; True error=31.25
e) Answer not given intentionally
f) $c_1 = \frac{3(b-a)(b+a)^2}{4(b^2+a^2+ab)}$; $x_1 = \frac{2(b^2+a^2+ab)}{3(b+a)}$
g) 447.50 (from the formula); 447.50 (from exact)
h) 9.067 (from the formula); 10 (from exact)
i) Not given intentionally

2) The upward velocity of a rocket is given by

$$v(t) = 200\ln(t+1) - 10t,\ t > 0$$

where t is given in seconds and v is given in m/s.
a) Use 2-point Gauss quadrature rule to calculate the displacement of the rocket from $t = 0$ to $t = 5$s.
b) What is the true value of the displacement of the rocket from $t = 0$ to $t = 5$s?
c) What is the true error in part (a)?
d) What is the relative true error in part (a)?
e) What is the absolute relative true error in percentage for part (a).
Answer:
$a)$ 1034.6 m
$b)$ 1025.1 m
$c)\ -9.4458\ m$
$d)\ -0.0092144$
$e)$ 0.92144%

3) A scientist uses the following quadrature rule to integrate functions with fixed limits of integration.
$\int_5^{15} f(x)\,dx \cong Cf(10)$,
If the quadrature rule is expected to give the exact value of the integral for the functions of the form be^x, then what is the value of C?
Answer: 148.41