EML3041 Computational Methods
Week Three
Session 03

Answer the free-response questions on a fresh sheets of paper. ***Solve the problem as if you were submitting them for a test. Submit at the end of the class.***

1) A Maclaurin series for a function is given by

$$f(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots \ldots \ldots \ldots \ldots$$

How many terms should be used in the series to consider at least 2 significant digits are correct in your answer $f(0.1)$? Show your work.

Answer: 2

2) Take the base-10 number: -0.435

a) A hypothetical computer stores base-10 numbers in fixed point binary format in 10-bit words. The first bit is used for the sign of the number, the next four bits for the integer part, and the last five bits for the fractional part. Represent the number in the fixed-point binary format. Show all work.

b) A hypothetical computer stores base-10 numbers in fixed point binary format in 10-bit words. The first bit is used for the sign of the number, the next four bits for the biased exponent, and the last five bits for the mantissa. Represent the number in the floating-point binary format. Show all work.

c) What is the machine epsilon for the computer in part (b)?

d) Verify that the absolute relative true error in representing (-0.435) in the computer in part (b) is less than the machine epsilon.

Answers: Not given intentionally.

3) The table below shows the calculated values of the first derivative of a function $f(x)$ at $x = 3$ using three divided difference methods.

	$f'(3)$		
h	**Forward divided difference method**	**Central divided difference method**	**Backward divided difference method**
0.100000	291.3571	271.0652	250.7734
0.050000	280.4363	270.3046	260.1730
0.025000	275.1787	270.1147	265.0506
0.012500	272.5990	270.0672	267.5354
0.006250	271.3212	270.0553	268.7895
0.003125	270.6853	270.0524	269.4194

a) Based on what you know about numerical differentiation, which one is the best estimate of $f'(3)$ in the table above. Write this best estimate and justify your answer.
b) What is the absolute relative approximate error in the value of $f'(3)$ for $h = 0.00625$ as found using the forward divided difference method?
c) How many significant digits can one trust in the value of $f'(3)$ for $h = 0.00625$ found using the forward divided difference method? Show your reasoning.
d) Can you find a better approximation of $f'(3)$ based on the values given in the table. Hint: Richardson's extrapolation formula.

Answers

a) Answer not given intentionally.
b) 0.47095%
c) Answer not given intentionally.
d) 270.0514