Simultaneous Linear Equations and Matrix Algebra

Major: All Engineering Majors

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Transforming Numerical Methods Education for STEM Undergraduates

1

How do you find what you need to study and do for the course? (CHECK ALL THOSE APPLY)

- What is handed out and said in class
- Weekly CANVAS announcements
- Go to CANVAS modules
- Go to Canvas syllabus link and see what is due

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2

How many of the problems did you do from the end of chapter problem sets in the textbook to prepare for Test 1?

- Almost 100%
- About 75%
- About 50%
- About 25%
- What is this problem set you talk about?

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3

Which of the following prerequisite concepts of matrix algebra were you exposed to before you signed up for the Computational Methods course (CHOOSE ALL THAT APPLY)

- What is a matrix
- Matrix Addition
- Matrix Multiplication
- Matrix Inverse
- Determinant

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4

Given $[C] = \begin{bmatrix} 13 & 11 \\ 19 & 23 \end{bmatrix} \begin{bmatrix} 13 & 17 & 19 \\ 23 & 7 & 29 \\ 31 & 37 & 41 \end{bmatrix}$,

what is the value of c_{21}**?**

- cannot be determined
- 298
- 534
- 776

Total Results: 0

5

Velocity vs Time

The following data is given for the velocity of the rocket as a function of time. To find the velocity at $t = 21s$, you are asked to use a quadratic polynomial $v(t) = at^2 + bt + c$ to approximate the velocity profile.

t (s)	0	14	15	20	30	35
v (m/s)	0	227	362	517	602	901

$$\begin{bmatrix} 196 & 14 & 1 \\ 225 & 15 & 1 \\ 400 & 20 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 227 \\ 362 \\ 517 \end{bmatrix}$$

$$\begin{bmatrix} 225 & 15 & 1 \\ 400 & 20 & 1 \\ 900 & 30 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 362 \\ 517 \\ 602 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 1 \\ 225 & 15 & 1 \\ 400 & 20 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 362 \\ 517 \end{bmatrix}$$

$$\begin{bmatrix} 400 & 20 & 1 \\ 900 & 30 & 1 \\ 1225 & 35 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 517 \\ 602 \\ 901 \end{bmatrix}$$

Total Results: 0

6

Physical Problems

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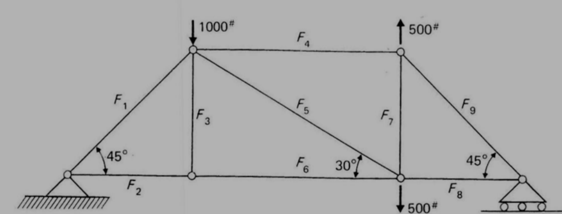
7

Truss Problem

$$\begin{bmatrix} 0.7071 & 0 & 0 & -1 & -0.8660 & 0 & 0 & 0 & 0 \\ 0.7071 & 0 & 1 & 0 & 0.5 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0.7071 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -0.7071 \\ 0 & 0 & 0 & 0 & 0.8660 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -0.5 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0.7071 \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_6 \\ F_7 \\ F_8 \\ F_9 \end{bmatrix} = \begin{bmatrix} 0 \\ -1000 \\ 0 \\ 0 \\ 500 \\ 0 \\ 0 \\ -500 \\ 0 \end{bmatrix}$$



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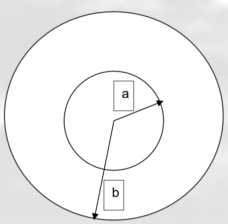
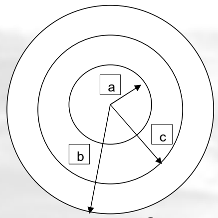
8

Pressure vessel problem





$$u_1 = c_1 r + \frac{c_2}{r}$$

$$u_1 = c_1 r + \frac{c_2}{r}$$
$$u_2 = c_3 r + \frac{c_4}{r}$$

$$\begin{bmatrix} 4.2857\times10^{7} & -9.2307\times10^{5} & 0 & 0 \\ 4.2857\times10^{7} & -5.4619\times10^{5} & -4.2857\times10^{7} & 5.4619\times10^{5} \\ -6.5 & -0.15384 & 6.5 & 0.15384 \\ 0 & 0 & 4.2857\times10^{7} & -3.6057\times10^{5} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} -7.887\times10^{3} \\ 0 \\ 0.007 \\ 0 \end{bmatrix}$$

Polynomial Regression

We are to fit the data

$$(T_1, \alpha_1), (T_2, \alpha_2), \ldots, (T_{n-1}, \alpha_{n-1}), (T_n, \alpha_n)$$

to the second order polynomial regression model

$$\alpha = a_0 + a_1 T + a_2 T^2$$

$$\begin{bmatrix} n & \left(\sum_{i=1}^{n} T_i\right) & \left(\sum_{i=1}^{n} T_i^2\right) \\ \left(\sum_{i=1}^{n} T_i\right) & \left(\sum_{i=1}^{n} T_i^2\right) & \left(\sum_{i=1}^{n} T_i^3\right) \\ \left(\sum_{i=1}^{n} T_i^2\right) & \left(\sum_{i=1}^{n} T_i^3\right) & \left(\sum_{i=1}^{n} T_i^4\right) \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{n} \alpha_i \\ \sum_{i=1}^{n} T_i \alpha_i \\ \sum_{i=1}^{n} T_i^2 \alpha_i \end{bmatrix}$$

END

Inverse of a matrix

Definition of Inverse

A matrix [B] is inverse of [A] if [B][A]=[I].

13

Application of Inverse

$$[A][X] = [C]$$

$$[A]^{-1}[A][X] = [A]^{-1}[C]$$

$$[I][X] = [A]^{-1}[C]$$

$$[X] = [A]^{-1}[C]$$

14

Find Inverse of Matrix

Find inverse of

$$\begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix} \qquad [A]^{-1} = \begin{bmatrix} a'_{11} & a'_{12} & a'_{13} \\ a'_{21} & a'_{22} & a'_{23} \\ a'_{31} & a'_{32} & a'_{33} \end{bmatrix}$$

$$\begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix} \begin{bmatrix} a'_{11} & a'_{12} & a'_{13} \\ a'_{21} & a'_{22} & a'_{23} \\ a'_{31} & a'_{32} & a'_{33} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

15

Setting up equations to find inverse

$$\begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix} \begin{bmatrix} a'_{11} & a'_{12} & a'_{13} \\ a'_{21} & a'_{22} & a'_{23} \\ a'_{31} & a'_{32} & a'_{33} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix} \begin{bmatrix} a'_{11} \\ a'_{21} \\ a'_{31} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \longrightarrow \begin{bmatrix} a'_{11} \\ a'_{21} \\ a'_{31} \end{bmatrix} = \begin{bmatrix} 0.04762 \\ -0.9524 \\ 4.571 \end{bmatrix}$$

$$\begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix} \begin{bmatrix} a'_{12} \\ a'_{22} \\ a'_{32} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \longrightarrow \begin{bmatrix} a'_{12} \\ a'_{22} \\ a'_{32} \end{bmatrix} = \begin{bmatrix} -0.08333 \\ 1.417 \\ -5.000 \end{bmatrix}$$

$$\begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix} \begin{bmatrix} a'_{13} \\ a'_{23} \\ a'_{33} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \longrightarrow \begin{bmatrix} a'_{13} \\ a'_{23} \\ a'_{33} \end{bmatrix} = \begin{bmatrix} 0.03571 \\ -0.4643 \\ 1.429 \end{bmatrix}$$

16

Putting the solutions in matrix

$$\begin{bmatrix} a'_{11} \\ a'_{21} \\ a'_{31} \end{bmatrix} = \begin{bmatrix} 0.04762 \\ -0.9524 \\ 4.571 \end{bmatrix} \quad \begin{bmatrix} a'_{12} \\ a'_{22} \\ a'_{32} \end{bmatrix} = \begin{bmatrix} -0.08333 \\ 1.417 \\ -5.000 \end{bmatrix} \quad \begin{bmatrix} a'_{13} \\ a'_{23} \\ a'_{33} \end{bmatrix} = \begin{bmatrix} 0.03571 \\ -0.4643 \\ 1.429 \end{bmatrix}$$

$$[A]^{-1} = \begin{bmatrix} 0.04762 & -0.08333 & 0.03571 \\ -0.9524 & 1.417 & -0.4643 \\ 4.571 & -5.000 & 1.429 \end{bmatrix}$$

$$\begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix} \begin{bmatrix} 0.04762 & -0.08333 & 0.03571 \\ -0.9524 & 1.417 & -0.4643 \\ 4.571 & -5.000 & 1.429 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

17

If $[A]_{n\times n}$ is the inverse of $[B]_{n\times n}$, then the following statements are true (Check all that apply)

- [B] is non-singular
- [B][A]=[I]
- [B] is inverse of [A}
- [A] is singular
- [A][B]=[I]

Total Results: 0

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18

END

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19

Solve a Set of Equations Using Inverse

$$[A][X] = [C] \implies [X] = [A]^{-1}[C]$$

$$\begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 106.8 \\ 177.2 \\ 279.2 \end{bmatrix}$$

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 0.04762 & -0.08333 & 0.03571 \\ -0.9524 & 1.417 & -0.4643 \\ 4.571 & -5.000 & 1.429 \end{bmatrix} \begin{bmatrix} 106.8 \\ 177.2 \\ 279.2 \end{bmatrix}$$

$$= \begin{bmatrix} 0.2917 \\ 19.67 \\ 1.15 \end{bmatrix}$$

20

Naive Gauss Elimination Synopsis

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21

Naive Gaussian Elimination

A method to solve simultaneous linear equations of the form [A][X]=[C]

Two parts
1. Forward Elimination
2. Back Substitution

22

Forward Elimination Part

The goal of forward elimination is to transform the coefficient matrix into an upper triangular matrix

$$\begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 106.8 \\ 177.2 \\ 279.2 \end{bmatrix}$$

$$\downarrow$$

$$\begin{bmatrix} 25 & 5 & 1 \\ 0 & -4.8 & -1.56 \\ 0 & 0 & 0.7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 106.8 \\ -96.21 \\ 0.735 \end{bmatrix}$$

23

Back Substitution

Solve each equation starting from the last equation

$$\begin{bmatrix} 25 & 5 & 1 \\ 0 & -4.8 & -1.56 \\ 0 & 0 & 0.7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 106.8 \\ -96.21 \\ 0.735 \end{bmatrix}$$

24

The goal of forward elimination steps in the Naive Gauss elimination method is to reduce the coefficient matrix to a (an) ________ matrix.

- Upper triangular
- Diagonal
- Lower triangular
- Identity

Total Results: 0

25

END

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26

Determinant of a Square Matrix Using Naïve Gauss Elimination

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27

Theorem of Determinants

If a multiple of one row of $[A]_{n\times n}$ is added or subtracted to another row of $[A]_{n\times n}$ to result in $[B]_{n\times n}$ then $\det(A) = \det(B)$

28

Theorem of Determinants

The determinant of an upper triangular, lower triangular or diagonal matrix $[A]_{n\times n}$ is given by

$$\det(\mathrm{A}) = a_{11} \times a_{22} \times \ldots \times a_{ii} \times \ldots \times a_{nn}$$
$$= \prod_{i=1}^{n} a_{ii}$$

29

Forward Elimination of a Square Matrix

Use forward elimination part to transform $[A]_{n\times n}$ to an upper triangular matrix, $[U]_{n\times n}$.

$$[A]_{n\times n} \rightarrow [U]_{n\times n}$$

$$\det(A) = \det(U)$$

30

Using Naive Gaussian Elimination method, find the determinant of the following square matrix.

$$\begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix}$$

31

Finding the Determinant

After forward elimination part

$$\begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 25 & 5 & 1 \\ 0 & -4.8 & -1.56 \\ 0 & 0 & 0.7 \end{bmatrix}$$

$$\begin{aligned} \det(\mathrm{A}) &= u_{11} \times u_{22} \times u_{33} \\ &= 25 \times (-4.8) \times 0.7 \\ &= -84.00 \end{aligned}$$

32

What does det(A)=0 and det(A)≠0 mean for [A][X]=[C]

$\det(A) = 0$ implies [A][X]=[C] has no solution or infinite solutions

$\det(A) \neq 0$ implies [A][X]=[C] has a unique solution.

33

The following system of equations

$$x + y = 2$$
$$6x + 6y = 12$$

has ___________ solution(s)

- no
- one
- more than one but a finite number of
- infinite

Total Results: 0

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34

If the determinant of a square matrix [A] is zero, then the following are (is) true (check all that apply)

- [A] does not have an inverse
- [A] has an inverse
- [A] is singular
- if [A][X]=[C] is a set of simultaneous linear equations, then [X] is unique
- if [A][X]=[C] is a set of simultaneous linear equations, then [X] is not unique

Total Results: 0

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35

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36