**EML3041 Computational Methods**
**Spring 2023**
**Week Five**

*Answer the free-response question starting on a fresh sheet of paper. Use as many sheets as possible of paper you need. Solve the problem as if you were submitting them for a test. Put your last name, first name, and the first letter of the last name in bold on the top of the page. Submit Q1 at the end of the class.*

1. Given the following set of equations in matrix form

$$\begin{bmatrix} 12 & 16 & 28 & 56 \\ 24 & 36 & 66 & 76 \\ 48 & 32 & 64 & 96 \\ 60 & 66 & 78 & 92 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 444 \\ 746 \\ 928 \\ 1066 \end{bmatrix}$$

a) At the *end of the first step* of forward elimination part in the Naïve Gauss elimination algorithm, the equations obtained in a matrix form on a given set of equations are as follows.

$$\begin{bmatrix} 12 & 16 & 28 & 56 \\ 0 & 4 & 10 & -36 \\ 0 & -32 & -48 & -128 \\ 0 & -14 & -62 & -188 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 444 \\ -142 \\ -848 \\ -1154 \end{bmatrix}$$

Conduct only the second step of forward elimination of the Naïve Gauss elimination method and show the result in matrix form. **Show your work for full credit and put your final answer in the box.**

b) At the end of the forward elimination part of Naïve Gauss Elimination, I obtain the following system of equations.

$$\begin{bmatrix} 12 & 16 & 28 & 56 \\ 0 & 4 & 10 & -36 \\ 0 & 0 & 32 & -416 \\ 0 & 0 & 0 & -665 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 444 \\ -142 \\ -1984 \\ -3325 \end{bmatrix}$$

Now using a computer that uses only **three** significant digits with **chopping**, what is the value of $x_3$ using back substitution **Show your work for full credit and put your final answer in the box.**

c) What is the determinant of the original coefficient matrix

$$\begin{bmatrix} 12 & 16 & 28 & 56 \\ 24 & 36 & 66 & 76 \\ 48 & 32 & 64 & 96 \\ 60 & 66 & 78 & 92 \end{bmatrix}$$

You can use previous parts of the problem to answer the question. **Show your work for full credit and put your final answer in the box.**

d) Does the inverse of the coefficient matrix exist? Yes or No. How did you come to this conclusion?

Answers

a) $$\begin{bmatrix} 12 & 16 & 28 & 56 \\ 0 & 4 & 10 & -36 \\ 0 & 0 & 32 & -416 \\ 0 & 0 & -27 & -314 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 444 \\ -142 \\ -1984 \\ -1651 \end{bmatrix}$$

b) 2.81 (The answer is not 3 as you are only allowed to carry 3 significant digits with chopping)

c) -1021440

d) Answer not given intentionally

2. An AMD chip uses 4 clock cycles to conduct one multiplication, 4 clock cycles to conduct one addition, 4 clock cycles to conduct one subtraction and 16 clock cycles to conduct one division. The computer uses a 2.5 GHz processor. The computational times (*CT*) are given for various parts of Naïve Gauss elimination and LU decomposition methods of solving simultaneous linear equations as below.

$T$ = Clock cycle time

$n$= Number of rows or columns of a square matrix

$CT|_{DE} = T\left(\frac{8n^3}{3} + 4n^2 - \frac{20n}{3}\right),$

$CT|_{FS} = T(4n^2 - 4n),$

$CT|_{BS} = T(4n^2 + 12n)$

$CT|_{FE} = T\left(\frac{8n^3}{3} + 8n^2 - \frac{32n}{3}\right)$

The computational time to find the inverse of a matrix by LU decomposition method as derived in class is given by

$$CT|_{inverse\ by\ LU\ decomposition\ method} = T\left(\frac{32n^3}{3} + 12n^2 - \frac{20n}{3}\right)$$

Answer the following questions. Please do not look at the final answer before attempting a question.

a) What is the clock cycle time? Give units.

b) How much time in seconds does it take to conduct one division?

c) How much time would it take to find the value of $2.03 \times 3.06 + 5.07 - 3.03/4.05 + 1.03/2.915$

d) Find the formula for the time it takes to multiply two matrices $[A]_{n\times n}[C]_{n\times 1}$ .

e) Estimate the computational time to find the solution of one set of 85 simultaneous linear equations using Naive Gaussian elimination.

f) Estimate the computational time to find the solution of one set of 85 simultaneous linear equations using LU decomposition method.

g) Estimate the computational time to find the solution of 1000 sets of 85 simultaneous linear equations using Naive Gaussian elimination *where the coefficient matrix is the same for all sets of equations*.

h) Estimate the computational time to find the solution of 1000 sets of 85 simultaneous linear equations using LU decomposition method *where the coefficient matrix is the same for all sets of equations*.

i) To find the solution of 1000 sets of 85 simultaneous linear equations *where the coefficient matrix stays the same*, a student claims that finding first the inverse of the coefficient matrix by LU decomposition and then using matrix multiplication for each set of equations would be more efficient than what one is doing in parts (g) and (h). Show if this claim is true.

**Answers**

(a) $T = 4 \times 10^{-10}$ seconds
(b) $6.4 \times 10^{-9}$ seconds
(c) $1.92 \times 10^{-8}$ seconds
(d) $T(8n^2 - 4n)$
(e) $6.89792 \times 10^{-4}$ seconds
(f) $6.89792 \times 10^{-4}$ seconds
(g) $0.689792$ seconds
(h) $0.0240584$ seconds
(i) Answer not given intentionally