Chapter 4. Simultaneous Linear Equations and Matrix Algebra

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If $[A]_{n\times n}$ is the inverse of $[B]_{n\times n}$, then the following statements are true (Check all that apply)

- [B] is non-singular
- [B][A]=[I]
- [B] is inverse of [A}
- [A] is singular
- [A][B]=[I]

Total Results: 0

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The following system of equations

$$x + y = 2$$

$$6x + 6y = 12$$

has ___________ solution(s)

- no
- one
- more than one but a finite number of
- infinite

Total Results: 0

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If the determinant of a square matrix [A] is zero, then the following are (is) true (check all that apply)

- [A] does not have an inverse
- [A] has an inverse
- [A] is singular
- if [A][X]=[C] is a set of simultaneous linear equations, then [X] is unique
- if [A][X]=[C] is a set of simultaneous linear equations, then [X] is not unique

Total Results: 0

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The goal of forward elimination steps in the Naive Gauss elimination method is to reduce the coefficient matrix to a (an) ________ matrix.

- Upper triangular
- Diagonal
- Lower triangular
- Identity

Total Results: 0

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LU Decomposition Method

Major: All Engineering Majors

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What makes equation sets (1) and (2) easier to solve than set (3)?

$$\begin{bmatrix} 25 & 5 & 1 \\ 0 & -4.8 & -1.56 \\ 0 & 0 & 0.7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 106.8 \\ -96.21 \\ 0.735 \end{bmatrix} \quad \ldots(1)$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2.56 & 1 & 0 \\ 5.76 & 3.5 & 1 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} 106.8 \\ 177.2 \\ 279.2 \end{bmatrix} \quad \ldots(2)$$

$$\begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 106.8 \\ 177.2 \\ 279.2 \end{bmatrix} \quad \ldots(3)$$

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LU Decomposition Method

[A][X] = [C]

1.Decompose [A] into [L] and [U]

2.Solve [L][Z] = [C] for [Z] by using forward substitution

3.Solve [U][X] = [Z] for [X] by using back substitution

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Using LU Decomposition to solve SLEs

Solve the following set of linear equations using LU Decomposition Method.

$$\begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 106.8 \\ 177.2 \\ 279.2 \end{bmatrix}$$

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1. Decomposing [A] to [L][U] form

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L U Form

$$[A] = [L][U]$$

$$\begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \ell_{21} & 1 & 0 \\ \ell_{31} & \ell_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

[*U*] is the same as the coefficient matrix at the end of the forward elimination part of Naïve Gauss Elimination method.

[*L*] is obtained using the *multipliers* that were used in the forward elimination part of Naïve Gauss Elimination method to make the corresponding elements to be zero.

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Finding the [*U*] matrix

Using the Forward Elimination Procedure of Gauss Elimination

$$\begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix}$$

Step 1: $\frac{64}{25} = 2.56; Row2 - Row1(2.56) = \begin{bmatrix} 25 & 5 & 1 \\ 0 & -4.8 & -1.56 \\ 144 & 12 & 1 \end{bmatrix}$

$$\frac{144}{25} = 5.76; Row3 - Row1(5.76) = \begin{bmatrix} 25 & 5 & 1 \\ 0 & -4.8 & -1.56 \\ 0 & -16.8 & -4.76 \end{bmatrix}$$

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Finding the [U] Matrix

Matrix after Step 1: $\begin{bmatrix} 25 & 5 & 1 \\ 0 & -4.8 & -1.56 \\ 0 & -16.8 & -4.76 \end{bmatrix}$

Step 2: $\frac{-16.8}{-4.8} = 3.5; Row3 - Row2(3.5) = \begin{bmatrix} 25 & 5 & 1 \\ 0 & -4.8 & -1.56 \\ 0 & 0 & 0.7 \end{bmatrix}$

$$[U] = \begin{bmatrix} 25 & 5 & 1 \\ 0 & -4.8 & -1.56 \\ 0 & 0 & 0.7 \end{bmatrix}$$

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Finding the [L] matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ \ell_{21} & 1 & 0 \\ \ell_{31} & \ell_{32} & 1 \end{bmatrix}$$

Using the multipliers used during the forward elimination part

From the first step of forward elimination

$$\begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix}$$

$$\ell_{21} = \frac{a_{21}}{a_{11}} = \frac{64}{25} = 2.56$$

$$\ell_{31} = \frac{a_{31}}{a_{11}} = \frac{144}{25} = 5.76$$

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Finding the [L] Matrix

From the second step of forward elimination

$$\begin{bmatrix} 25 & 5 & 1 \\ 0 & -4.8 & -1.56 \\ 0 & -16.8 & -4.76 \end{bmatrix}$$

$$\ell_{32} = \frac{a_{32}}{a_{22}} = \frac{-16.8}{-4.8} = 3.5$$

$$[L] = \begin{bmatrix} 1 & 0 & 0 \\ 2.56 & 1 & 0 \\ 5.76 & 3.5 & 1 \end{bmatrix}$$

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Does [L][U] = [A]?

$$[L][U] = \begin{bmatrix} 1 & 0 & 0 \\ 2.56 & 1 & 0 \\ 5.76 & 3.5 & 1 \end{bmatrix} \begin{bmatrix} 25 & 5 & 1 \\ 0 & -4.8 & -1.56 \\ 0 & 0 & 0.7 \end{bmatrix} = \begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix}$$

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2. Solving [L][Z]=[C]

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Solving [L][Z]=[C]

$$\begin{bmatrix} 1 & 0 & 0 \\ 2.56 & 1 & 0 \\ 5.76 & 3.5 & 1 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} 106.8 \\ 177.2 \\ 279.2 \end{bmatrix}$$

$$z_1 = 106.8$$
$$2.56z_1 + z_2 = 177.2$$
$$5.76z_1 + 3.5z_2 + z_3 = 279.2$$

$$z_1 = 106.8$$

$$z_2 = 177.2 - 2.56z_1 = 177.2 - 2.56(106.8) = -96.2$$

$$z_3 = 279.2 - 5.76z_1 - 3.5z_2 = 279.2 - 5.76(106.8) - 3.5(-96.21) = 0.735$$

$$[Z] = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} 106.8 \\ -96.21 \\ 0.735 \end{bmatrix}$$

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3. Solving [U][X]=[Z]

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Solving [U][X]=[Z]

$$\begin{bmatrix}25 & 5 & 1\\0 & -4.8 & -1.56\\0 & 0 & 0.7\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}106.8\\-96.21\\0.735\end{bmatrix}$$

$$\begin{aligned}25x_1+5x_2+x_3&=106.8\\-4.8x_2-1.56x_3&=-96.21\\0.7x_3&=0.735\end{aligned}$$

$$\begin{aligned}0.7x_3&=0.735\\x_3&=\frac{0.735}{0.7}\\&=1.050\end{aligned}$$

$$\begin{aligned}&-4.8x_2-1.56x_3=-96.21\\x_2&=\frac{-96.21+1.56x_3}{-4.8}\\&=\frac{-96.21+1.56(1.050)}{-4.8}\\&=19.70\end{aligned}$$

$$\begin{aligned}&25x_1+5x_2+x_3=106.8\\x_1&=\frac{106.8-5x_2-x_3}{25}\\&=\frac{106.8-5(19.70)-1.050}{25}\\&=0.2900\end{aligned}$$

$$\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}0.2900\\19.70\\1.050\end{bmatrix}$$

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Reviewing the LU Decomposition example

$$\begin{bmatrix}25 & 5 & 1\\64 & 8 & 1\\144 & 12 & 1\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}106.8\\177.2\\279.2\end{bmatrix}$$

$$\begin{bmatrix}25 & 5 & 1\\64 & 8 & 1\\144 & 12 & 1\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\2.56 & 1 & 0\\5.76 & 3.5 & 1\end{bmatrix}\begin{bmatrix}25 & 5 & 1\\0 & -4.8 & -1.56\\0 & 0 & 0.7\end{bmatrix}$$

$$\begin{bmatrix}1 & 0 & 0\\2.56 & 1 & 0\\5.76 & 3.5 & 1\end{bmatrix}\begin{bmatrix}z_1\\z_2\\z_3\end{bmatrix}=\begin{bmatrix}106.8\\177.2\\279.2\end{bmatrix}\text{ gives }\begin{bmatrix}z_1\\z_2\\z_3\end{bmatrix}=\begin{bmatrix}106.8\\-96.208\\0.76\end{bmatrix}$$

$$\begin{bmatrix}25 & 5 & 1\\0 & -4.8 & -1.56\\0 & 0 & 0.7\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}106.8\\-96.208\\0.76\end{bmatrix}\text{ gives }\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}0.290472\\19.6905\\1.08571\end{bmatrix}$$

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THE END

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LU decomposition method for solving a set of equations of form uses the following steps

- $[L][X]=[Z]$ followed by $[U][Z]=[C]$
- $[L][Z]=[C]$ followed by $[U][X]=[Z]$
- $[U][Z]=[C]$ followed by $[L][X]=[Z]$
- $[U][X]=[Z]$ followed by $[L][Z]=[C]$

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To solve a set of equations $[A][X]=[C]$ by LU decomposition, the first set of equations to be solved are $[L][Z]=[C]$. The vector $[Z]$ is ________. (Check all that apply)

- Same as $[L]^{-1}[C]$
- Same as the solution vector $[X]$
- Same as the right-hand side vector one gets at the end of the forward elimination steps of Naive Gauss Elimination for $[A][X]=[C]$
- Same as the vector $[C]$

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Given the decomposition of a square matrix $[A]=[L][U]$, where $[L]$ has ones in the diagonal, the determinant of $[A]$ is

- Product of diagonal elements of $[A]$
- Product of diagonal elements of $[U]$
- Sum of the diagonal elements of $[A]$
- Sum of the diagonal elements of $[U]$

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THE END

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Comparing Naïve Gauss Elimination Method with LU Decomposition

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Naïve Gaussian Elimination

[A][X] = [C]

1.Conduct forward elimination to get

[U][X] = [Z]

2. Conduct back substitution to solve

[U][X] = [Z] for [X]

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LU Decomposition Method

[A][X] = [C]

1.Decompose [A] into [L] and [U], [A]=[L][U]

2.Solve [L][Z] = [C] for [Z] by using forward substitution

3.Solve [U][X] = [Z] for [X] by using back substitution

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Is LU Decomposition Method better than Gaussian Elimination Method?

Solve $[A][X] = [B]$

T = clock cycle time (s)
$n \times n$ = size of the matrix
CT= computational time (s)

Naive Gauss Elimination Method	LU Decomposition Method
Forward Elimination $CT\vert_{FE} = T\left(\frac{8n^3}{3} + 8n^2 - \frac{32n}{3}\right)$	**Decomposition to LU** $CT\vert_{DE} = T\left(\frac{8n^3}{3} + 4n^2 - \frac{20n}{3}\right)$
Back Substitution $CT\vert_{BS} = T(4n^2 + 12n)$	**Forward Substitution** $CT\vert_{FS} = T(4n^2 - 4n)$
	Back Substitution $CT\vert_{BS} = T(4n^2 + 12n)$

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Is LU Decomposition better than Gaussian Elimination?

To solve $[A][X] = [B]$

Time taken by methods

Gaussian Elimination	LU Decomposition
$T\left(\frac{8n^3}{3} + 12n^2 + \frac{4n}{3}\right)$	$T\left(\frac{8n^3}{3} + 12n^2 + \frac{4n}{3}\right)$

T = clock cycle time and $n \times n$ = size of the matrix

So both methods are equally efficient.

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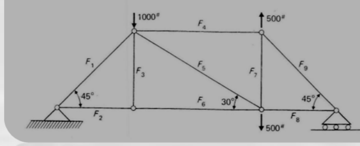
THE END

Truss Problem

$$\begin{bmatrix} 0.7071 & 0 & 0 & -1 & -0.8660 & 0 & 0 & 0 & 0 \\ 0.7071 & 0 & 1 & 0 & 0.5 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0.7071 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -0.7071 \\ 0 & 0 & 0 & 0 & 0.8660 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -0.5 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0.7071 \end{bmatrix} F = \begin{bmatrix} 0 \\ -1000 \\ 0 \\ 0 \\ 500 \\ 0 \\ 0 \\ -500 \\ 0 \end{bmatrix}$$



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Finding the inverse of a square matrix

[B] is inverse of a square matrix [*A*] if

$$[A][B] = [I] \quad \textbf{OR} \quad [B][A] = [I]$$

Two ways to show what inverse means?

$$\begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Finding the inverse of a square matrix

How can LU Decomposition be used to find the inverse?

$$[A][B] = [I]$$

First Column of [B]

$$[A]\begin{bmatrix} b_{11} \\ b_{21} \\ \vdots \\ b_{n1} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

Second Column of [B] ....

$$[A]\begin{bmatrix} b_{12} \\ b_{22} \\ \vdots \\ b_{n2} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ \vdots \\ 0 \end{bmatrix}$$

Last Column of [B]

$$[A]\begin{bmatrix} b_{1n} \\ b_{2n} \\ \vdots \\ b_{nn} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}$$

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Example: Inverse of a Matrix

Find the inverse of a square matrix [*A*]

$$[A] = \begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix}$$

Using the decomposition procedure, the [*L*] and [*U*] matrices are found to be

$$[A] = [L][U] = \begin{bmatrix} 1 & 0 & 0 \\ 2.56 & 1 & 0 \\ 5.76 & 3.5 & 1 \end{bmatrix}\begin{bmatrix} 25 & 5 & 1 \\ 0 & -4.8 & -1.56 \\ 0 & 0 & 0.7 \end{bmatrix}$$

Solving for each column of [*B*] requires two steps

1) Solve [*L*] [*Z*] = [*C*] for [*Z*]
2) Solve [*U*] [*X*] = [*Z*] for [*X*]

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Finding 1st column of inverse: LZ=C

$$\begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix}\begin{bmatrix} b_{11} \\ b_{21} \\ b_{31} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\textit{Step 1: } [L][Z] = [C] \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 2.56 & 1 & 0 \\ 5.76 & 3.5 & 1 \end{bmatrix}\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{aligned} z_1 &= 1 \\ 2.56z_1 + z_2 &= 0 \\ 5.76z_1 + 3.5z_2 + z_3 &= 0 \end{aligned}$$

$$\begin{aligned} z_1 &= 1 \\ z_2 &= 0 - 2.56z_1 \\ &= 0 - 2.56(1) \\ &= -2.56 \\ z_3 &= 0 - 5.76z_1 - 3.5z_2 \\ &= 0 - 5.76(1) - 3.5(-2.56) \\ &= 3.2 \end{aligned}$$

$$[Z] = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -2.56 \\ 3.2 \end{bmatrix}$$

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Finding 1st column of inverse: UX=Z

$$\textit{Step 2: } [U][X] = [Z] \rightarrow \begin{bmatrix} 25 & 5 & 1 \\ 0 & -4.8 & -1.56 \\ 0 & 0 & 0.7 \end{bmatrix}\begin{bmatrix} b_{11} \\ b_{21} \\ b_{31} \end{bmatrix} = \begin{bmatrix} 1 \\ -2.56 \\ 3.2 \end{bmatrix}$$

$$\begin{aligned} 25b_{11} + 5b_{21} + b_{31} &= 1 \\ -4.8b_{21} - 1.56b_{31} &= -2.56 \\ 0.7b_{31} &= 3.2 \end{aligned}$$

$$\begin{aligned} b_{31} &= \frac{3.2}{0.7} \\ &= 4.571 \\ b_{21} &= \frac{-2.56 + 1.560b_{31}}{-4.8} \\ &= \frac{-2.56 + 1.560(4.571)}{-4.8} \\ &= -0.9524 \\ b_{11} &= \frac{1 - 5b_{21} - b_{31}}{25} \\ &= \frac{1 - 5(-0.9524) - 4.571}{25} \\ &= 0.04762 \end{aligned}$$

$$\begin{bmatrix} b_{11} \\ b_{21} \\ b_{31} \end{bmatrix} = \begin{bmatrix} 0.04762 \\ -0.9524 \\ 4.571 \end{bmatrix}$$

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Inverse of a Matrix

$$\begin{bmatrix}25 & 5 & 1\\64 & 8 & 1\\144 & 12 & 1\end{bmatrix}\begin{bmatrix}b_{11} & b_{12} & b_{13}\\b_{21} & b_{22} & b_{23}\\b_{31} & b_{32} & b_{33}\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

First Column	Second Column	Third Column
$\begin{bmatrix}25 & 5 & 1\\64 & 8 & 1\\144 & 12 & 1\end{bmatrix}\begin{bmatrix}b_{11}\\b_{21}\\b_{31}\end{bmatrix}=\begin{bmatrix}1\\0\\0\end{bmatrix}$	$\begin{bmatrix}25 & 5 & 1\\64 & 8 & 1\\144 & 12 & 1\end{bmatrix}\begin{bmatrix}b_{12}\\b_{22}\\b_{32}\end{bmatrix}=\begin{bmatrix}0\\1\\0\end{bmatrix}$	$\begin{bmatrix}25 & 5 & 1\\64 & 8 & 1\\144 & 12 & 1\end{bmatrix}\begin{bmatrix}b_{13}\\b_{23}\\b_{33}\end{bmatrix}=\begin{bmatrix}0\\0\\1\end{bmatrix}$
$\begin{bmatrix}b_{11}\\b_{21}\\b_{31}\end{bmatrix}=\begin{bmatrix}0.04762\\-0.9524\\4.571\end{bmatrix}$	$\begin{bmatrix}b_{12}\\b_{22}\\b_{32}\end{bmatrix}=\begin{bmatrix}-0.08333\\1.417\\-5.000\end{bmatrix}$	$\begin{bmatrix}b_{13}\\b_{23}\\b_{33}\end{bmatrix}=\begin{bmatrix}0.03571\\-0.4643\\1.429\end{bmatrix}$

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Example: Inverse of a Matrix

$$\begin{bmatrix}b_{11}\\b_{21}\\b_{31}\end{bmatrix}=\begin{bmatrix}0.04762\\-0.9524\\4.571\end{bmatrix} \quad \begin{bmatrix}b_{12}\\b_{22}\\b_{32}\end{bmatrix}=\begin{bmatrix}-0.08333\\1.417\\-5.000\end{bmatrix} \quad \begin{bmatrix}b_{13}\\b_{23}\\b_{33}\end{bmatrix}=\begin{bmatrix}0.03571\\-0.4643\\1.429\end{bmatrix}$$

$$[A]^{-1}=\begin{bmatrix}0.04762 & -0.08333 & 0.03571\\-0.9524 & 1.417 & -0.4643\\4.571 & -5.000 & 1.429\end{bmatrix}$$

To check your work, check the following

$$[A][A]^{-1} = [I] = [A]^{-1}[A]$$

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To find inverse of [A]

<u>Time taken by Gaussian Elimination</u>

$$= n(CT|_{FE} + CT|_{BS})$$

$$= n\times T\left(\frac{8n^3}{3}+8n^2-\frac{32n}{3}\right) + n\times T(4n^2+12n)$$

$$= T\left(\frac{8n^4}{3}+12n^3+\frac{4n^2}{3}\right)$$

<u>Time taken by LU Decomposition</u>

$$= CT|_{DE} + n\times CT|_{FS} + n\times CT|_{BS}$$

$$= 1\times T\left(\frac{8n^3}{3}+4n^2-\frac{20n}{3}\right) + n\times T(4n^2-4n) + n\times T(4n^2+12n)$$

$$= T\left(\frac{32n^3}{3}+12n^2-\frac{20n}{3}\right)$$

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To find inverse of [A]

<u>Time taken by Gaussian Elimination</u>

$$T\left(\frac{8n^4}{3}+12n^3+\frac{4n^2}{3}\right)$$

<u>Time taken by LU Decomposition</u>

$$T\left(\frac{32n^3}{3}+12n^2-\frac{20n}{3}\right)$$

Table 1 Comparing computational times of finding inverse of a matrix using LU decomposition and Naive Gauss elimination.

n	10	100	1000	10000
$\frac{CT_{\text{inverse GE}}}{CT_{\text{inverse LU}}}$	3.288	25.84	250.8	2501

For large n, $\frac{CT_{\text{inverse GE}}}{CT_{\text{inverse LU}}} \approx \frac{n}{4}$

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Time Taken by Back Substitution

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ 0 & a'_{22} & a'_{23} & \cdots & a'_{2n} \\ 0 & 0 & a''_{33} & \cdots & a''_{3n} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & 0 & 0 & a_{nn}^{(n-1)} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b'_2 \\ b''_3 \\ \vdots \\ b_n^{(n-1)} \end{bmatrix}$$

$$x_n = \frac{b_n^{(n-1)}}{a_{nn}^{(n-1)}}$$

$$x_i = \frac{b_i^{(i-1)} - \sum_{j=i+1}^{n} a_{ij}^{(i-1)} x_j}{a_{ii}^{(i-1)}} \text{ for } i = n-1, \ldots, 1$$

```
n = length(B);
X(n)=B(n)/A(n,n)
for i=n-1:-1:1
X(i) = B(i);
    for j=(i+1):1:n
        X(i) = X(i) - A(i, j)*X(j);
    end
X(i) = X(i)/A(i, i);
end
```

Back Substitution CT

$CT|_{BS} = T(4n^2 + 12n)$

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How many times does the variable 'a' get assigned the value of 2?

```
for j=4:1:11
    a=2
end
```

- 4
- 7
- 8
- 11

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How many times does the variable 'a' get assigned the value of 2?

```
for j=i+1:1:n
    a=2
end
```

- n-i
- n-i-1
- n
- i+1

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How many times does the variable 'a' get assigned the value of 2?

```
for i=3: -1: 1
    for j=i+1: 1: 4
        a=2
    end
end
```

- 6
- 7
- 8
- 12

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What is the sum of 5+6+7+...+122?

Hint: $S = \frac{n}{2}(a_1 + a_n)$

" 7493 "

" 7493.00000 "

" 7493! "

" 7493 "

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Time Taken by Back Substitution

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ 0 & a'_{22} & a'_{23} & \cdots & a'_{2n} \\ 0 & 0 & a''_{33} & \cdots & a''_{3n} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & 0 & 0 & a_{nn}^{(n-1)} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b'_2 \\ b''_3 \\ \vdots \\ b_n^{(n-1)} \end{bmatrix}$$

$$x_n = \frac{b_n^{(n-1)}}{a_{nn}^{(n-1)}}$$

$$x_i = \frac{b_i^{(i-1)} - \sum_{j=i+1}^{n} a_{ij}^{(i-1)} x_j}{a_{ii}^{(i-1)}} \text{ for } i = n-1, \ldots, 1$$

```
n = length(B);
X(n)=B(n)/A(n,n)
for i=n-1:-1:1
X(i) = B(i);
   for j=(i+1):1:n
      X(i) = X(i) - A(i, j)*X(j);
   end
X(i) = X(i)/A(i, i);
end
```

Back Substitution CT

$CT|_{BS} = T(4n^2 + 12n)$

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```
n = length(B);
X(n)=B(n)/A(n,n)
for i=n-1:-1:1
X(i) = B(i);
   for j=(i+1):1:n
      X(i) = X(i) - A(i, j)*X(j);
   end
X(i) = X(i)/A(i, i);
end
```

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THE END

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Increasing the precision of numbers from single to double in the Naive Gaussian elimination method

- avoids division by zero
- decreases round-off error
- allows equations with a noninvertible coefficient matrix to be solved

53

Someone is asking you to use LU decomposition method to solve m sets of (n equations, n unknowns) of the form [A][X]=[C]. If the coefficient matrix [A] stays the same in all sets, how many times does one need to decompose the matrix [A]?

- 1
- m
- n
- mn

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There are two parts in Naive Gaussian Elimination - forward elimination and back substitution. If we are solving n equations for n unknowns, how many steps are in the forward elimination part?

- n
- n-1
- n+1

55

Naive Gaussian elimination has these inherent numerical methods errors (Check all that apply)

- Round off errors
- Truncation errors

56

LU decomposition method is computationally more efficient than Naive Gauss elimination for (Check all that apply)

57

Using 3 significant digits with *chopping* at all stages, the result for the following calculation is

$$x_1 = \frac{6.095 - 3.456 \times 1.99}{8}$$

-0.0988
-0.0978
-0.0969
-0.0962

58

Division by zero during the forward elimination part in *Naïve Gaussian* elimination for [A][X]=[C] implies the coefficient matrix [A] _______________

is invertible

is not invertible

cannot be determined to be invertible or not

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Every A matrix that has inverse can be decomposed to LU or PLU

$$\begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2.56 & 1 & 0 \\ 5.76 & 3.5 & 1 \end{bmatrix} \begin{bmatrix} 25 & 5 & 1 \\ 0 & -4.8 & -1.56 \\ 0 & 0 & 0.7 \end{bmatrix}$$

$$A = PLU$$

$$\begin{bmatrix} 0 & 1 & 0 \\ -8 & 8 & 1 \\ 2 & -2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -4 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & -2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

P is a so-called permutation matrix

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THE END

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