## **EML3041** Computational Methods

## Spring 2023

## Week 8

Answer each question on a fresh sheet of paper. *Solve the problem as if you were submitting them for a test.* Identify each part separately. You will be told at the end of the class which problems to submit.

1. The following incomplete y vs. x data is given

x	1	2	4	6	7
У	5	11	??	??	32

The data is fit by a quadratic spline interpolant given by the following piecewise quadratics

$$f(x) = \begin{cases} ax - 1, 1 \le x \le 2\\ -2x^2 + 14x - 9, 2 \le x \le 4\\ bx^2 + cx + d, 4 \le x \le 6\\ 25x^2 - 303x + 928, 6 \le x \le 7 \end{cases}$$

where *a*, *b*, *c*, and *d*, are constants.

- a) Estimate f(2).
- b) Estimate f(4).
- c) Estimate f'(3).
- d) Estimate f'(4).
- e) Estimate  $\int_{1.6}^{3.7} f(x) dx$

f) Estimate *b*. You can use internet (google search for "wolfram widget 3 equation solver" or "wolfram widget 2 equation solver" depending on how many equations you need to solve) to solve sets of equations.

Answer

- a) 11
- b) 15
- c) 2
- d) -2
- e) 28.0147
- f) -0.25

2) A robot follows a path generated by a quadratic interpolant through three consecutive data points (2, 4), (3, 9) and (4, 16) from x = 2 to x = 4. The quadratic interpolant is given as a courtesy to you as  $y = x^2$ . The length of a curve is given by  $L = \int_a^b \sqrt{1 + (dy/dx)^2} dx$ 

a) Find the approximate length of the interpolant path from x = 2 to x = 4. You can approximate a general definite integral by an approximation given below.

$$\int_{a}^{b} f(x)dx \cong \frac{(b-a)}{6} \left[ f(a) + 4f\left(\frac{a+b}{2}\right) + f(b) \right]$$

b) Find the approximate length of the interpolant path from x = 2 to x = 4. You can use three straight lines to approximate the curve, each of whose x-coordinate is equally spaced.

Answer:

a) 12.17

b) 12.17

3) The following (x, y) data is given:

x	15	18	22
у	24	35	25

A first-order polynomial is chosen as an interpolant for the first two data points as

y = a + b(x - 15),  $15 \le x \le 18$ , Find the value of b.

## Answer: 3.6667

4) The following data of the velocity of a body as a function of time is given as follows.

Time (s)	0	15	18	22	24
Velocity(m/s)	22	24	37	25	123

Using quadratic interpolation, the interpolant

 $v(t) = 9.50000t^2 - 383t + 3853$ 

approximates the velocity of the body between 18 and 24 seconds. From this information, can you find out when the velocity of the body is 26 m/s during the above time interval? If

so, find the times when the velocity of the body is 26 m/s during the above time interval.

Answer: t=22.03 s,18.28 s