Integration

$$I = 4\int_0^1 \sqrt{1-x^2}\,dx$$

As difficult a problem as thou
finding quadrature of a circle

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1

One Point Rule

A scientist would derive one-point Gauss Quadrature rule based on getting exact results for integration for the function $g(x) = a_0 + a_1 x$. The one-point rule approximation for the general integral $\int_a^b f(x)dx$ is _________

$\frac{b-a}{2}[f(a)+f(b)]$

$(b-a)f\left(\frac{a+b}{2}\right)$

$\frac{b-a}{2}\left[f\left(\frac{b-a}{2}\left\{-\frac{1}{\sqrt{3}}\right\}+\frac{b+a}{2}\right)+f\left(\frac{b-a}{2}\left\{\frac{1}{\sqrt{3}}\right\}+\frac{b+a}{2}\right)\right]$

$(b-a)f(a)$

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Two-Point Gauss Quadrature Rule

$$\int_a^b f(x)dx \approx c_1 f(x_1) + c_2 f(x_2)$$

Let it be exact for $a_0 + a_1 x + a_2 x^2 + a_3 x^3$

$$\int_a^b f(x)dx \approx c_1 f(x_1) + c_2 f(x_2)$$

$$= \frac{b-a}{2} f\left(\frac{b-a}{2}\left(-\frac{1}{\sqrt{3}}\right) + \frac{b+a}{2}\right) + \frac{b-a}{2} f\left(\frac{b-a}{2}\left(\frac{1}{\sqrt{3}}\right) + \frac{b+a}{2}\right)$$

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One-Point Gauss Quadrature Rule

$$\int_a^b f(x)dx \approx c_1 f(x_1)$$

Let the formula be exact for $a_0 + a_1 x$

$$\int_a^b f(x)dx \approx c_1 f(x_1) = (b-a) f\left(\frac{b+a}{2}\right)$$

Could Gauss have derived the formula by letting it be exact for $a_0 + a_1 x + a_2 x^2$?

Could Gauss have derived the formula by letting it be exact for $a_1 x + a_2 x^2$?

5

Would $f(x) = a_0 + a_1 x + a_2 x^2$ work for one-point rule

$$\int_a^b f(x)dx \approx c_1 f(x_1)$$

Let the above formula be exact for $a_0 + a_1 x + a_2 x^2$

Exact: $\int_a^b (a_0 + a_1 x + a_2 x^2)dx = a_0(b-a) + a_1 \frac{b^2-a^2}{2} + a_2 \frac{b^3-a^3}{3}$

From formula: $c_1 f(x_1) = c_1(a_0 + a_1 x_1 + a_2 x_1^2) = a_0(c_1) + a_1(c_1 x_1) + a_2(c_1 x_1^2)$

Equating the coefficients of a_0, a_1, a_2

$$c_1 = b - a \qquad (Eqn\ 1)$$

$$c_1 x_1 = \frac{b^2-a^2}{2} \qquad (Eqn\ 2)$$

$$c_1 x_1^2 = \frac{b^3-a^3}{3} \qquad (Eqn\ 3)$$

6

A 2-point Gauss quad rule will give the exact definite integral value of the following integrands. Choose all that apply.

$6x^4$

$2x$

$2 + 3x + 3x^2 + 5x^3 + 6x^4$

$2 + 3x + 3x^2 + 5x^3$

$2 + 5x^2$

$5x^3$

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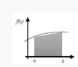
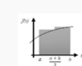
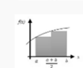
Which of these represents a single application of the trapezoidal rule of integration? Choose all that apply.







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In Gauss quadrature rule, the number of function evaluations for the 8-point rule is

8

9

17

9

For integrating any third order polynomial, the two-point Gauss quadrature rule will give you the same results as

1-segment trapezoidal rule

2-segment trapezoidal rule

3-segment trapezoidal rule

none of the above

10

In Gauss quadrature rule, the number of function evaluations for the 8-point rule is

8

9

17

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What is the highest order of polynomial that can be integrated exactly by a 5-point Gauss quadrature rule?

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A function is being integrated from a lower limit of 2 to an upper limit of 10. A person is willing to only give you the value of the function at only two points. Which two points would you choose? Separate the answers with a comma

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$$\int_{5}^{10} f(x)dx$$ **is exactly**

$$\int_{-1}^{1} f(2.5x+7.5)dx$$

$$2.5\int_{-1}^{1} f(2.5x+7.5)dx$$

$$5\int_{-1}^{1} f(5x+5)dx$$

$$5\int_{-1}^{1} (2.5x+7.5)f(x)dx$$

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