In-class Exercise

Background: A cantilever beam (Figure 1) with a rectangular cross-section was subjected to an end point load. Two strain gages were placed on the top of the beam as shown in Figure 2, and a dial gage was placed under the beam on precision tracks as shown in Figure 3.

Let us present the mathematical model for relating the deflection of the beam as a function of position for a fixed load and the normal strain on top of the beam as a function of the applied load.

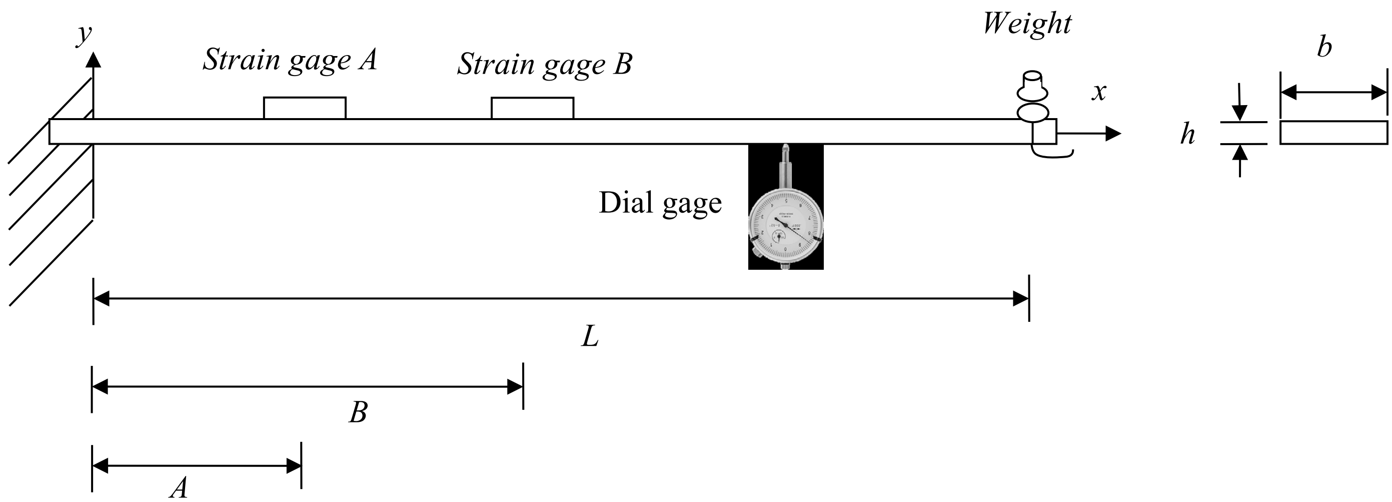

Figure 1: Bending a beam.

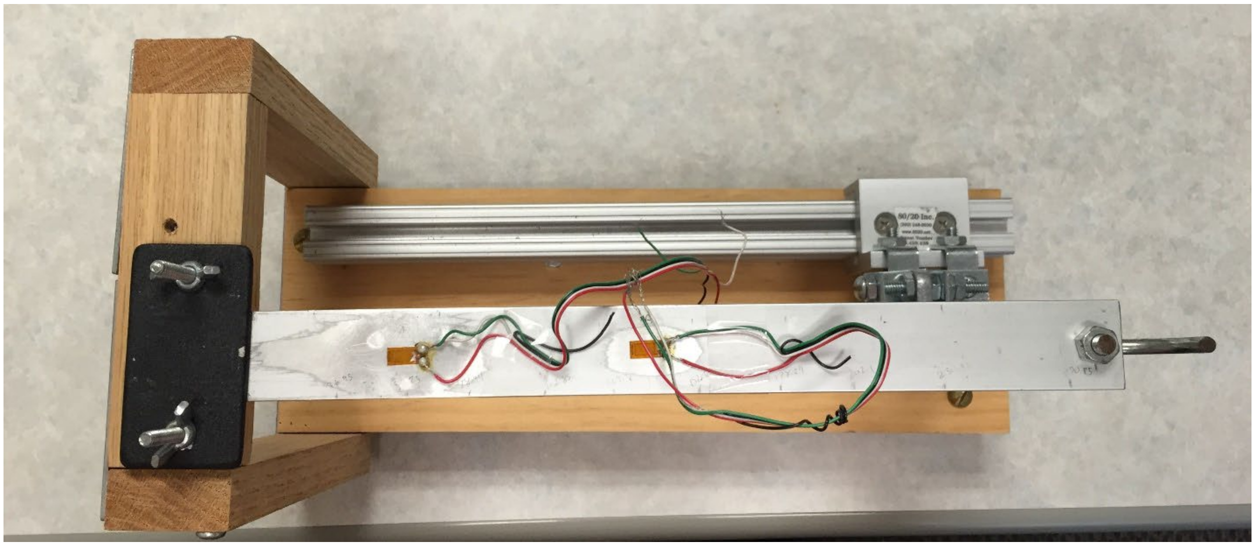

Figure 2: Strain Gages on Top of Beam

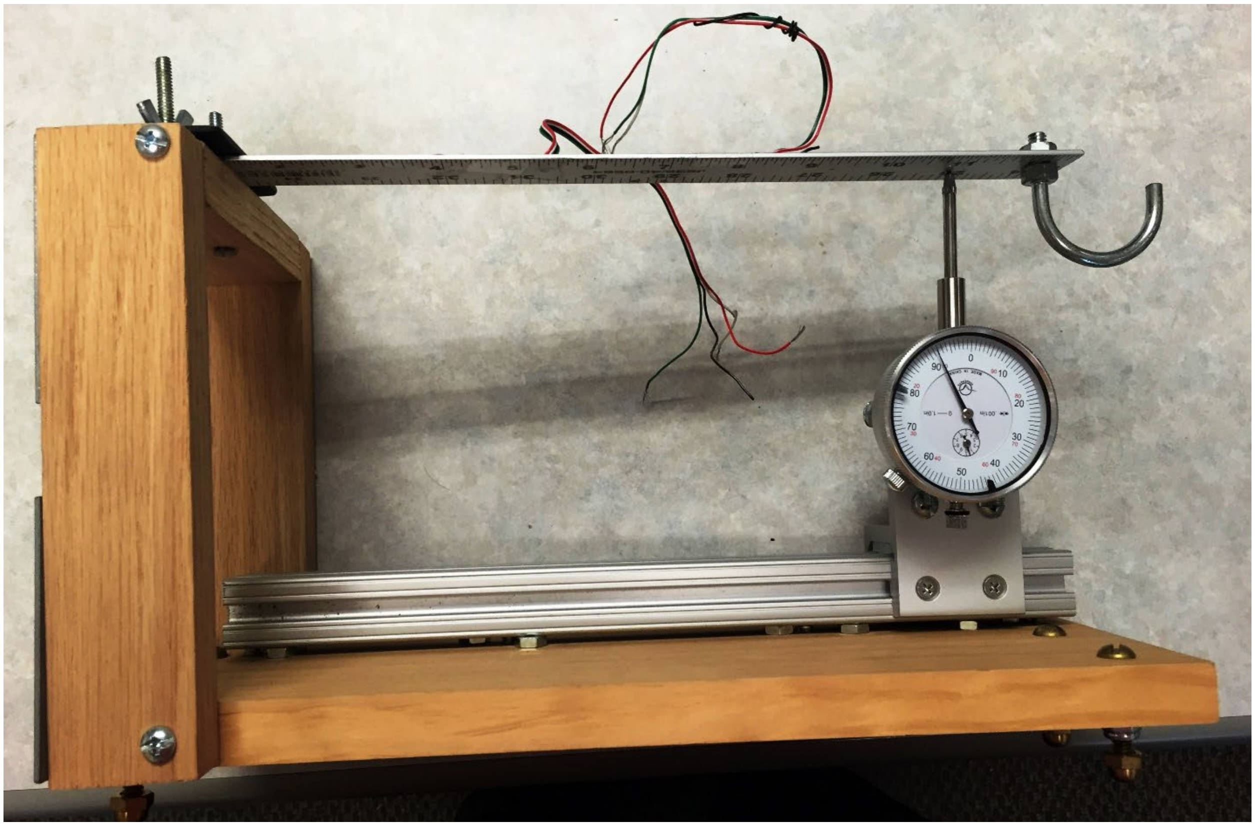

Figure 3: Dial Gage Under the Beam

The deflection of a beam is given by

$$\frac{d^2v}{dx^2} = \frac{M(x)}{EI} \tag{1}$$

where

v = deflection of beam, m

$M(x)$ = bending moment at location x, N-m

E = Young's Modulus, Pa

I = second moment of area, m^4

The bending moment at location, x due to an end load, P in a cantilever beam is given by

$$M(x) = P(L - x) \tag{2}$$

where

L = length of beam, m

P = force applied at end, N

The second moment of area of a rectangular cross section is given by

$$I = \frac{bh^3}{12} \tag{3}$$

where

b = width of beam, m

h = thickness of beam, m

The normal strain along the length of the beam is measured by the strain gages at location x and would be given by

$$\varepsilon = \frac{M(x)\frac{h}{2}}{EI}$$

Data Given

Length of beam, L = 278 mm
Height of beam, h = 1.92 mm
Width of beam, b = 28.5 mm
Distance from fixed end to Strain Gage A = 58 mm
Distance from fixed end to Strain Gage B = 135 mm
Young's modulus, E of beam material = 70 GPa.

Table 1: Deflection vs position data (**<u>note the units</u>**)

Position from end, *x* (mm)	Deflection, *v* (in)
0	0
37	0.0295
58	0.0770
79	0.1305
103	0.2045
125	0.2850
152	0.3805
178	0.4895
202	0.5630
231	0.6600
271	0.7690

Problem Statement

Use as many methods as you can think of to estimate the weight that is applied at the end? Use SI system throughout your calculations.

Hints:

1) Think about the numerical methods you learned in Numerical Differentiation (Chapter 2) to find second derivative of a continuous function given at discrete points with use of interpolation (Chapter 5).
2) Think about what you learned in Chapter 6 (Regression).
3) How did you calculate deflection from the general equation (1) in your mechanics of solids course?