SAMPLE COMPETENCY QUESTIONS Fall 2022

1. A scientist finds that regressing the y vs x data given below to the straight-line $y = a_0 + a_1 x$ results in a perfect fit.

x	1	3	11	20
y	3	7	23	?

The missing value for y at $x = 20$ most nearly is
(A) 1.000
(B) 2.000
(C) 40.00
(D) 41.00

2. The velocity, v of a rocket is given as a function of time, t by $v = a_0 + a_1 t + a_2 t^2, 1 < t < 3$ and is based on the following v vs. t values

t	1	2	3
v	2.5	4	10

The set of equations that would solve for constants a_0, a_1, a_2 would be

(A) $\begin{bmatrix} 0 & 1 & 1 \\ 0 & 2 & 4 \\ 0 & 3 & 9 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 2.5 \\ 4 \\ 10 \end{bmatrix}$

(B) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 2.5 \\ 4 \\ 10 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 2.5 \\ 4 \\ 10 \end{bmatrix}$

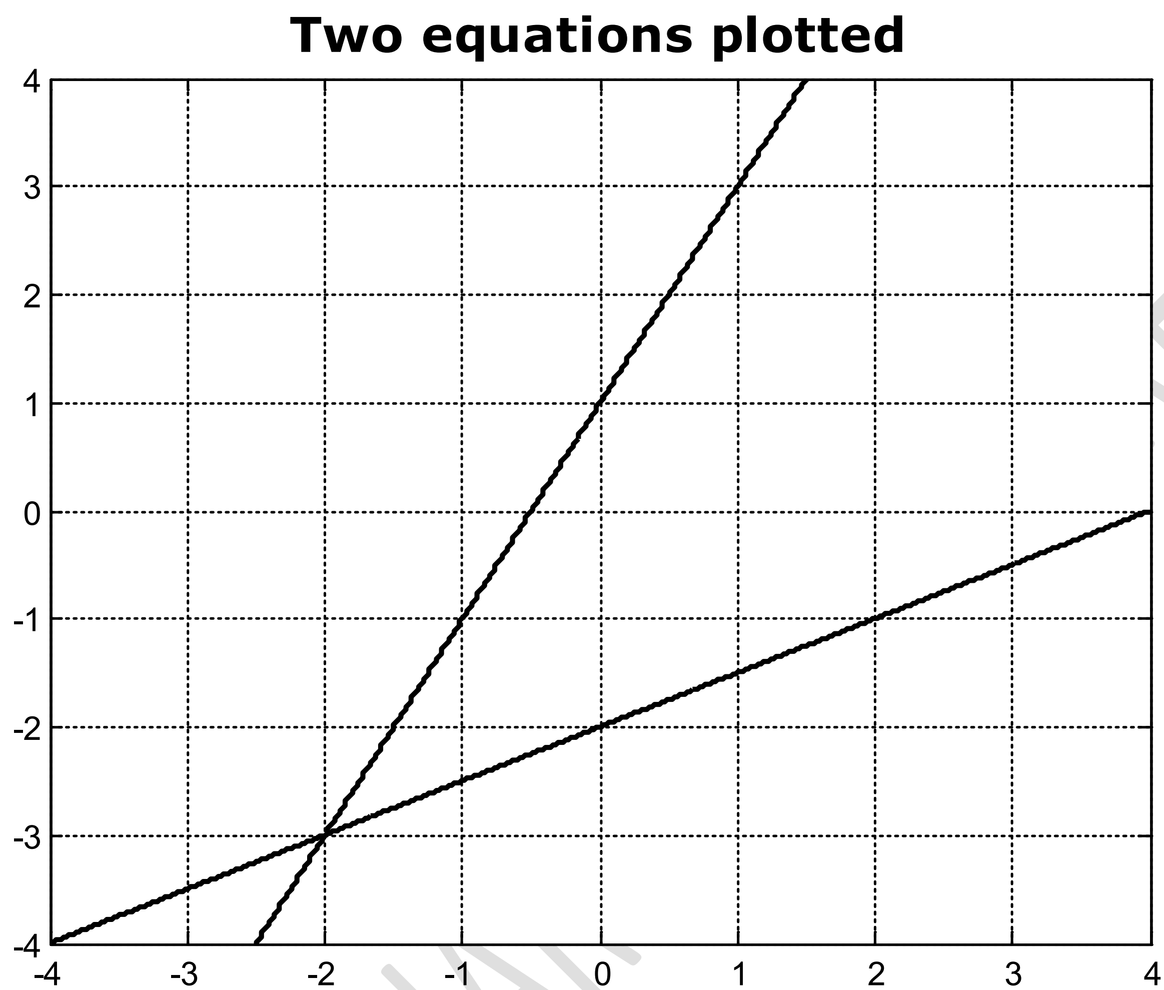
(D) $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 3 & 3 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 2.5 \\ 4 \\ 10 \end{bmatrix}$

3. To find the inverse of a square matrix, given is that

P = total computational time by using Naïve Gaussian elimination method
Q = total computational time by using Gaussian elimination method with partial pivoting
R = total computational time by using LU decomposition method

The amount of computational time required to find the inverse of a 1000×1000 matrix satisfies the following inequality

(A) P > Q > R
(B) R > P > Q
(C) Q > P > R
(D) R > Q > P



4. [A] is the coefficient matrix, [X] is the unknown vector, and [C] is the right-hand side vector. Which augmented matrix [A⋮C] of [A][X]=[C] matrix form represents the two equations plotted above?

(A) $\begin{bmatrix} 1 & 0 & 3 \\ -3 & -3 & 4 \end{bmatrix}$

(B) $\begin{bmatrix} 2 & 2 & -3 \\ 0 & 1 & 4 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 2 & -4 \\ 3 & -2 & -2 \end{bmatrix}$

(D) $\begin{bmatrix} -2 & 1 & 1 \\ 1 & -2 & 4 \end{bmatrix}$

5. The Newton-Raphson method of finding real roots of the nonlinear equation $f(x) = 0$can be derived by using the first two terms of the Taylor series. One would then expect that adding one more term of the Taylor series to the Newton-Raphson method would result in higher accuracy of the results for the same number of iterations as the Newton-Raphson method. However, this new approach of adding one more term of Taylor series is *not* suitable mostly because (*choose the most appropriate answer*)

(A) one has to solve a quadratic equation to get the new estimate of the root of the equation.
(B) the estimate of the roots may turn out to be complex numbers.
(C) one needs to find the first derivative of $f(x)$.
(D) one needs to find the second derivative of $f(x)$.