

Bring it together

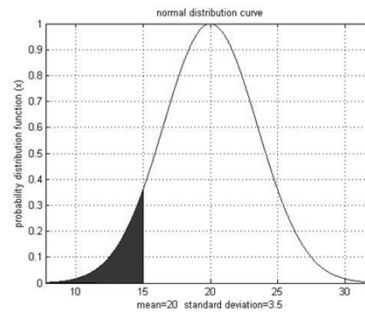
Computational Methods
EML3041

Why do we use numerical methods?

Why use Numerical Methods?

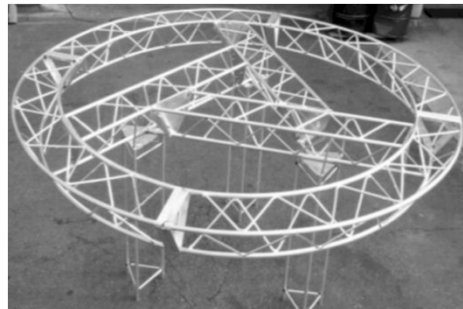
- To solve problems that cannot be solved exactly

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-\frac{u^2}{2}} du$$

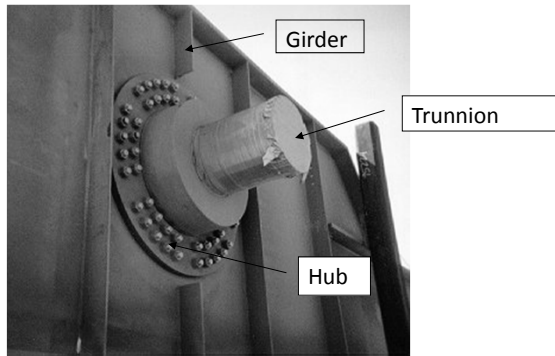


Why use Numerical Methods?

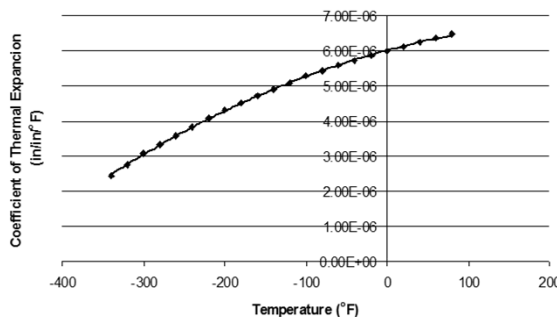
- To solve problems that have exact solutions but are otherwise intractable!



Assembling a fulcrum for a bascule bridge



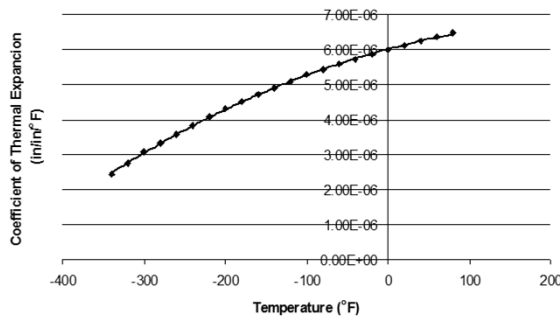
What model should I use to calculate contraction of trunnion?



$$\Delta D = D \alpha \Delta T$$

$$\Delta D = D \int_{T_{room}}^{T_{fluid}} \alpha dT$$

Finding the fluid temperature to get enough contraction



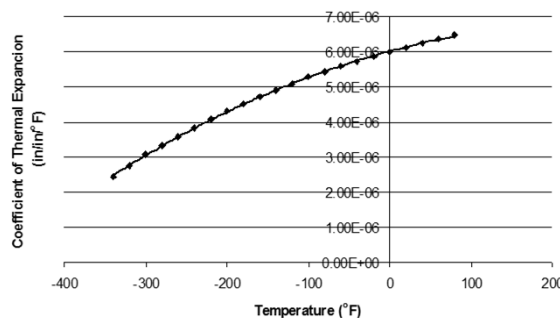
$$\Delta D = D \int_{T_{room}}^{T_{fluid}} \alpha dT$$

$$\alpha = -1.2278 \times 10^{-11} T^2 + 6.1946 \times 10^{-9} T + 6.0150 \times 10^{-6}$$

$$-0.015 = 12.363 \int_{80}^{T_{fluid}} (-1.2278 \times 10^{-11} T^2 + 6.1946 \times 10^{-9} T + 6.015 \times 10^{-6}) dT$$

$$\hat{f}(T_f) = -0.50598 \times 10^{-10} T_f^3 + 0.38292 \times 10^{-7} T_f^2 + 0.74363 \times 10^{-4} T_f + 0.88318 \times 10^{-2} = 0$$

Finding the expression for thermal expansion coefficient



$$\Delta D = D \int_{T_{room}}^{T_{fluid}} \alpha dT$$

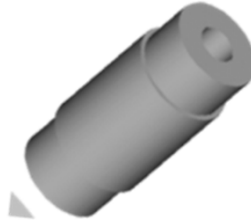
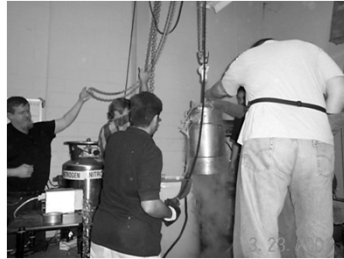
$$\alpha = a_0 + a_1 T + a_2 T^2$$

$$S_r = \sum_{i=1}^n (\alpha_i - \{a_0 + a_1 T_i + a_2 T_i^2\})^2$$

$$\begin{bmatrix} n \\ \left(\sum_{i=1}^n T_i \right) \\ \left(\sum_{i=1}^n T_i^2 \right) \end{bmatrix} \begin{bmatrix} \left(\sum_{i=1}^n T_i \right) \\ \left(\sum_{i=1}^n T_i^2 \right) \\ \left(\sum_{i=1}^n T_i^3 \right) \end{bmatrix} \begin{bmatrix} \left(\sum_{i=1}^n T_i^2 \right) \\ \left(\sum_{i=1}^n T_i^3 \right) \\ \left(\sum_{i=1}^n T_i^4 \right) \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^n \alpha_i \\ \sum_{i=1}^n T_i \alpha_i \\ \sum_{i=1}^n T_i^2 \alpha_i \end{bmatrix}$$

$$\alpha = -1.2278 \times 10^{-11} T^2 + 6.1946 \times 10^{-9} T + 6.0150 \times 10^{-6}$$

What is the temperature of the trunnion after half an hour?



$$mc \frac{d\theta}{dt} = -hA(\theta - \theta_a), \quad \theta(0) = \theta_{room}$$

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How long does it take a trunnion to cool down?



$$mc \frac{d\theta}{dt} = -hA(\theta - \theta_a), \quad \theta(0) = \theta_{room}$$

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Example

A solid steel shaft at room temperature of 27°C is needed to be contracted so that it can be shrunk-fit into a hollow hub. It is placed in a refrigerated chamber that is maintained at −33°C. The rate of change of temperature of the solid shaft θ is given by

$$\frac{d\theta}{dt} = -5.33 \times 10^{-6} \left(-3.69 \times 10^{-6} \theta^4 + 2.33 \times 10^{-5} \theta^3 + 1.35 \times 10^{-3} \theta^2 \right) (\theta + 33)$$

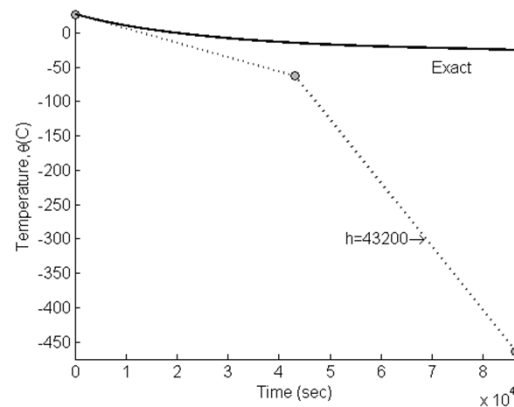
$$\theta(0) = 27^\circ\text{C}$$

Using Euler's method, find the temperature of the steel shaft after 86400 seconds. Take a step size of $h = 43200$ seconds.

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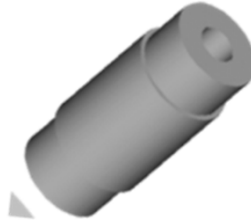
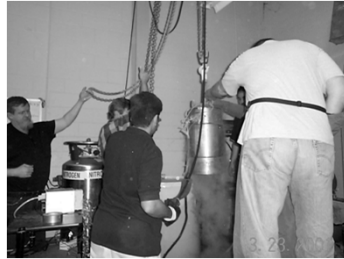
Temperature vs. Time



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What is the rate of change of heat stored in the cylinder?



$$mc \frac{d\theta}{dt} = -hA(\theta - \theta_a), \quad \theta(0) = \theta_{room}$$

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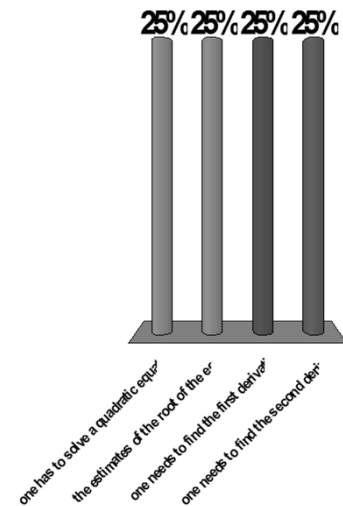
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Can you identify?

- We talked about 7 mathematical procedures in this course. Write them down.
- Now connect the problem just discussed to these 7 mathematical procedures on each of the previous slides.

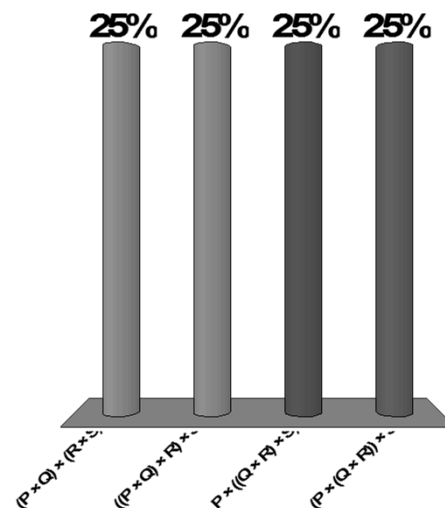
The Newton-Raphson method of finding real roots of the nonlinear equation $f(x)=0$ can be derived by using the first two terms of the Taylor series. One would then expect that adding one more term of the Taylor series to the Newton-Raphson method would result in higher accuracy of the results for the same number of iterations as the Newton-Raphson method. However, this new approach of adding one more term of Taylor series is **not feasible** because

- A. one has to solve a quadratic equation to get the new estimate of the root of the equation.
- B. the estimates of the root of the equation may turn out to be complex numbers
- C. one needs to find the first derivative of $f(x)$.
- D. one needs to find the second derivative of $f(x)$.



It is desired to compute the following matrix product, $P \times Q \times R \times S$. The sizes of the four matrices are: P is 1×4 , Q is 4×20 , R is 20×5 , and S is 5×1 . Amongst the choices given below, the most efficient (the least number of floating point operations of add, subtract, multiplication and divide) way to compute the matrix product is

- 1) $(P \times Q) \times (R \times S)$
- 2) $((P \times Q) \times R) \times S$
- 3) $P \times ((Q \times R) \times S)$
- 4) $(P \times (Q \times R)) \times S$



THE END