Introduction to Scientific Computing

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Transforming Numerical Methods Education for STEM Undergraduates

- **Welcome to EML3041: Computational Methods.**
- **My name is Dr. Kaw.**
- **Sign the attendance sheet as it gets passed around.**
- **Have you gotten your textbooks and TI30Xa calculator!**
- **Introduce yourself to the person on your left and right. Say one thing (nothing personal) about yourself to them.**
- **At home, go thru the CANVAS modules.**
- **All is well that ends well; well-begun is half-done.**
- **No cell phones, laptops or other distractions. Daydreamers and sleepers will not be disturbed.**

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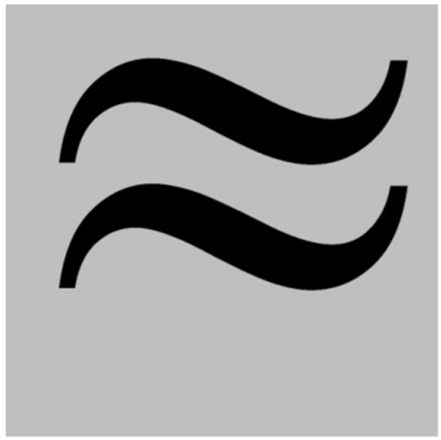
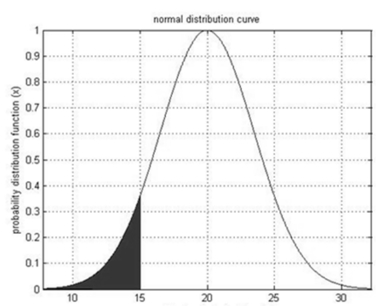
Why use Numerical Methods?



Why use Numerical Methods?

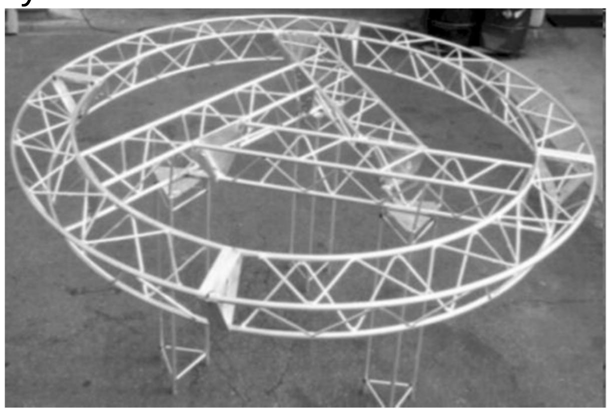
- To solve problems that cannot be solved exactly

$$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-\frac{u^2}{2}}\,du$$



Why use Numerical Methods?

- To solve problems that are intractable to solve exactly!



Steps in Solving an Engineering Problem

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How do we solve an engineering problem?

Problem Description

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Mathematical Model

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Solution of Mathematical Model

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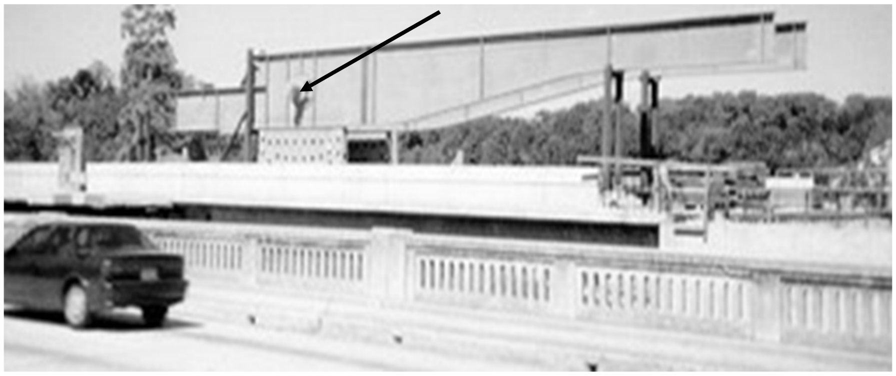
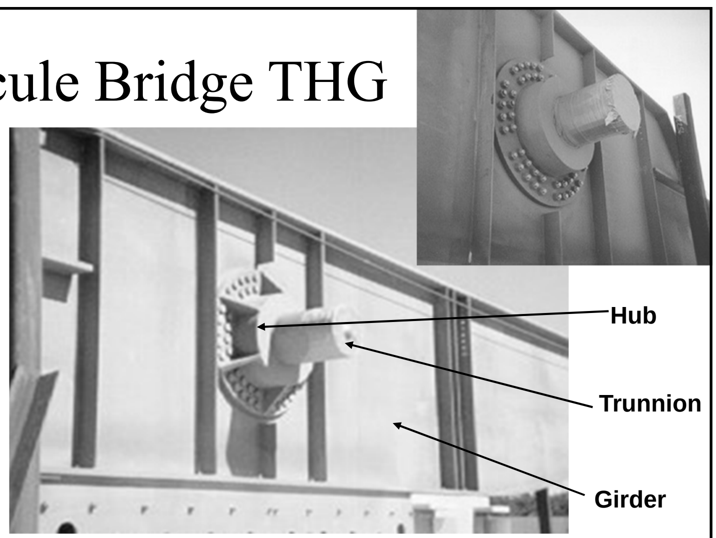
Using the Solution

Example of Solving an Engineering Problem

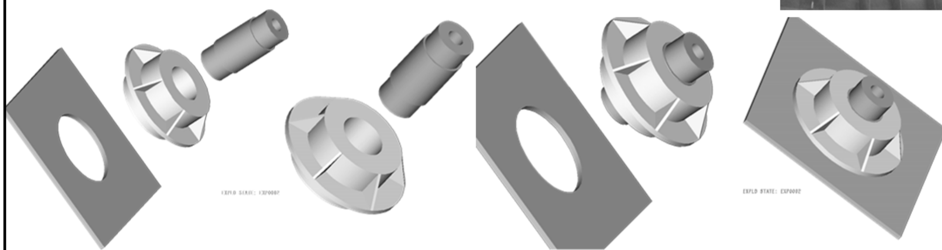
Bascule Bridge THG



Bascule Bridge THG



Trunnion-Hub-Girder Assembly Procedure



Simulation

Step1. Trunnion immersed in dry-ice/alcohol

Step2. Trunnion warm-up in hub

Step3. Trunnion-Hub immersed in dry-ice/alcohol

Step4. Trunnion-Hub warm-up into girder

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Problem

After Cooling, the Trunnion Got Stuck in Hub

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Why did it get stuck?

Magnitude of contraction needed in the trunnion was 0.015” or more. Did it contract enough?

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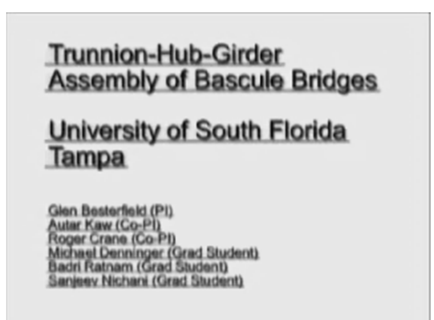
Video of Assembly Process



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Consultant calculations

$$\Delta D = D \times \alpha \times \Delta T$$

$$D = 12.363''$$
$$\alpha = 6.47 \times 10^{-6} in/in/^{o} F$$
$$\Delta T = -108 - 80 = -188^{o} F$$

$$\Delta D = (12.363)(6.47 \times 10^{-6})(-188)$$
$$= -0.01504''$$

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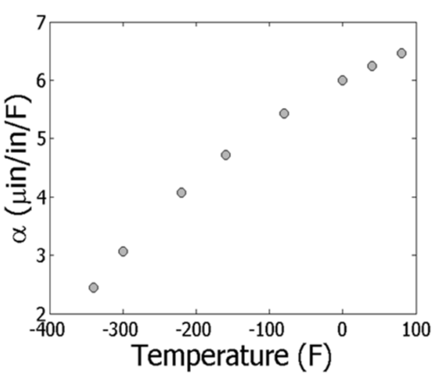
Is the formula used correct?

$$\Delta D = D \times \alpha \times \Delta T$$



T(°F)	α (μin/in/°F)
-340	2.45
-300	3.07
-220	4.08
-160	4.72
-80	5.43
0	6.00
40	6.24
80	6.47

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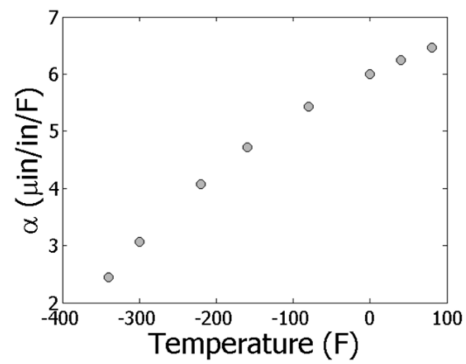
The Correct Model Would Account for Varying Thermal Expansion Coefficient



$$\Delta D = D\int_{T_a}^{T_c} \alpha(T)\,dT$$

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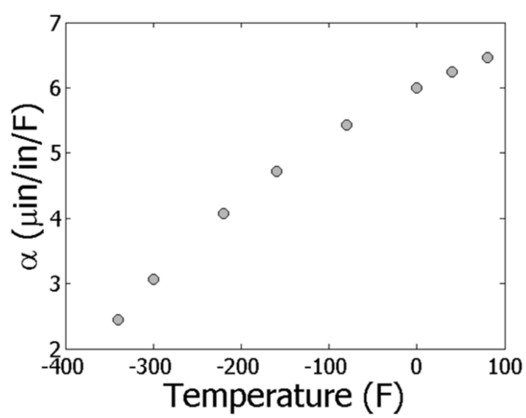
Can You Roughly Estimate the Contraction?

$$\Delta D = D\int_{T_a}^{T_c} \alpha(T)\,dT$$

T_a=80°F; T_c=-108°F; D=12.363”



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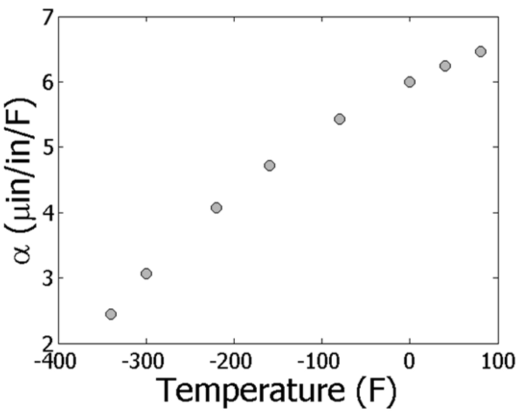
Can You Find a Better Estimate for the Contraction?

$$\Delta D = D\int_{T_a}^{T_c}\alpha(T)dT$$

$T_a = 80^oF$
$T_c = -108^oF$
$D = 12.363"$

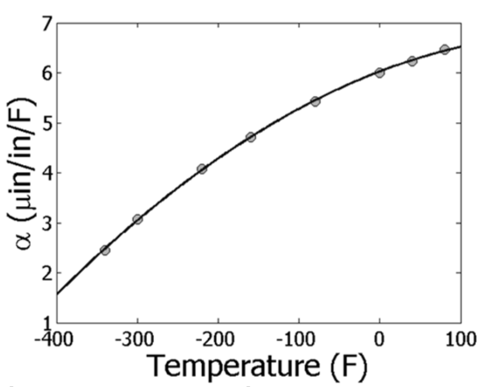


Estimating Contraction Accurately

Change in diameter (ΔD) by cooling it in dry ice/alcohol is given by

$$\Delta D = D\int_{T_a}^{T_c}\alpha(T)dT$$

$T_a = 80^oF$
$T_c = -108^oF$
$D = 12.363"$



$$\alpha = -1.2278\times10^{-5}T^2 + 6.1946\times10^{-3}T + 6.0150$$

$$\Delta D = -0.0137"$$

So what is the solution to the problem?

One solution is to immerse the trunnion in liquid nitrogen which has a boiling point of -321°F as opposed to the dry-ice/alcohol temperature of -108°F.

$$\Delta D = -0.0244''$$

Revisiting steps to solve a problem

1) Problem Statement: Trunnion got stuck in the hub.
2) Modeling: Developed a new model

$$\Delta D = D\int_{T_a}^{T_c} \alpha(T)dT$$

3) Solution: 1) Used trapezoidal rule OR b) Used regression and integration.
4) Implementation: Cool the trunnion in liquid nitrogen.

THE END

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Introduction to Numerical Methods

Mathematical Procedures

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Mathematical Procedures

- Nonlinear Equations
- Differentiation
- Simultaneous Linear Equations
- Curve Fitting
  - Interpolation
  - Regression
- Integration
- Ordinary Differential Equations
- Other Advanced Mathematical Procedures:
  - Partial Differential Equations
  - Optimization
  - Fast Fourier Transforms

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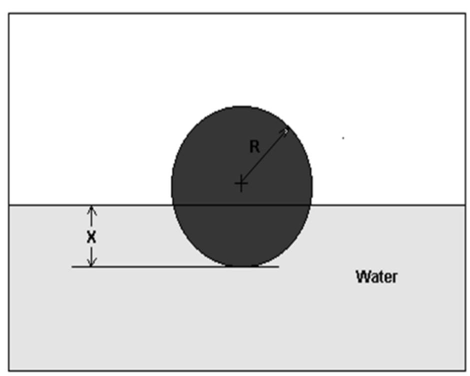
Nonlinear Equations

How much of the floating ball is under water?

Diameter=0.11m

Specific Gravity=0.6



$$x^3 - 0.165x^2 + 3.993\times10^{-4} = 0$$

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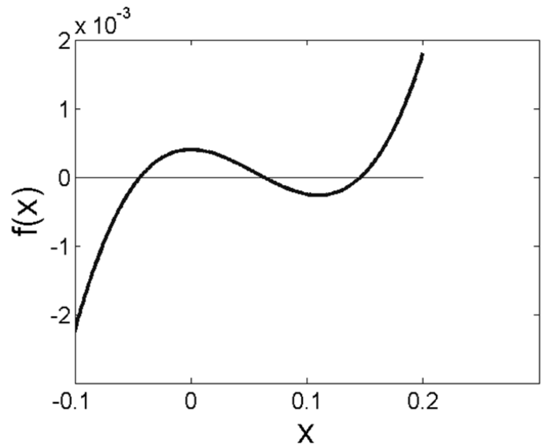
Nonlinear Equations

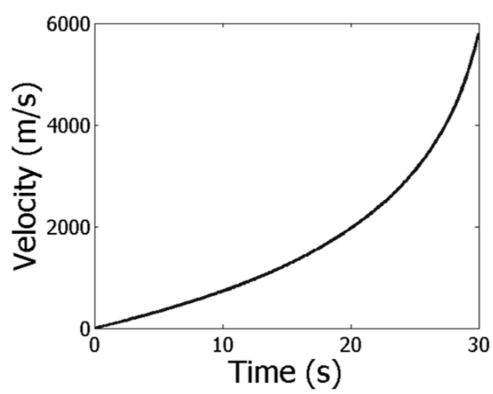
How much of the floating ball is under the water?



$$f(x) = x^3 - 0.165x^2 + 3.993\times10^{-4} = 0$$

Differentiation

What is the acceleration at t=7 seconds?

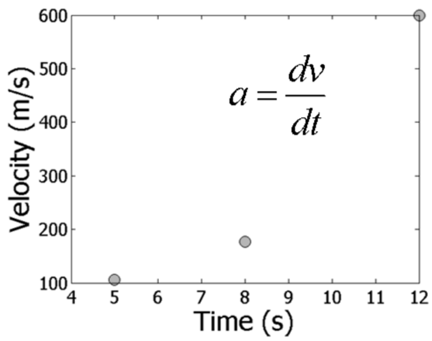


$$v(t) = 2200\ \ln\left(\frac{16\times10^4}{16\times10^4 - 5000t}\right) - 9.8t \qquad a = \frac{dv}{dt}$$

Differentiation

What is the acceleration at t=7 seconds?

Time (s)	5	8	12
Vel (m/s)	106	177	600



$$a = \frac{dv}{dt}$$

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Simultaneous Linear Equations

Find the velocity profile, given

Time (s)	5	8	12
Vel (m/s)	106	177	600

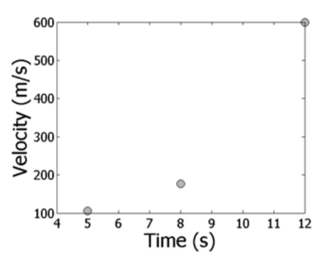
$$v(t) = at^2 + bt + c, \; 5 \le t \le 12$$

Three simultaneous linear equations

$$25a + 5b + c = 106$$

$$64a + 8b + c = 177$$

$$144a + 12b + c = 600$$



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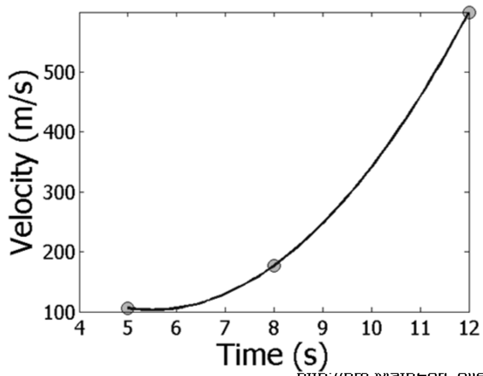
Interpolation

What is the velocity of the rocket at t=7 seconds?

Time (s)	5	8	12
Vel (m/s)	106	177	600



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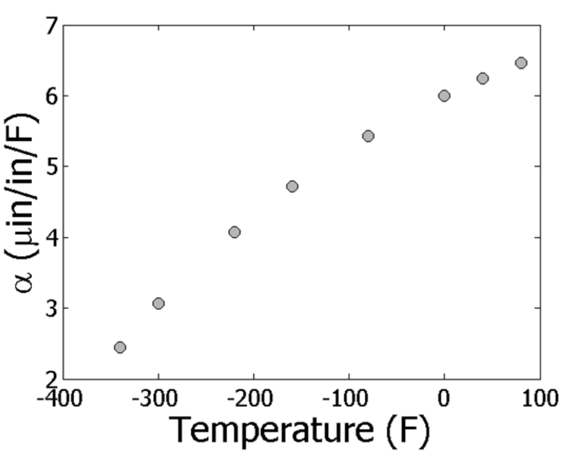
Regression

Thermal expansion coefficient data for cast steel



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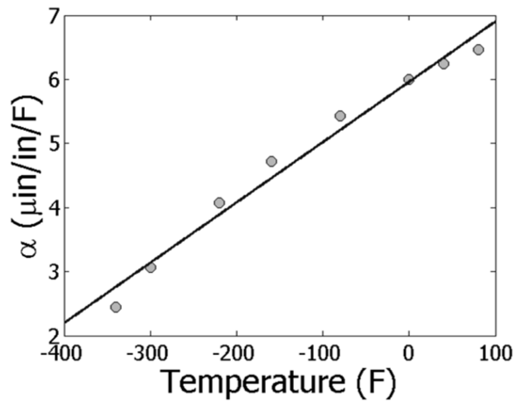
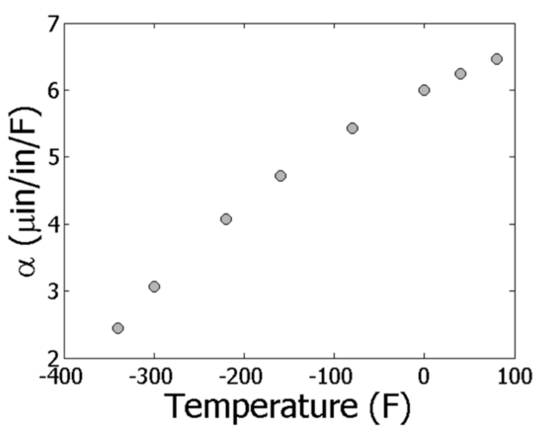
Regression (cont)



Integration

Finding the diametric contraction in a steel shaft when dipped in liquid nitrogen.

$$\Delta D = D \int_{T_{room}}^{T_{fluid}} \alpha \, dT$$



Ordinary Differential Equations

How long does it take a trunnion to cool down?

$$mc\frac{d\theta}{dt} = -hA(\theta - \theta_a), \quad \theta(0) = \theta_{room}$$

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THE END

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