Partial derivatives primer

If someone gives a function $f(x)$ of one variable x, then we already know how to find the derivative $f'(x)$. However, functions can be of more than one variable. How can we then calculate the rate of change of function with respect to each variable? This is simply done by defining partial derivatives, where one finds the derivatives with respect to one variable while treating other variables as constants. For example, for a function $f(x, y)$, the partial derivative with respect to x is defined as

$$\frac{\partial f}{\partial x} = \lim_{\Delta x \to 0} \frac{f(x + \Delta x, y) - f(x, y)}{\Delta x}$$

and the partial derivative with respect to y is defined as

$$\frac{\partial f}{\partial y} = \lim_{\Delta y \to 0} \frac{f(x, y + \Delta y) - f(x, y)}{\Delta y}$$

Example

Given

$$f(x, y) = 2x^3 y^2 + 7x^2 y^2,$$

find

$$\frac{\partial f}{\partial x} \text{ and } \frac{\partial f}{\partial y}.$$

Solution

$$f(x, y) = 2x^3 y^2 + 7x^2 y^2$$

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}(2x^3 y^2 + 7x^2 y^2)$$

Since all variables other than x are considered to be constant,

$$\frac{\partial f}{\partial x} = 2y^2 \frac{\partial}{\partial x}(x^3) + 7y^2 \frac{\partial}{\partial x}(x^2)$$

$$= 2y^2(3x^2) + 7y^2(2x)$$

$$= 6x^2 y^2 + 14xy^2$$

Now

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}(2x^3 y^2 + 7x^2 y^2)$$

Since all the variables other than y are considered to be constant,

$$\frac{\partial f}{\partial y} = 2x^3 \frac{\partial}{\partial y}(y^2) + 7x^2 \frac{\partial}{\partial y}(y^2)$$

$$= 2x^3(2y) + 7x^2(2y)$$

$$= 4x^3 y + 14x^2 y$$