$$2\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 5y = 11e^{-x}$$

$$y(0) = 7, \quad \frac{dy}{dx}(0) = 13$$

Find $y(0.5)$ using $h = 0.25$

Let $\frac{dy}{dx} = z$, then

$$2\frac{dz}{dx} + 3z + 5y = 11e^{-x}$$

$$\frac{dz}{dx} = \frac{11e^{-x} - 3z - 5y}{2}$$

ODEs

$$\frac{dy}{dx} = z = f_1(x, y, z), \quad y(0) = 7$$

$$\frac{dz}{dx} = \frac{11e^{-x} - 3z - 5y}{2} = f_2(x, y, z)$$

$$z(0) = 13$$

$$y_{i+1} = y_i + \left(\frac{1}{2}k_1^y + \frac{1}{2}k_2^y\right)h$$

$$z_{i+1} = z_i + \left(\frac{1}{2}k_1^z + \frac{1}{2}k_2^z\right)h$$

$$k_1^y = f_1(x_i, y_i, z_i)$$

$$k_2^y = f_1(x_i+h, y_i+k_1^y h, z_i+k_1^z h)$$
$$k_1^z = f_2(x_i, y_i, z_i)$$
$$k_2^y = f_2(x_i+h, y_i+k_1^y h, z_i+k_1^z h)$$

$i=0$

$x_0=0$, $y_0=7$, $z_0=13$

$$k_1^y = f_1(x_0, y_0, z_0)$$
$$= f_1(0, 7, 13)$$
$$= 13$$
$$k_1^z = f_2(x_0, y_0, z_0)$$
$$= f_2(0, 7, 13)$$
$$= \frac{11e^{-0} - 3(13) - 5(7)}{2}$$
$$= -31.5$$
$$k_2^y = f_1(x_0+h, y_0+k_1^y h, z_0+k_1^z h)$$
$$= f_1(0+0.25, 7+13\times 0.25, 13+(-31.5)\times 0.25)$$
$$= f_1(0.25, 10.25, 5.125)$$
$$= 5.125$$
$$k_2^z = f_2(x_0+h, y_0+k_1^y h, z_0+k_1^z h)$$

$= f_2(0.25, 10.25, 5.125)$

$= \frac{11e^{-0.25} - 3(5.125) - 5(10.25)}{2}$

$= -29.03$

$y_1 = y_0 + \left(\frac{1}{2}k_1^y + \frac{1}{2}k_2^y\right)h$

$= 7 + \left(\frac{1}{2}(13) + \frac{1}{2}(5.125)\right) * 0.25$

$= 9.266$

$z_1 = z_1 + \left(\frac{1}{2}k_1^z + \frac{1}{2}k_2^z\right)h$

$= 13 + \left(\frac{1}{2}(-31.5) + \frac{1}{2}(-29.03)\right) * 0.25$

$= 5.434$

$i=1,\ x_1 = 0.25,\ y_1 = 9.266,\ z_1 = 5.434$

$k_1^y = f_1(x_1, y_1, z_1)$

$= f_1(0.25, 9.266, 5.434)$

$= 5.434$

$k_1^z = f_2(x_1, y_1, z_1)$

$= f_2(0.25, 9.266, 5.434)$

$= \frac{11e^{-0.25} - 3(5.434) - 5(9.266)}{2}$

$= -27.03$ (not readed)

$$k_2^y = f_1(x_1+h, y_1+k_1^y h, z_1+k_1^z h)$$

$$= f_1(0.25+0.25,\ 9.266+5.434\times 0.25,\ 5.434+(-27.03)\times 0.25)$$

$$= f_1(0.5, 10.62, -1.324)$$

$$= -1.324$$

$$k_2^z = f_2(0.5, 10.62, -1.324)$$

$$= \frac{11e^{-0.5} - 3(1.324) - 5(10.62)}{2}$$

$$= -21.23 \text{ (Not needed)}$$

$$y_2 = y_1 + \left(\frac{1}{2}k_1^y + \frac{1}{2}k_2^y\right) * 0.25$$

$$= 9.266 + \left(\frac{1}{2}(5.434) + \frac{1}{2}(-1.324)\right) \times 0.25$$

$$= 9.780$$

$$\approx y(0.5)$$