Conceptual Questions
Chapter 04.01 Simultaneous Linear Equations

Last Name _____ First Name_____ Date ____ Group#_____ Last Name Initial__

1) A square matrix [A] is upper triangular if

Individual Attempt	**Group Attempt**
A. $a_{ij} = 0, i > j$	A. $a_{ij} = 0, i > j$
B. $a_{ij} = 0, j > i$	B. $a_{ij} = 0, j > i$
C. $a_{ij} \neq 0, i > j$	C. $a_{ij} \neq 0, i > j$
D. $a_{ij} \neq 0, j > i$	D. $a_{ij} \neq 0, j > i$

Justification/ Work _____

2) The following system of equations

$x + y = 2,$
$6x + 6y = 12.$

has _____solution(s).

Individual Attempt	**Group Attempt**
A. no	A. no
B. one	B. one
C. more than one but a finite number of	C. more than one but a finite number of
D. infinite	D. infinite

Justification/ Work _____

PS. The questions in this handout have been taken from tests from previous years

Conceptual Questions
Chapter 04.01 Simultaneous Linear Equations

Last Name _____ First Name_____ Date ____ Group#_____ Last Name Initial__

3) The following data is given for the velocity of the rocket as a function of time. To find the velocity at $t = 21s$, you are asked to use a quadratic polynomial $v(t) = at^2 + bt + c$ to approximate the velocity profile.

t (s)	0	14	15	20	30	35
v (m/s)	0	227.04	362.78	517.35	602.97	901.67

Individual Attempt	**Group Attempt**
A. $\begin{bmatrix} 196 & 14 & 1 \\ 225 & 15 & 1 \\ 400 & 20 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 227.04 \\ 362.78 \\ 517.35 \end{bmatrix}$	A. $\begin{bmatrix} 176 & 14 & 1 \\ 225 & 15 & 1 \\ 400 & 20 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 227.04 \\ 362.78 \\ 517.35 \end{bmatrix}$
B. $\begin{bmatrix} 225 & 15 & 1 \\ 400 & 20 & 1 \\ 900 & 30 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 362.78 \\ 517.35 \\ 602.97 \end{bmatrix}$	B. $\begin{bmatrix} 225 & 15 & 1 \\ 400 & 20 & 1 \\ 900 & 30 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 362.78 \\ 517.35 \\ 602.97 \end{bmatrix}$
C. $\begin{bmatrix} 0 & 0 & 1 \\ 225 & 15 & 1 \\ 400 & 20 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 362.78 \\ 517.35 \end{bmatrix}$	C. $\begin{bmatrix} 0 & 0 & 1 \\ 225 & 15 & 1 \\ 400 & 20 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 362.78 \\ 517.35 \end{bmatrix}$
D. $\begin{bmatrix} 400 & 20 & 1 \\ 900 & 30 & 1 \\ 1225 & 35 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 517.35 \\ 602.97 \\ 901.67 \end{bmatrix}$	D. $\begin{bmatrix} 400 & 20 & 1 \\ 900 & 30 & 1 \\ 1225 & 35 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 517.35 \\ 602.97 \\ 901.67 \end{bmatrix}$

Justification/ Work _____

PS. The questions in this handout have been taken from tests from previous years

Free Response Questions
Chapter 04.01 Introduction to Matrix Algebra

Chapter 04.01 (Set One)

1) By any scientific method, find the second column of the inverse of

$$\begin{bmatrix} 1 & 2 & 0 \\ 4 & 5 & 0 \\ 0 & 0 & 13 \end{bmatrix}$$

2) Solve $[A][X] = [B]$ for $[X]$ if

$$[A]^{-1} = \begin{bmatrix} 10 & -7 & 0 \\ 2 & 2 & 5 \\ 2 & 0 & 6 \end{bmatrix} \text{ and } [B] = \begin{bmatrix} 7 \\ 2.5 \\ 6.012 \end{bmatrix}$$

3) Let $[A]$ be a 3×3 matrix. Suppose

$$[X] = \begin{bmatrix} 7 \\ 2.5 \\ 6.012 \end{bmatrix}$$

is a solution to the homogeneous set of equations $[A][X] = [0]$ (the right hand side is a zero vector of order 3×1). Does $[A]$ have an inverse? Justify your answer.

Answers

Chapter 04.01 (Set One)

1) $\begin{bmatrix} 0.667 \\ -0.333 \\ 0 \end{bmatrix}$

2) $\begin{bmatrix} 52.5 \\ 49.06 \\ 50.072 \end{bmatrix}$

3) Answer is No, but prove it.

PS. The questions in this handout have been taken from tests from previous years

Conceptual Questions
Chapter 04.06 Naïve-Gauss Elimination Method (Part 1)

Last Name _____ First Name_____ Date ____ Group#_____ Last Name Initial__

1) Using 3 significant digit with *chopping* at all stages, the result for the following calculation is

$$x_1 = \frac{6.095 - 3.456 \times 1.99}{8}$$

Individual Attempt	**Group Attempt**
A. -0.0988 B. -0.0978 C. -0.0969 D. -0.0962	A. -0.0988 B. -0.0978 C. -0.0969 D. -0.0962

Justification/ Work _____

PS. The questions in this handout have been taken from tests from previous years

Conceptual Questions
Chapter 04.06 Naïve-Gauss Elimination Method (Part 1)

Last Name _____ First Name_____ Date ____ Group#_____ Last Name Initial__

2) Using 3 significant digits with *rounding-off* at all stages, the result for the following calculation is

$$x_1 = \frac{6.095 - 3.456 \times 1.99}{8}$$

Individual Attempt	**Group Attempt**
A. -0.0988 B. -0.0978 C. -0.0969 D. -0.0962	A. -0.0988 B. -0.0978 C. -0.0969 D. -0.0962

Justification/ Work _____

3) Division by zero during forward elimination steps in *Naïve Gaussian elimination* for $[A][X] = [C]$ implies the coefficient matrix $[A]$.

Individual Attempt	**Group Attempt**
A. is invertible B. is not invertible C. cannot be determined to be invertible or not	A. is invertible B. is not invertible C. cannot be determined to be invertible or not

Justification/ Work _____

PS. The questions in this handout have been taken from tests from previous years

Conceptual Questions
Chapter 04.06 Gaussian Elimination with Partial Pivoting (Part 2)

Last Name _____ First Name_____ Date ____ Group#_____ Last Name Initial__

1) One of the pitfalls of Naïve Gauss Elimination method is

Individual Attempt	**Group Attempt**
A. large truncation error B. large round-off error C. not able to solve equations with a noninvertible coefficient matrix	A. large truncation error B. large round-off error C. not able to solve equations with a noninvertible coefficient matrix

Justification/ Work _____

2) Increasing the precision of numbers from single to double in the Naïve Gaussian elimination method

Individual Attempt	**Group Attempt**
A. avoids division by zero B. decreases round-off error C. allows equations with a noninvertible coefficient matrix to be solved	A. avoids division by zero B. decreases round-off error C. allows equations with a noninvertible coefficient matrix to be solved

Justification/ Work _____

PS. The questions in this handout have been taken from tests from previous years

Conceptual Questions
Chapter 04.06 Gaussian Elimination with Partial Pivoting (Part 2)

Last Name _____ First Name_____ Date ____ Group#_____ Last Name Initial__

3) Division by zero during forward elimination steps in *Gaussian elimination with partial pivoting* of the set of equations $[A][X]=[C]$ implies the coefficient matrix $[A]$.

Individual Attempt	**Group Attempt**
A. is invertible B. is not invertible C. cannot be determined to be invertible or not	A. is invertible B. is not invertible C. cannot be determined to be invertible or not

Justification/ Work _____

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Free Response Questions
Chapter 04.06 Gauss Elimination

Chapter 04.06 (Set One)

1) Using forward elimination, find the determinant of the matrix below

$$\begin{bmatrix} 2 & 5 & 7 \\ 13 & 12 & 3 \\ 11 & 7 & 32 \end{bmatrix}$$

2) What is the value of a_{32} of the coefficient matrix A at the end of the first step of forward elimination of Gauss elimination with partial pivoting?

$$\begin{bmatrix} 5 & 6 & 10 \\ 8 & 12 & 11 \\ 16 & 4 & 13 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 21 \\ 19 \\ 37 \end{bmatrix}$$

3) What is the minimum number of zero elements in a 418×418 coefficient matrix at the end of 100 steps of forward elimination?

Chapter 04.06 (Set Two)

1) At the end of Gauss Elimination steps on a set of three equations, I obtain the following system of equations.

$$\begin{bmatrix} 10 & -7 & 0 \\ 0 & 2.567 & 5 \\ 0 & 0 & 6.022 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 7 \\ 2.5 \\ 6.012 \end{bmatrix}$$

Now using a computer that uses only **three** significant digits with **chopping**, what is the value of unknowns using back substitution? **Show all your intermediate work.**

2) At the *end of the first step* of forward elimination in the Gauss elimination with partial pivoting method algorithm, the equations obtained in matrix form on a given set of equations are as follows.

$$\begin{bmatrix} 2 & 4 & 6 & 10 \\ 0 & 16 & 24 & 16 \\ 0 & 32 & 42 & 17 \\ 0 & 24 & 36 & 29 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 1 \\ 24 \\ 64 \\ 96 \end{bmatrix}$$

Conduct only the *second step of forward elimination* of Gauss elimination with partial pivoting method and show the result in matrix form. **Show your work for full credit and put your final answer in the box.**

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Free Response Questions
Chapter 04.06 Gauss Elimination

3) Find the determinant of this matrix by a method learned in this class (cofactor method is not allowed). **Show your work for full credit and put your final answer in the box.**

$$\begin{bmatrix} 2 & 4 & 6 & 10 \\ 0 & 16 & 24 & 16 \\ 0 & 32 & 42 & 17 \\ 0 & 24 & 36 & 29 \end{bmatrix}$$

Answers

Chapter 04.06 (Set One)

1) -1476.0
2) 4.75
3) 36750

Chapter 04.06 (Set Two)

1) $x_1 = 0.020, x_2 = -0.972, x_3 = 0.998$

2) $$\begin{bmatrix} 2 & 4 & 6 & 10 \\ 0 & 32 & 42 & 17 \\ 0 & 0 & 3 & 7.5 \\ 0 & 0 & 4.5 & 16.25 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 1 \\ 64 \\ -8 \\ 48 \end{bmatrix}$$

3) -960

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