Conceptual Questions
Chapter 04.07 LU Decomposition Part-1

Last Name _________ First Name________ Date ____ Group#_____ Last Name Initial__

1) LU decomposition method is computationally more efficient than Naïve Gauss elimination for solving

Individual Attempt	**Group Attempt**
A. a single set of simultaneous linear equations B. multiple sets of simultaneous linear equations with different coefficient matrices and same right hand side vectors. C. multiple sets of simultaneous linear equations with same coefficient matrix and different right hand side vectors	A. a single set of simultaneous linear equations B. multiple sets of simultaneous linear equations with different coefficient matrices and same right hand side vectors. C. multiple sets of simultaneous linear equations with same coefficient matrix and different right hand side vectors

Justification/ Work ____

PS. The questions in this handout have been taken from tests from previous years

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2) LU decomposition method for solving a set of equations of form $[A][X]=[C]$ uses the following steps

Individual Attempt	**Group Attempt**
A. $[L][X]=[Z]$ followed by $[U][Z]=[C]$ B. $[L][Z]=[C]$ followed by $[U][X]=[Z]$ C. $[U][Z]=[C]$ followed by $[L][X]=[Z]$ D. $[U][X]=[Z]$ followed by $[L][Z]=[C]$	A. $[L][X]=[Z]$ followed by $[U][Z]=[C]$ B. $[L][Z]=[C]$ followed by $[U][X]=[Z]$ C. $[U][Z]=[C]$ followed by $[L][X]=[Z]$ D. $[U][X]=[Z]$ followed by $[L][Z]=[C]$

Justification/ Work ____

3) Given the decomposition of a square matrix $[A]=[L][U]$, where $[L]$ has 1's in the diagonal, the determinant of A is

Individual Attempt	**Group Attempt**
A. Product of diagonal elements of $[A]$ B. Product of diagonal elements of $[U]$ C. Sum of the diagonal elements of $[A]$ D. Sum of diagonal elements of $[U]$	A. Product of diagonal elements of $[A]$ B. Product of diagonal elements of $[U]$ C. Sum of the diagonal elements of $[A]$ D. Sum of diagonal elements of $[U]$

Justification/ Work ____

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Conceptual Questions
Chapter 04.07 LU Decomposition Part-2

Last Name _________ First Name________ Date ____ Group#_____ Last Name Initial__

1) If you have a $n \times n$ matrix [A], and T is clock cycle time, the computation time for decomposing the matrix [A] to LU is approximately proportional to

Individual Attempt	**Group Attempt**
A. $\frac{8nT}{3}$ B. $\frac{8n^2T}{3}$ C. $\frac{8n^3T}{3}$	D. $\frac{8nT}{3}$ E. $\frac{8n^2T}{3}$ F. $\frac{8n^3T}{3}$

Justification/ Work ____

2) For a given 1700×1700 matrix [A], assume that it takes about 16 seconds to find the inverse of [A] by the use of the [L][U] decomposition method. The approximate time in seconds that all the forward substitutions take out of the 16 seconds is

Individual Attempt	**Group Attempt**
A. 4 B. 6 C. 8 D. 12	A. 4 B. 6 C. 8 D. 12

Justification/ Work ____

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Conceptual Questions
Chapter 04.07 LU Decomposition Part-2

Last Name _________ First Name________ Date ____ Group#_____ Last Name Initial__

3) For a given 1700×1700 matrix $[A]$, assume that it takes about 16 seconds to find the inverse of $[A]$ by the use of the $[L][U]$ decomposition method. The approximate time in seconds that the LU decomposition takes out of the 16 seconds is

Individual Attempt	**Group Attempt**
A. 4 B. 6 C. 8 D. 12	A. 4 B. 6 C. 8 D. 12

Justification/ Work ____

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Free Response Questions
Chapter 04.07 LU Decomposition Method

Chapter 04.07 (Set One)

1) A 2000×2000 matrix $[A]$ is decomposed into a LU form by using the forward elimination steps of Gaussian elimination. It is given that it takes 16 seconds of computational time to conduct the LU decomposition of $[A]$.

a) Find the approximate computational time in seconds it would take to do the LU decomposition of a 6000×6000 matrix $[B]$.

b) Find the approximate computational time it would take to find the inverse of the 2000×2000 matrix $[A]$ by using LU decomposition route.

2) Given the LU decomposition of a 3×3 matrix $[A]$ is

$$[A] = \begin{bmatrix} 1 & 4 & 6 \\ 4 & 21 & 32 \\ 5 & 30 & 59 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 5 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 4 & 6 \\ 0 & 5 & 8 \\ 0 & 0 & 13 \end{bmatrix}$$

find the second column of the inverse of $[A]$ using the LU decomposition method of solving simultaneous linear equations.

3) As part of finding the LU decomposition of a square matrix $[A]$,

$$[A] = \begin{bmatrix} 16 & 8 & 4 & 2 \\ 2 & 4 & 8 & 16 \\ 16 & 2 & 4 & 8 \\ 32 & 16 & 9 & 16 \end{bmatrix},$$

at the end of the 1st step of forward elimination, the matrix $[A]$ becomes

$$\begin{bmatrix} 16 & 8 & 4 & 2 \\ 0 & 3 & 7.5 & 15.75 \\ 0 & -6 & 0 & 6 \\ 0 & 0 & 1 & 12 \end{bmatrix}$$

The multipliers used to make a_{21}, a_{31}, and a_{41} zero in the 1st step of forward elimination were 0.1250, 1.000 and 2.000, respectively.

Find the $[L][U]$ decomposition of the $[A]$ matrix, given that the $[L]$ matrix has 1 as its diagonal elements.

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Free Response Questions
Chapter 04.07 LU Decomposition Method

Answers
Chapter 04.07 (Set One)

1 a) 432 s

b) 64 s

2) $\begin{bmatrix} -0.8615 \\ 0.4462 \\ -0.1538 \end{bmatrix}$

3) $[U] = \begin{bmatrix} 16 & 8 & 4 & 2 \\ 0 & 3 & 7.5 & 15.75 \\ 0 & 0 & 15 & 37.5 \\ 0 & 0 & 0 & 9.5 \end{bmatrix}, [L] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.125 & 1 & 0 & 0 \\ 1 & -2 & 1 & 0 \\ 2 & 0 & 0.06666 & 1 \end{bmatrix}$

Chapter 04.07 (Set Two)

1) For a given 891×891 matrix A, assume that it takes about 2 seconds to find the inverse of A by the use of the LU decomposition method, that is, decomposing A once to LU, and then doing both forward substitution and back substitution 891 times by using the 891 columns of the identity matrix as the right-hand side vectors. What would be the approximate time (in seconds) that it will take to find the inverse by repeated use of Naïve gauss elimination method, that is, doing forward elimination and back substitution 891 times by using the 891 columns of the identity matrix as the right-hand side vectors?

2) To solve a set of equations $[A][X]=[B]$ by LU decomposition requires back substitution $[U][X]=[Z]$ given by the algorithm below.

$x(n)=Z(n)/U(n,n)$
for $i=(n-1):-1:1$
$sum=Z(i)$
for $j=(i+1):1:n$
$sum=sum-U(i,j)*x(j)$
end
$x(i)=sum/U(i,i)$
end

If $n=141$, where n is the number of equations, what is the sum of the total number of multiplication and division operations in the algorithm?

3) Given the set of equations $[A][X]=[C]$

$$\begin{bmatrix} 9 & 5 & 7 \\ 13 & 37 & 3 \\ 11 & 7 & 29 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 22 \\ 15 \\ 16 \end{bmatrix}$$

What is the value of l_{32} of the $[A]=[L][U]$ decomposition with all 1's in the diagonal of the $[L]$ matrix?

Answers
Chapter 04.07 (Set Two)

1) 445.5
2) 10011
3) 0.0299