Work for October 3 and 4, 2019

Chapters

04.01 Introduction to Matrix Algebra
04.06 Gaussian Elimination
04.07 LU Decomposition

Sequence of Work in Class for Thursday

1. Finish conceptual questions from 04.01.
2. Do the concept questions 04.06 – Part 1 by yourself. Give justification and show work.
3. Redo the concept questions of 04.06 – Part 1 with your group of 2.
4. Do concept handout questions from 04.06 – Part 2 by yourself. Give justification and show work.
5. Redo the concept questions of 04.06 – Part 2 with your group of 2.
6. Finish free-response questions from 04.06.

Sequence of Work in Class for Friday

1. Do concept handout questions from 04.07 – Part 1 by yourself. Give justification and show work.
2. Redo the concept questions of 04.07 – Part 1 with your group of 2.
3. Do concept handout questions from 04.07 – Part 2 by yourself. Give justification and show work.
4. Redo the concept questions of 04.07 – Part 2 with your group of 2.
5. Finish free-response questions from 04.07.

What If I Have Finished The Work For Day

1. Solve the multiple-choice questions at end of textbook chapters 04.06, 04.07
2. Solve the problem-set questions at end of textbook chapters 04.06, 04.07

Conceptual Questions
Chapter 04.01 Simultaneous Linear Equations

Last Name _________ First Name________ Date ____ Group#_____ Last Name Initial__

1) A square matrix [A] is upper triangular if

Individual Attempt	**Group Attempt**
A. $a_{ij} = 0, i > j$ B. $a_{ij} = 0, j > i$ C. $a_{ij} \neq 0, i > j$ D. $a_{ij} \neq 0, j > i$	A. $a_{ij} = 0, i > j$ B. $a_{ij} = 0, j > i$ C. $a_{ij} \neq 0, i > j$ D. $a_{ij} \neq 0, j > i$

Justification/ Work ____

2) The following system of equations

$x + y = 2,$
$6x + 6y = 12.$

has __________solution(s).

Individual Attempt	**Group Attempt**
A. no B. one C. more than one but a finite number of D. infinite	A. no B. one C. more than one but a finite number of D. infinite

Justification/ Work ____

PS. The questions in this handout have been taken from tests from previous years

Last Name _________ First Name________ Date ____ Group#_____ Last Name Initial__

3) The following data is given for the velocity of the rocket as a function of time. To find the velocity at $t = 21s$, you are asked to use a quadratic polynomial $v(t) = at^2 + bt + c$ to approximate the velocity profile.

t (s)	0	14	15	20	30	35
v (m/s)	0	227.04	362.78	517.35	602.97	901.67

Individual Attempt	**Group Attempt**
A. $\begin{bmatrix} 196 & 14 & 1 \\ 225 & 15 & 1 \\ 400 & 20 & 1 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 227.04 \\ 362.78 \\ 517.35 \end{bmatrix}$	A. $\begin{bmatrix} 176 & 14 & 1 \\ 225 & 15 & 1 \\ 400 & 20 & 1 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 227.04 \\ 362.78 \\ 517.35 \end{bmatrix}$
B. $\begin{bmatrix} 225 & 15 & 1 \\ 400 & 20 & 1 \\ 900 & 30 & 1 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 362.78 \\ 517.35 \\ 602.97 \end{bmatrix}$	B. $\begin{bmatrix} 225 & 15 & 1 \\ 400 & 20 & 1 \\ 900 & 30 & 1 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 362.78 \\ 517.35 \\ 602.97 \end{bmatrix}$
C. $\begin{bmatrix} 0 & 0 & 1 \\ 225 & 15 & 1 \\ 400 & 20 & 1 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 362.78 \\ 517.35 \end{bmatrix}$	C. $\begin{bmatrix} 0 & 0 & 1 \\ 225 & 15 & 1 \\ 400 & 20 & 1 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 362.78 \\ 517.35 \end{bmatrix}$
D. $\begin{bmatrix} 400 & 20 & 1 \\ 900 & 30 & 1 \\ 1225 & 35 & 1 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 517.35 \\ 602.97 \\ 901.67 \end{bmatrix}$	D. $\begin{bmatrix} 400 & 20 & 1 \\ 900 & 30 & 1 \\ 1225 & 35 & 1 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 517.35 \\ 602.97 \\ 901.67 \end{bmatrix}$

Justification/ Work ____

PS. The questions in this handout have been taken from tests from previous years

Free Response Questions
Chapter 04.01 Introduction to Matrix Algebra

Chapter 04.01 (Set One)

1) By any scientific method, find the second column of the inverse of

$$\begin{bmatrix} 1 & 2 & 0 \\ 4 & 5 & 0 \\ 0 & 0 & 13 \end{bmatrix}$$

2) Solve $[A][X] = [B]$ for $[X]$ if

$$[A]^{-1} = \begin{bmatrix} 10 & -7 & 0 \\ 2 & 2 & 5 \\ 2 & 0 & 6 \end{bmatrix} \text{ and } [B] = \begin{bmatrix} 7 \\ 2.5 \\ 6.012 \end{bmatrix}$$

3) Let $[A]$ be a 3×3 matrix. Suppose

$$[X] = \begin{bmatrix} 7 \\ 2.5 \\ 6.012 \end{bmatrix}$$

is a solution to the homogeneous set of equations $[A][X] = [0]$ (the right hand side is a zero vector of order 3×1). Does $[A]$ have an inverse? Justify your answer.

Answers

Chapter 04.01 (Set One)

1) $\begin{bmatrix} 0.667 \\ -0.333 \\ 0 \end{bmatrix}$

2) $\begin{bmatrix} 52.5 \\ 49.06 \\ 50.072 \end{bmatrix}$

3) Answer is No, but prove it.

PS. The questions in this handout have been taken from tests from previous years

Conceptual Questions
Chapter 04.06 Naïve-Gauss Elimination Method (Part 1)

Last Name _________ First Name________ Date ____ Group#_____ Last Name Initial__

1) Using 3 significant digit with *chopping* at all stages, the result for the following calculation is

$$x_1 = \frac{6.095 - 3.456 \times 1.99}{8}$$

Individual Attempt	**Group Attempt**
A. -0.0988 B. -0.0978 C. -0.0969 D. -0.0962	A. -0.0988 B. -0.0978 C. -0.0969 D. -0.0962

Justification/ Work ____

PS. The questions in this handout have been taken from tests from previous years

Conceptual Questions
Chapter 04.06 Naïve-Gauss Elimination Method (Part 1)

Last Name _________ First Name________ Date ____ Group#_____ Last Name Initial__

2) Using 3 significant digits with *rounding-off* at all stages, the result for the following calculation is

$$x_1 = \frac{6.095 - 3.456 \times 1.99}{8}$$

Individual Attempt	**Group Attempt**
A. -0.0988 B. -0.0978 C. -0.0969 D. -0.0962	A. -0.0988 B. -0.0978 C. -0.0969 D. -0.0962

Justification/ Work ____

3) Division by zero during forward elimination steps in *Naïve Gaussian elimination* for $[A][X]=[C]$ implies the coefficient matrix $[A]$.

Individual Attempt	**Group Attempt**
A. is invertible B. is not invertible C. cannot be determined to be invertible or not	A. is invertible B. is not invertible C. cannot be determined to be invertible or not

Justification/ Work ____

PS. The questions in this handout have been taken from tests from previous years

Conceptual Questions
Chapter 04.06 Gaussian Elimination with Partial Pivoting (Part 2)

Last Name _________ First Name________ Date ____ Group#_____ Last Name Initial__

1) One of the pitfalls of Naïve Gauss Elimination method is

Individual Attempt	**Group Attempt**
A. large truncation error B. large round-off error C. not able to solve equations with a noninvertible coefficient matrix	A. large truncation error B. large round-off error C. not able to solve equations with a noninvertible coefficient matrix

Justification/ Work ____

2) Increasing the precision of numbers from single to double in the Naïve Gaussian elimination method

Individual Attempt	**Group Attempt**
A. avoids division by zero B. decreases round-off error C. allows equations with a noninvertible coefficient matrix to be solved	A. avoids division by zero B. decreases round-off error C. allows equations with a noninvertible coefficient matrix to be solved

Justification/ Work ____

PS. The questions in this handout have been taken from tests from previous years

Conceptual Questions
Chapter 04.06 Gaussian Elimination with Partial Pivoting (Part 2)

Last Name _________ First Name________ Date ____ Group#_____ Last Name Initial__

3) Division by zero during forward elimination steps in *Gaussian elimination with partial pivoting* of the set of equations $[A][X]=[C]$ implies the coefficient matrix $[A]$.

Individual Attempt	**Group Attempt**
A. is invertible B. is not invertible C. cannot be determined to be invertible or not	A. is invertible B. is not invertible C. cannot be determined to be invertible or not

Justification/ Work ____

PS. The questions in this handout have been taken from tests from previous years

Free Response Questions
Chapter 04.06 Gauss Elimination

Chapter 04.06 (Set One)

1) Using forward elimination, find the determinant of the matrix below

$$\begin{bmatrix} 2 & 5 & 7 \\ 13 & 12 & 3 \\ 11 & 7 & 32 \end{bmatrix}$$

2) What is the value of a_{32} of the coefficient matrix A at the end of the first step of forward elimination of Gauss elimination with partial pivoting?

$$\begin{bmatrix} 5 & 6 & 10 \\ 8 & 12 & 11 \\ 16 & 4 & 13 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 21 \\ 19 \\ 37 \end{bmatrix}$$

3) What is the minimum number of zero elements in a 418×418 coefficient matrix at the end of 100 steps of forward elimination?

Chapter 04.06 (Set Two)

1) At the end of Gauss Elimination steps on a set of three equations, I obtain the following system of equations.

$$\begin{bmatrix} 10 & -7 & 0 \\ 0 & 2.567 & 5 \\ 0 & 0 & 6.022 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 7 \\ 2.5 \\ 6.012 \end{bmatrix}$$

Now using a computer that uses only **three** significant digits with **chopping**, what is the value of unknowns using back substitution? **Show all your intermediate work.**

2) At the *end of the first step* of forward elimination in the Gauss elimination with partial pivoting method algorithm, the equations obtained in matrix form on a given set of equations are as follows.

$$\begin{bmatrix} 2 & 4 & 6 & 10 \\ 0 & 16 & 24 & 16 \\ 0 & 32 & 42 & 17 \\ 0 & 24 & 36 & 29 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 1 \\ 24 \\ 64 \\ 96 \end{bmatrix}$$

Conduct only the *second step of forward elimination* of Gauss elimination with partial pivoting method and show the result in matrix form. **Show your work for full credit and put your final answer in the box.**

PS. The questions in this handout have been taken from tests from previous years

Free Response Questions
Chapter 04.06 Gauss Elimination

3) Find the determinant of this matrix by a method learned in this class (cofactor method is not allowed). **Show your work for full credit and put your final answer in the box.**

$$\begin{bmatrix} 2 & 4 & 6 & 10 \\ 0 & 16 & 24 & 16 \\ 0 & 32 & 42 & 17 \\ 0 & 24 & 36 & 29 \end{bmatrix}$$

Answers

Chapter 04.06 (Set One)

1) -1476.0
2) 4.75
3) 36750

Chapter 04.06 (Set Two)

1) $x_1 = 0.020$, $x_2 = -0.972$, $x_3 = 0.998$

2) $\begin{bmatrix} 2 & 4 & 6 & 10 \\ 0 & 32 & 42 & 17 \\ 0 & 0 & 3 & 7.5 \\ 0 & 0 & 4.5 & 16.25 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 1 \\ 64 \\ -8 \\ 48 \end{bmatrix}$

3) -960

PS. The questions in this handout have been taken from tests from previous years

Conceptual Questions
Chapter 04.07 LU Decomposition Part-1

Last Name _________ First Name________ Date ____ Group#_____ Last Name Initial__

1) LU decomposition method is computationally more efficient than Naïve Gauss elimination for solving

Individual Attempt	**Group Attempt**
A. a single set of simultaneous linear equations B. multiple sets of simultaneous linear equations with different coefficient matrices and same right hand side vectors. C. multiple sets of simultaneous linear equations with same coefficient matrix and different right hand side vectors	A. a single set of simultaneous linear equations B. multiple sets of simultaneous linear equations with different coefficient matrices and same right hand side vectors. C. multiple sets of simultaneous linear equations with same coefficient matrix and different right hand side vectors

Justification/ Work ____

Conceptual Questions
Chapter 04.07 LU Decomposition Part-1

Last Name _________ First Name________ Date ____ Group#_____ Last Name Initial__

2) LU decomposition method for solving a set of equations of form $[A][X]=[C]$ uses the following steps

Individual Attempt	**Group Attempt**
A. $[L][X]=[Z]$ followed by $[U][Z]=[C]$ B. $[L][Z]=[C]$ followed by $[U][X]=[Z]$ C. $[U][Z]=[C]$ followed by $[L][X]=[Z]$ D. $[U][X]=[Z]$ followed by $[L][Z]=[C]$	A. $[L][X]=[Z]$ followed by $[U][Z]=[C]$ B. $[L][Z]=[C]$ followed by $[U][X]=[Z]$ C. $[U][Z]=[C]$ followed by $[L][X]=[Z]$ D. $[U][X]=[Z]$ followed by $[L][Z]=[C]$

Justification/ Work ____

3) Given the decomposition of a square matrix $[A]=[L][U]$, where $[L]$ has 1's in the diagonal, the determinant of A is

Individual Attempt	**Group Attempt**
A. Product of diagonal elements of $[A]$ B. Product of diagonal elements of $[U]$ C. Sum of the diagonal elements of $[A]$ D. Sum of diagonal elements of $[U]$	A. Product of diagonal elements of $[A]$ B. Product of diagonal elements of $[U]$ C. Sum of the diagonal elements of $[A]$ D. Sum of diagonal elements of $[U]$

Justification/ Work ____

PS. The questions in this handout have been taken from tests from previous years

Conceptual Questions
Chapter 04.07 LU Decomposition Part-2

Last Name _________ First Name________ Date ____ Group#_____ Last Name Initial__

1) If you have a $n \times n$ matrix [A], and T is clock cycle time, the computation time for decomposing the matrix [A] to LU is approximately proportional to

Individual Attempt	**Group Attempt**
A. $\frac{8nT}{3}$ B. $\frac{8n^2T}{3}$ C. $\frac{8n^3T}{3}$	D. $\frac{8nT}{3}$ E. $\frac{8n^2T}{3}$ F. $\frac{8n^3T}{3}$

Justification/ Work ____

2) For a given 1700×1700 matrix [A], assume that it takes about 16 seconds to find the inverse of [A] by the use of the [L][U] decomposition method. The approximate time in seconds that all the forward substitutions take out of the 16 seconds is

Individual Attempt	**Group Attempt**
A. 4 B. 6 C. 8 D. 12	A. 4 B. 6 C. 8 D. 12

Justification/ Work ____

PS. The questions in this handout have been taken from tests from previous years

Conceptual Questions
Chapter 04.07 LU Decomposition Part-2

Last Name _________ First Name________ Date ____ Group#_____ Last Name Initial__

3) For a given 1700×1700 matrix $[A]$, assume that it takes about 16 seconds to find the inverse of $[A]$ by the use of the $[L][U]$ decomposition method. The approximate time in seconds that the LU decomposition takes out of the 16 seconds is

Individual Attempt	**Group Attempt**
A. 4 B. 6 C. 8 D. 12	A. 4 B. 6 C. 8 D. 12

Justification/ Work ____

PS. The questions in this handout have been taken from tests from previous years

Free Response Questions

Chapter 04.07 LU Decomposition Method

Chapter 04.07 (Set One)

1) A 2000×2000 matrix $[A]$ is decomposed into a LU form by using the forward elimination steps of Gaussian elimination. It is given that it takes 16 seconds of computational time to conduct the LU decomposition of $[A]$.

a) Find the approximate computational time in seconds it would take to do the LU decomposition of a 6000×6000 matrix $[B]$.

b) Find the approximate computational time it would take to find the inverse of the 2000×2000 matrix $[A]$ by using LU decomposition route.

2) Given the LU decomposition of a 3×3 matrix $[A]$ is

$$[A] = \begin{bmatrix} 1 & 4 & 6 \\ 4 & 21 & 32 \\ 5 & 30 & 59 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 5 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 4 & 6 \\ 0 & 5 & 8 \\ 0 & 0 & 13 \end{bmatrix}$$

find the second column of the inverse of $[A]$ using the LU decomposition method of solving simultaneous linear equations.

3) As part of finding the LU decomposition of a square matrix $[A]$,

$$[A] = \begin{bmatrix} 16 & 8 & 4 & 2 \\ 2 & 4 & 8 & 16 \\ 16 & 2 & 4 & 8 \\ 32 & 16 & 9 & 16 \end{bmatrix},$$

at the end of the 1st step of forward elimination, the matrix $[A]$ becomes

$$\begin{bmatrix} 16 & 8 & 4 & 2 \\ 0 & 3 & 7.5 & 15.75 \\ 0 & -6 & 0 & 6 \\ 0 & 0 & 1 & 12 \end{bmatrix}$$

The multipliers used to make a_{21}, a_{31}, and a_{41} zero in the 1st step of forward elimination were 0.1250, 1.000 and 2.000, respectively.

Find the $[L][U]$ decomposition of the $[A]$ matrix, given that the $[L]$ matrix has 1 as its diagonal elements.

PS. The questions in this handout have been taken from tests from previous years

Answers

Chapter 04.07 (Set One)

1 a) 432 s

b) 64 s

2) $\begin{bmatrix} -0.8615 \\ 0.4462 \\ -0.1538 \end{bmatrix}$

3) $[U]=\begin{bmatrix} 16 & 8 & 4 & 2 \\ 0 & 3 & 7.5 & 15.75 \\ 0 & 0 & 15 & 37.5 \\ 0 & 0 & 0 & 9.5 \end{bmatrix}, [L]=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.125 & 1 & 0 & 0 \\ 1 & -2 & 1 & 0 \\ 2 & 0 & 0.06666 & 1 \end{bmatrix}$

Chapter 04.07 (Set Two)

1) For a given 891×891 matrix A, assume that it takes about 2 seconds to find the inverse of A by the use of the LU decomposition method, that is, decomposing A once to LU, and then doing both forward substitution and back substitution 891 times by using the 891 columns of the identity matrix as the right-hand side vectors. What would be the approximate time (in seconds) that it will take to find the inverse by repeated use of Naïve gauss elimination method, that is, doing forward elimination and back substitution 891 times by using the 891 columns of the identity matrix as the right-hand side vectors?

Free Response Questions
Chapter 04.07 LU Decomposition Method

2) To solve a set of equations $[A][X]=[B]$ by LU decomposition requires back substitution $[U][X]=[Z]$ given by the algorithm below.

$x(n) = Z(n)/U(n,n)$
for $i=(n-1):-1:1$
$sum = Z(i)$
for $j=(i+1):1:n$
$sum = sum - U(i,j)*x(j)$
end
$x(i) = sum/U(i,i)$
end

If $n=141$, where n is the number of equations, what is the sum of the total number of multiplication and division operations in the algorithm?

3) Given the set of equations $[A][X]=[C]$

$$\begin{bmatrix} 9 & 5 & 7 \\ 13 & 37 & 3 \\ 11 & 7 & 29 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 22 \\ 15 \\ 16 \end{bmatrix}$$

What is the value of l_{32} of the $[A]=[L][U]$ decomposition with all 1's in the diagonal of the $[L]$ matrix?

Answers
Chapter 04.07 (Set Two)

1) 445.5
2) 10011
3) 0.0299