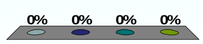
Simultaneous Linear Equations

http://nm.MathForCollege.com

Numerical Methods for the STEM undergraduate

The size of matrix $\begin{bmatrix} 4 & 6 & 7 & 8 \\ 9 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \end{bmatrix}$ is

1. 3×4
2. 4×3
3. 3×3
4. 4×4



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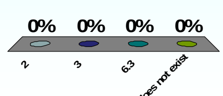
The c_{32} entity of the matrix

$$[C] = \begin{bmatrix} 4.1 & 61 & 7 & 8 \\ 9 & 2 & 3 & 4 \\ 5 & 6.3 & 7.2 & 8.9 \end{bmatrix}$$

1. 2
2. 3
3. 6.3
4. does not exist



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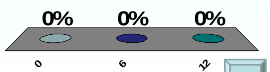
Given

$$[A]=\begin{bmatrix}3 & 6 & 2\\ 5 & -9 & 3\end{bmatrix} \quad [B]=\begin{bmatrix}2 & 6 & 3\\ -8 & 9.2 & 6\end{bmatrix}$$

then if [C]=[A]+[B], c_{12}=

1. 0
2. 6
3. 12



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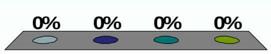
A square matrix [A] is lower triangular if

1. $a_{ij} = 0, i > j$
2. $a_{ij} = 0, j > i$
3. $a_{ij} \neq 0, i > j$
4. $a_{ij} \neq 0, j > i$



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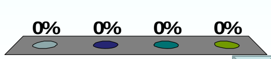
An identity matrix [I] needs to satisfy the following

1. $I_{ij} = 0, i \neq j$
2. $I_{ij} = 1, i = j$
3. matrix is square
4. all of the above



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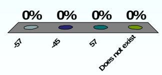
Given

$$[A]=\begin{bmatrix}4 & -6 & 3\\ 1 & 2 & -8\\ 6 & -5 & -9\end{bmatrix}, [B]=\begin{bmatrix}4 & 3\\ 9 & 7\\ 4 & -5\end{bmatrix}$$

then if $[C]=[A][B]$, then c_{31}=______.

1. -57
2. -45
3. 57
4. Does not exist



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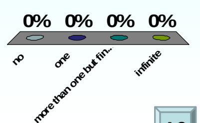
The following system of equations

$x + y=2$

$6x + 6y=12$

has __________solution(s).

1. no
2. one
3. more than one but finite number of
4. infinite

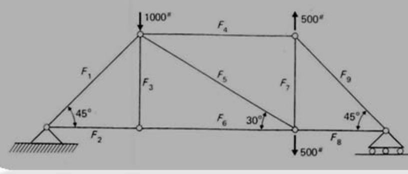


10

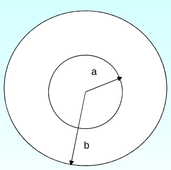
PHYSICAL PROBLEMS

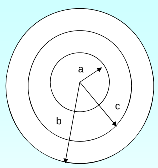
Truss Problem

$$\begin{bmatrix} 0.7071 & 0 & 0 & -1 & -0.8660 & 0 & 0 & 0 & 0 \\ 0.7071 & 0 & 1 & 0 & 0.5 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0.7071 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -0.7071 \\ 0 & 0 & 0 & 0 & 0.8660 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -0.5 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0.7071 \end{bmatrix} F = \begin{bmatrix} 0 \\ -1000 \\ 0 \\ 0 \\ 500 \\ 0 \\ 0 \\ -500 \\ 0 \end{bmatrix}$$



Pressure vessel problem





$$u_1 = c_1 r + \frac{c_2}{r}$$

$$u_2 = c_3 r + \frac{c_4}{r}$$

$$\begin{bmatrix} 4.2857\times10^7 & -9.2307\times10^5 & 0 & 0 \\ 4.2857\times10^7 & -5.4619\times10^5 & -4.2857\times10^7 & 5.4619\times10^5 \\ -6.5 & -0.15384 & 6.5 & 0.15384 \\ 0 & 0 & 4.2857\times10^7 & -3.6057\times10^5 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} -7.887\times10^3 \\ 0 \\ 0.007 \\ 0 \end{bmatrix}$$

Polynomial Regression

We are to fit the data to the polynomial regression model

$$\alpha = a_0 + a_1 T + a_2 T^2$$

$$\begin{bmatrix} n & \left(\sum_{i=1}^{n} T_i\right) & \left(\sum_{i=1}^{n} T_i^2\right) \\ \left(\sum_{i=1}^{n} T_i\right) & \left(\sum_{i=1}^{n} T_i^2\right) & \left(\sum_{i=1}^{n} T_i^3\right) \\ \left(\sum_{i=1}^{n} T_i^2\right) & \left(\sum_{i=1}^{n} T_i^3\right) & \left(\sum_{i=1}^{n} T_i^4\right) \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{n} \alpha_i \\ \sum_{i=1}^{n} T_i\,\alpha_i \\ \sum_{i=1}^{n} T_i^2 \alpha_i \end{bmatrix}$$

12

Simultaneous Linear Equations

Gaussian Elimination

(Naïve and the Not That So Innocent Also)

http://numericalmethods.eng.usf.edu

Transforming Numerical Methods Education for the STEM undergraduate

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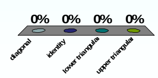
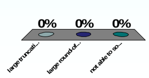
The goal of forward elimination steps in Naïve Gauss elimination method is to reduce the coefficient matrix to a (an) ________________ matrix.

1. diagonal
2. identity
3. lower triangular
4. upper triangular



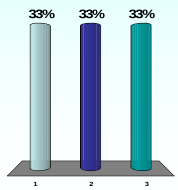
One of the pitfalls of Naïve Gauss Elimination method is

1. large truncation error
2. large round-off error
3. not able to solve equations with a noninvertible coefficient matrix



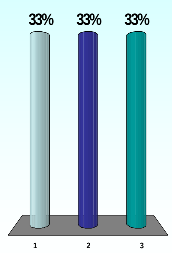
Increasing the precision of numbers from single to double in the Naïve Gaussian elimination method

1. avoids division by zero
2. decreases round-off error
3. allows equations with a noninvertible coefficient matrix to be solved



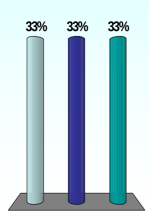
Division by zero during forward elimination steps in *Naïve Gaussian elimination* for [A][X]=[C] implies the coefficient matrix [A]

1. is invertible
2. is not invertible
3. cannot be determined to be invertible or not



Division by zero during forward elimination steps in *Gaussian elimination with partial pivoting* of the set of equations [A][X]=[C] implies the coefficient matrix [A]

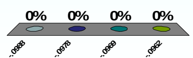
1. is invertible
2. is not invertible
3. cannot be determined to be invertible or not



Using 3 significant digit with *chopping* at all stages, the result for the following calculation is

$$x_1 = \frac{6.095 - 3.456 \times 1.99}{8}$$

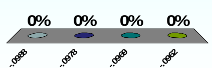
A. -0.0988
B. -0.0978
C. -0.0969
D. -0.0962



Using 3 significant digits with *rounding-off* at all stages, the result for the following calculation is

$$x_1 = \frac{6.095 - 3.456 \times 1.99}{8}$$

A. -0.0988
B. -0.0978
C. -0.0969
D. -0.0962



Simultaneous Linear Equations

LU Decomposition

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Truss Problem

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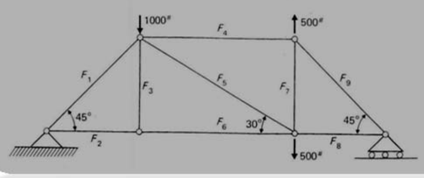
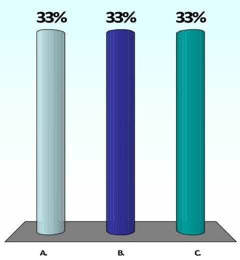


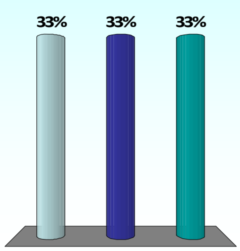
If you have n equations and n unknowns, the computation time for forward substitution is approximately proportional to

A. $4n$
B. $4n^2$
C. $4n^3$



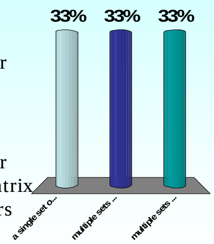
If you have a nxn matrix, the computation time for decomposing the matrix to LU is approximately proportional to

A. $8n/3$
B. $8n^2/3$
C. $8n^3/3$



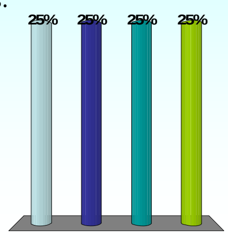
LU decomposition method is computationally more efficient than Naïve Gauss elimination for solving

1. a single set of simultaneous linear equations
2. multiple sets of simultaneous linear equations with different coefficient matrices and same right hand side vectors.
3. multiple sets of simultaneous linear equations with same coefficient matrix and different right hand side vectors



For a given 1700 x 1700 matrix [A], assume that it takes about 16 seconds to find the inverse of [A] by the use of the [L][U] decomposition method. Now you try to use the Gaussian Elimination method to accomplish the same task. It will now take approximately ____ seconds.

A. 4
B. 64
C. 6800
D. 27200



For a given 1700 x 1700 matrix [A], assume that it takes about 16 seconds to find the inverse of [A] by the use of the [L][U] decomposition method. The approximate time in seconds that all the forward substitutions take out of the 16 seconds is

1. 4
2. 6
3. 8
4. 12