Interpolation

Reading Between the Lines

WHAT IS INTERPOLATION ?

Given (x_0,y_0), (x_1,y_1), …… (x_n,y_n), find the value of 'y' at a value of 'x' that is not given.

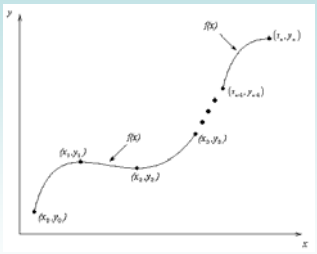


Figure Interpolation of discrete data.

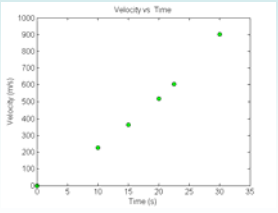
APPLIED PROBLEMS

FLY ROCKET FLY, FLY ROCKET FLY

The upward velocity of a rocket is given as a function of time in table below. Find the velocity and acceleration at t=16 seconds.

Table Velocity as a function of time.

t, (s)	$v(t)$, (m/s)
0	0
10	227.04
15	362.78
20	517.35
22.5	602.97
30	901.67

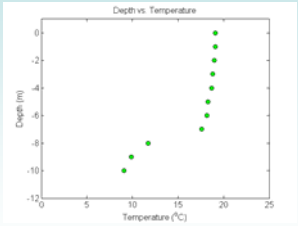


Velocity vs. time data for the rocket example

Bass Fishing Gets Technical

To maximize a catch of bass in a lake, it is suggested to throw the line to the depth of the thermocline. The characteristic feature of this area is the sudden change in temperature..

Temperature T (°C)	Depth z (m)
19.1	0
19.1	-1
19	-2
18.8	-3
18.7	-4
18.3	-5
18.2	-6
17.6	-7
11.7	-8
9.9	-9
9.1	-10



Temperature vs. Depth of a Lake

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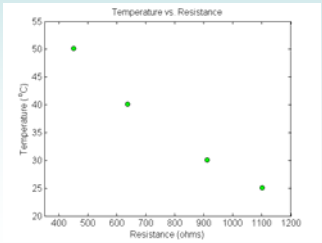
Thermistor Calibration

Thermistors are based on change in resistance of a material with temperature. A manufacturer of thermistors makes the following observations on a thermistor. Determine the calibration curve for thermistor.

$$\frac{1}{T} = a_0 + a_1[\ln R] + a_1[\ln R]^2 + a_1[\ln R]^3$$

R (Ω)	T(°C)
1101.0	25.113
911.3	30.131
636.0	40.120
451.1	50.128



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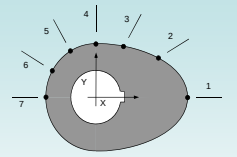
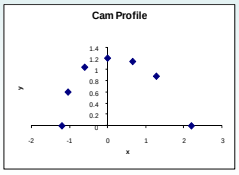
Following the Cam

A curve needs to be fit through the given points to fabricate the cam.

Point	x (in.)	y (in.)
1	2.20	0.00
2	1.28	0.88
3	0.66	1.14
4	0.00	1.20
5	–0.60	1.04
6	–1.04	0.60
7	–1.20	0.00





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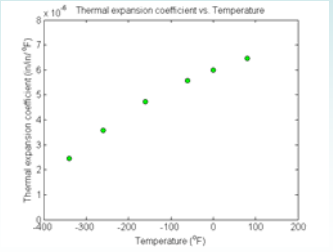
Thermal Expansion Coefficient Profile

A trunnion is cooled 80°F to − 108°F. Given below is the table of the coefficient of thermal expansion vs. temperature. Determine the coefficient of thermal expansion profile as a function of temperature.

Temperature (°F)	Thermal Expansion Coefficient (in/in/°F)
80	6.47×10^{-6}
0	6.00×10^{-6}
−60	5.58×10^{-6}
−160	4.72×10^{-6}
−260	3.58×10^{-6}
−340	2.45×10^{-6}



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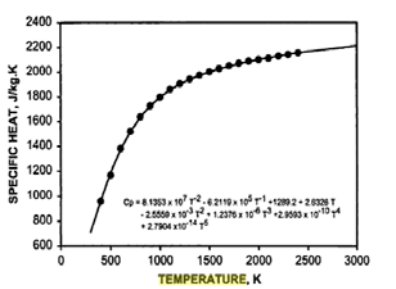
Specific Heat of Carbon

A graphite block needs to be pyrolized by heating it up from room temperature of 300K to 1800K. How much heat is required to do so?

Temperature (K)	Specific Heat (J/kg-K)
200	420
400	1070
600	1370
1000	1820
1500	2000
2000	2120

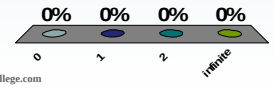


THE END

BACKGROUND OF INTERPOLATION

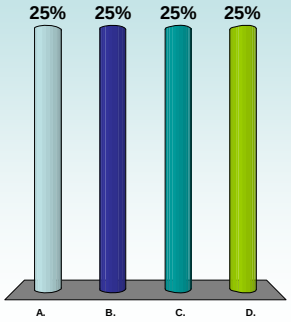
The number of different polynomials that can go though two fixed points (x_1,y_1) and (x_2,y_2) is

A. 0
B. 1
C. 2
D. infinite



Given n+1 data points, a unique polynomial of degree _____ passes through the n+1 data points

A. n+1
B. n+1 or less
C. n
D. n or less



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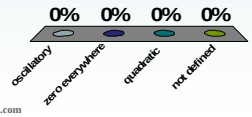
If a polynomial of degree n has more than n zeros, then the polynomial is

A. oscillatory
B. zero everywhere
C. quadratic
D. not defined



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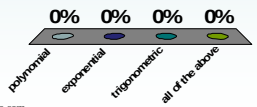
The following type of functions can be used for interpolation

A. polynomial
B. exponential
C. trigonometric
D. all of the above



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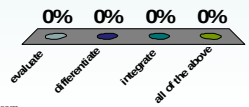
Polynomials are most commonly used functions for interpolation because they are easy to

A. evaluate
B. differentiate
C. integrate
D. all of the above



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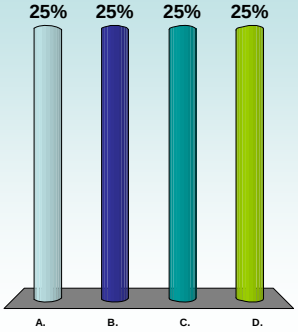
The length of a straight line path from (1, 2.2) to (4, 6.2) is

A. 3.0
B. 4.0
C. 5.0
D. 25.0



THE END

DIRECT METHOD

The following *x*-*y* data is given

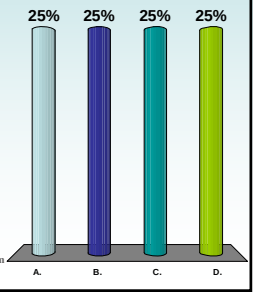
x	15	18	22
y	24	37	25

A first order polynomial is chosen as an interpolant for the first two data points as

$$b_0 + b_1(x-15), 15 \le x \le 18$$

The value of b_1 is most nearly

A. -1.048
B. 0.1433
C. 4.333
D. 24.00



The polynomial that passes through the following *x*-*y* data

x	18	22	24
y	24	25	123

is given by

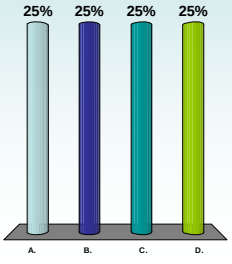
$$8.125x^2 - 324.75x + 3237,\ 18 \le x \le 24$$

The corresponding polynomial using Newton's divided difference polynomial method is given by

$$b_0 + b_1(x-18) + b_2(x-18)(x-22),\ 18 \le x \le 24$$

The value of b_2 is

A. 0.2500
B. 8.125
C. 24.00
D. not obtainable with the information given

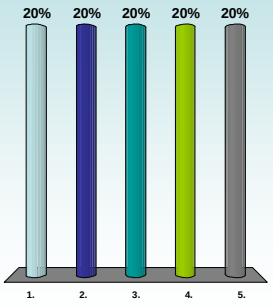


THE END

SPLINE INTERPOLATION

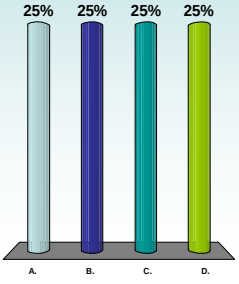
Spring Break is here soon. Rate your answer to this question - Will you miss coming to class during Spring Break week?

1. Strongly agree
2. Agree
3. Take the 5th
4. Disagree
5. Strongly disagree



Given n data points of y vs x for conducting quadratic spline interpolation, the x-data needs to be

A. equally spaced
B. in ascending or descending order
C. integers
D. positive



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A robot path on an x-y plane is found by interpolating 3 data points given below.

x	4	6	7
y	42	22	15

The interpolant is $y(x) = x^2 - 20x + 106,\ 4 \le x \le 7$

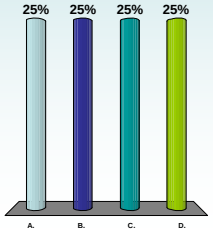
The length of the path from x=4 to x=7 is

A. $\sqrt{(6-4)^2+(22-42)^2}+\sqrt{(7-6)^2+(15-22)^2}$

B. $\int_4^7 \sqrt{1+(x^2-20x+106)^2}\,dx$

C. $\int_4^7 \sqrt{1+(2x-20)^2}\,dx$

D. $\int_4^7 (x^2-20x+106)dx$



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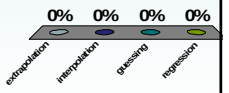
Given n+1 data points $(x_0,y_0),(x_1,y_1),\ldots,(x_{n-1},y_{n-1}),\ (x_n,y_n)$, and assume you pass a function $f(x)$ through all the data points. If now the value of the function $f(x)$ is required to be found outside the range of given x-data, the procedure is called

A. extrapolation
B. interpolation
C. guessing
D. regression



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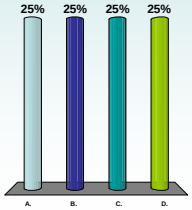
In quadratic spline interpolation,

A. the first derivatives of the splines are continuous at the interior data points
B. the second derivatives of the splines are continuous at the interior data points
C. the first or the second derivatives of the splines are continuous at the interior data points
D. the first and second derivatives are continuous at the interior data points



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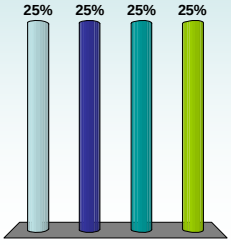
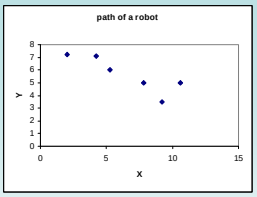
In cubic spline interpolation

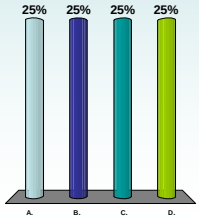
A. the first derivatives of the splines are continuous at the interior data points
B. the second derivatives of the splines are continuous at the interior data points
C. the first and the second derivatives of the splines are continuous at the interior data points
D. the first or the second derivatives of the splines are continuous at the interior data points



A robot needs to follow a path that passes through six points as shown in the figure. To find the shortest path that is also smooth you would recommend



A. Pass a 5th order polynomial through the data
B. Pass linear splines through the data
C. Pass quadratic splines through the data
D. Regress the data to a 2nd order polynomial



THE END

BONUS QUESTIONS ON INTERPOLATION

The following incomplete y vs. x data is given

x	1	2	4	6	7
y	5	11	????	????	32

The data is fit by quadratic spline interpolants given by

$f(x) = ax - 1, 1 \le x \le 2$

$f(x) = -2x^2 + 14x - 9, 2 \le x \le 4$

$f(x) = bx^2 + cx + d, 4 \le x \le 6$

$f(x) = 25x^2 - 303x + 928, 6 \le x \le 7$

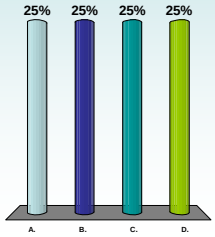
At $x=6$, the first derivative is continuous gives the equation

A. $2bx + c = 50x - 303$

B. $12b + c = -3$

C. $36b + 6c + d = 10$

D. $36x^2 + 6x + d = 25x^2 - 303x + 928$



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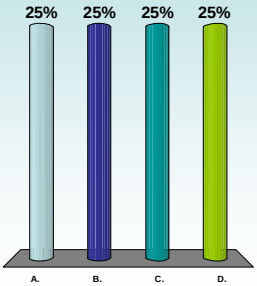
Given three data points (1,6), (3,28), (10,231), it is found that the function $y=2x^2+3x+1$ passes through the three data points. Your estimate of y at $x=2$ is most nearly

A. 6

B. 15

C. 17

D. 28



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The following data of the velocity of a body is given as a function of time

Time (s)	4	6	7	8	11
Velocity (m/s)	42	22	15	12	10

Using quadratic interpolation, the interpolant

$$v(t) = t^2 - 20t + 106, \quad 4 \le t \le 7,$$

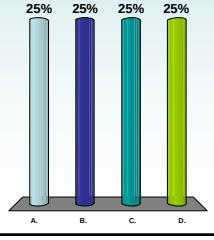
approximates the velocity of the body from t=4 to t=7 s. From this information, at what time in seconds is the velocity of the body 20 m/s

A. 6.26

B. 6.29

C. 6.44

D. cannot be found



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The following incomplete y vs. x data is given

x	1	2	4	6	7
y	5	11	????	????	32

The data is fit by quadratic spline interpolants given by

$f(x) = ax - 1, 1 \le x \le 2$

$f(x) = -2x^2 + 14x - 9, 2 \le x \le 4$

$f(x) = bx^2 + cx + d, 4 \le x \le 6$

$f(x) = 25x^2 - 303x + 928, 6 \le x \le 7$

where a, b, c, d, e, f, g are constants.

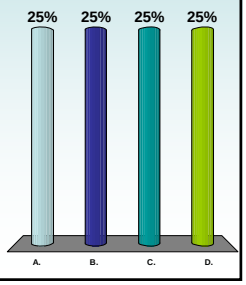
What is the value of $\int_{1.5}^{3.5} f(x)dx$?

A. 23.50

B. 25.67

C. 26.42

D. 28.00



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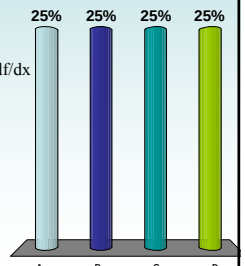
$f(x) = -2x^2 + 14x - 9, 2 \le x \le 4$

$f(x) = bx^2 + cx + d, 4 \le x \le 6$

$f(x) = ex^2 - fx + g, 6 \le x \le 7$

where a, b, c, d, e, f, g are constants. The value of df/dx at x=2.6 most nearly is

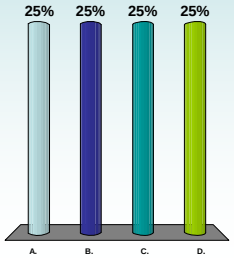
A. -144.5
B. -4.000
C. 3.600
D. 12.20



The following velocity vs time data is given. To find the velocity at t=14.9s, the three time data points you would choose for second order polynomial interpolation are

Time (s)	0	15	18	22	24
Velocity (m/s)	22	24	37	25	123

A. 0, 15, 18
B. 15, 18, 22
C. 0, 15, 22
D. 0, 18, 24



The data of velocity vs time is given.
The velocity in m/s at t=16s using linear interpolation is

Time (s)	0	15	18	22	24
Velocity (m/s)	22	24	37	25	123

A. 27.867
B. 28.333
C. 30.429
D. 43.000