

# Integration

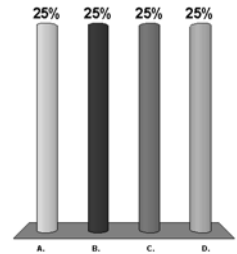
$$\int_0^1 \sqrt{1-x^2} dx$$

This is not your father's area?

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The economy is so bad that the following is happening with Snap, Crackle and Pop

- A. They are thinking of replacing all three of them with Pow
- B. Kelloggs hired a "cereal" killer to kill them all
- C. Snap is spreading rumors that "Pop was a rolling stone"
- D. They are selling smack, crack and pot, respectively.



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Ask me what I should already know  
The pre-requisite questions

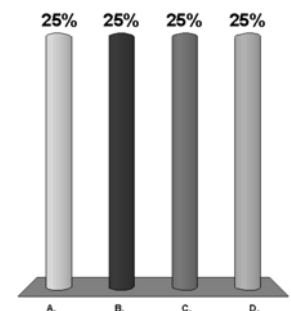
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The velocity of a body is given as  $v(t)=t^2$ . Given that

$$\int_3^6 t^2 dt = 63$$

the location of the body at  $t=6$  is

- A. 27
- B. 54
- C. 63
- D. Cannot be determined



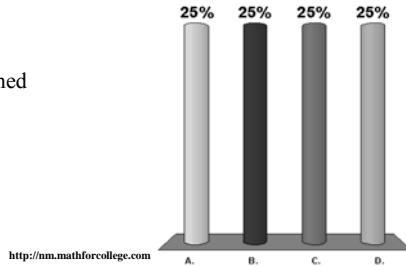
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The velocity of a body is given as  $v(t)=t^2$ . Given that

$$\int_3^6 t^2 dt = 63$$

the distance covered by the body between  $t=3$  and  $t=6$  is

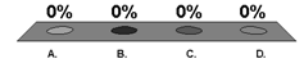
- A. 27
- B. 54
- C. 63
- D. Cannot be determined



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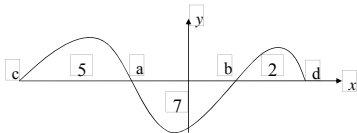
The exact mean value of the function  $f(x)$  from  $a$  to  $b$  is

- A.  $\frac{f(a)+f(b)}{2}$
- B.  $\frac{f(a)+2f\left(\frac{a+b}{2}\right)+f(b)}{4}$
- C.  $\int_a^b f(x)dx$
- D.  $\frac{\int_a^b f(x)dx}{(b-a)}$

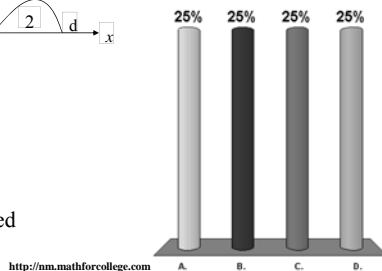


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Given the  $f(x)$  vs  $x$  curve, and the magnitude of the areas as shown, the value of  $\int_c^b f(x)dx$

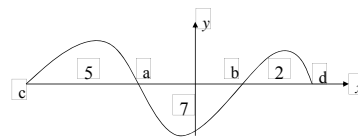


- A. -2
- B. 2
- C. 12
- D. Cannot be determined

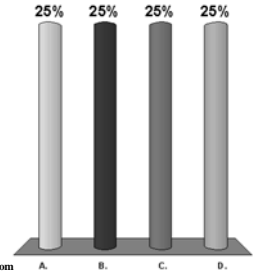


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Given the  $f(x)$  vs  $x$  curve, and the magnitude of the areas as shown, the value of  $\int_b^c f(x)dx$



- A. -2
- B. 2
- C. 12
- D. Cannot be determined



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Given the  $f(x)$  vs  $x$  curve, and the magnitude of the areas as shown, the value of  $\int_0^a f(x) dx$

A. 5  
B. 12  
C. 14  
D. Cannot be determined

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Given the  $f(x)$  vs  $x$  curve, and the magnitude of the areas as shown, the value of  $\int_0^b f(x) dx$

A. -7  
B. -2  
C. 12  
D. Cannot be determined

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Given the  $f(x)$  vs  $x$  curve, and the magnitude of the areas as shown, the value of  $\int_a^b f(x) dx$

A. -7  
B. -2  
C. 7  
D. 12

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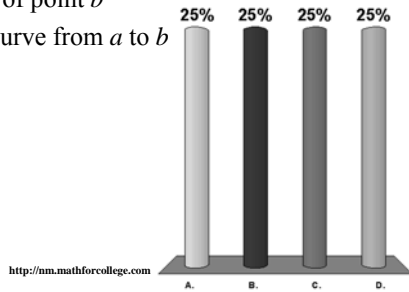
The area of a circle of radius  $a$  can be found by the following integral

A.  $\int_0^a (a^2 - x^2) dx$   
 B.  $\int_0^{2\pi} \sqrt{a^2 - x^2} dx$   
 C.  $4 \int_0^a \sqrt{a^2 - x^2} dx$   
 D.  $\int_0^a \sqrt{a^2 - x^2} dx$

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Physically, integrating  $\int_a^b f(x) dx$  means finding the

- A. Area under the curve from  $a$  to  $b$
- B. Area to the left of point  $a$
- C. Area to the right of point  $b$
- D. Area above the curve from  $a$  to  $b$



## PHYSICAL EXAMPLES

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### Distance Covered By Rocket



$$x = \int_{t_0}^{t_1} \left[ u \log_e \left( \frac{m_0}{m_0 - qt} \right) - gt \right] dt$$

$$x = \int_8^{30} \left( 2000 \ln \left[ \frac{140000}{140000 - 2100t} \right] - 9.8t \right) dt$$

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### Concentration of Benzene



$$c(x,t) = \frac{c_0}{2} \left[ \operatorname{erfc} \left( \frac{x-ut}{2\sqrt{Dt}} \right) + e^{\frac{ux}{D}} \operatorname{erfc} \left( \frac{x+ut}{2\sqrt{Dt}} \right) \right]$$

$u$  = velocity of ground water flow  
in the  $x$ -direction (m/s)

$D$  = dispersion coefficient ( $\text{m}^2$ )

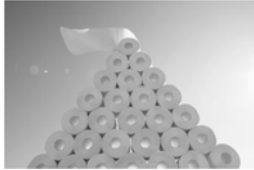
$C_0$  = initial concentration ( $\text{kg}/\text{m}^3$ )

$$\operatorname{erfc}(x) = \int_{\infty}^x e^{-z^2} dz$$

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Is Wal\*\*\* “short shifting” you?

$$P(y \geq a) = \int_a^{\infty} f(y) dy = \int_a^{\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{y-\mu}{\sigma}\right)^2} dy$$



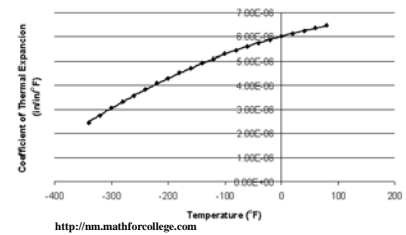
Roll	Number of sheets
1	253
2	250
3	251
4	252
5	253
6	253
7	252
8	254
9	252
10	252

$$P(y \geq 250) = \int_{250}^{\infty} 0.3515 e^{-0.3881(y-252)^2} dy$$

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## Calculating diameter contraction for trunnion-hub problem

$$\Delta D = D \int_{T_{room}}^{T_{fluid}} \alpha dT$$



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## 7.02 Trapezoidal Rule

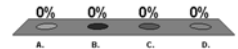
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The morning after learning  
trapezoidal rule...

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You are happy because

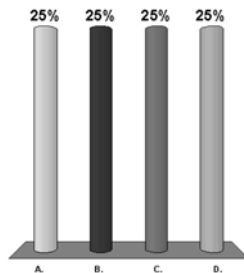
- A. You are thinking about your free will
- B. You are delusional
- C. You have to see my pretty face for only three more weeks
- D. All of the above



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Two-segment trapezoidal rule of integration is exact for integration of polynomials of order of at most

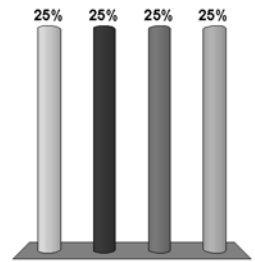
- A. 1
- B. 2
- C. 3
- D. 4



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In trapezoidal rule, the number of segments needed to get the exact value for a general definite integral

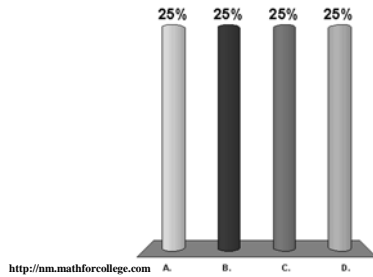
- A. 1
- B. 2
- C. 1 googol
- D. infinite



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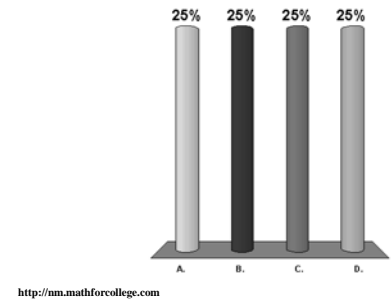
In trapezoidal rule, the number of points at which function is evaluated for 8 segments is

- A. 8
- B. 9
- C. 16
- D. 17



In trapezoidal rule, the number of function evaluations for 8 segments is

- A. 8
- B. 9
- C. 16
- D. 17

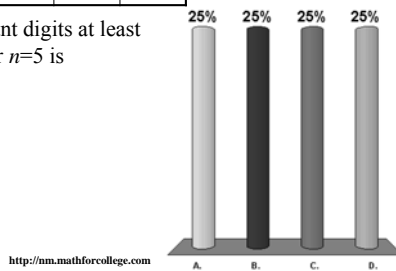


The distance covered by a rocket from  $t=8$  to  $t=34$  seconds is calculated using multiple segment trapezoidal rule by integrating a velocity function. Below is given the estimated distance for different number of segments,  $n$ .

$n$	1	2	3	4	5
Value	16520	15421	15212	15138	15104

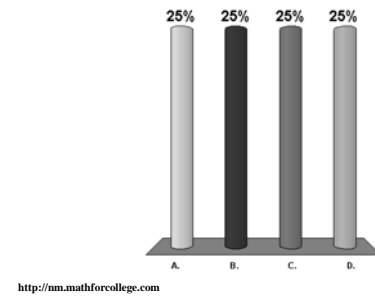
The number of significant digits at least correct in the answer for  $n=5$  is

- A. 1
- B. 2
- C. 3
- D. 4



The value of  $\int_5^9 x^2 dx$  by using one segment trapezoidal rule is most nearly

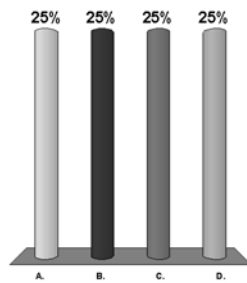
- A. 201.33
- B. 212.00
- C. 424.00
- D. 742.00



The velocity vs time is given below. A good estimate of the distance in meters covered by the body between  $t=0.5$  and 1.2 seconds is

$t(s)$	0	0.5	1.2	1.5	1.8
$v(m/s)$	0	213	256	275	300

- A.  $213 \cdot 0.7$
- B.  $256 \cdot 0.7$
- C.  $256 \cdot 1.2 - 213 \cdot 0.5$
- D.  $\frac{1}{2} \cdot (213 + 256) \cdot 0.7$



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## 7.05 Gauss Quadrature Rule

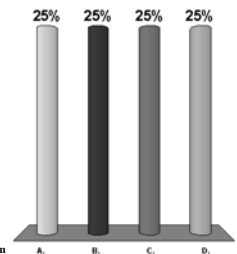
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The morning after learning  
Gauss Quadrature Rule...

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Autar Kaw is looking for a stage name.  
Please vote your choice.

- A. The last mindbender
- B. K (formerly known as Kaw)
- C. Kid Cuddi
- D. Kaw & Saki



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$\int_5^{10} f(x) dx$  is exactly

A.  $\int_{-1}^1 f(2.5x+7.5) dx$       25%    25%    25%    25%

B.  $2.5 \int_{-1}^1 f(2.5x+7.5) dx$

C.  $5 \int_{-1}^1 f(5x+5) dx$

D.  $5 \int_{-1}^1 (2.5x+7.5) f(x) dx$

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A scientist would derive one-point Gauss Quadrature Rule based on getting exact results of integration for function  $f(x)=a_0+a_1x$ . The one-point rule approximation for the integral  $\int f(x) dx$  is

A.  $\frac{b-a}{2}[f(a)+f(b)]$       20%    20%    20%    20%    20%

B.  $(b-a)f\left(\frac{a+b}{2}\right)$

C.  $\frac{b-a}{2} \left[ f\left(\frac{b-a}{2}\left\{\frac{-1}{\sqrt{3}}\right\} + \frac{b+a}{2}\right) + f\left(\frac{b-a}{2}\left\{\frac{1}{\sqrt{3}}\right\} + \frac{b+a}{2}\right) \right]$

D.  $(b-a)f(a)$

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For integrating any first order polynomial, the one-point Gauss quadrature rule will give the same results as

A. 1-segment trapezoidal rule      25%    25%    25%    25%

B. 2-segment trapezoidal rule

C. 3-segment trapezoidal rule

D. All of the above

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A scientist can derive a one-point quadrature rule for integrating definite integrals based on getting exact results of integration for the following function

A.  $a_0+a_1x+a_2x^2$

B.  $a_1x+a_2x^2$

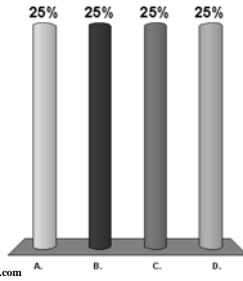
C.  $a_1x$

D.  $a_2x^2$

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For integrating any third order polynomial, the two-point Gauss quadrature rule will give the same results as

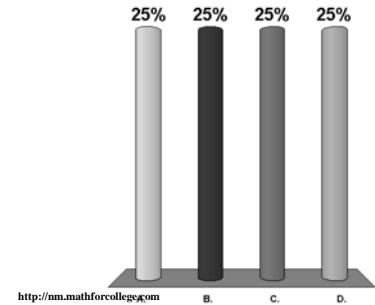
- A. 1-segment trapezoidal rule
- B. 2-segment trapezoidal rule
- C. 3-segment trapezoidal rule
- D. None of the above



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The highest order of polynomial for which the  $n$ -point Gauss-quadrature rule would give an exact integral is

- A.  $n$
- B.  $n+1$
- C.  $2n-1$
- D.  $2n$



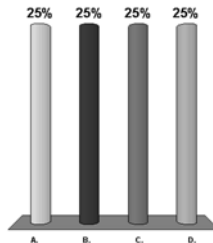
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You are asked to estimate the water flow rate in a pipe of radius 2m at a remote area location with a harsh environment. You already know that velocity varies along the radial location, but do not know how it varies. The flow rate,  $Q$  is given by

$$Q = \int_0^2 2\pi r V dr$$

To save money, you are allowed to put only two velocity probes (these probes send the data to the central office in New York, NY via satellite) in the pipe. Radial location,  $r$  is measured from the center of the pipe, that is  $r=0$  is the center of the pipe and  $r=2$ m is the pipe radius. The radial locations you would suggest for the two velocity probes for the most accurate calculation of the flow rate are

- A. 0, 2
- B. 1, 2
- C. 0, 1
- D. 0.42, 1.58



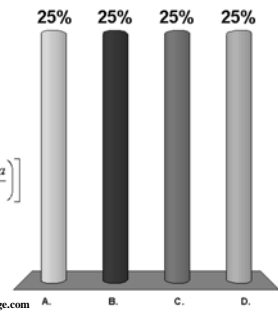
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A scientist an approximate formula for integration as

$$\int_a^b f(x) dx \approx c_1 f(x_1), \text{ where } a \leq x_1 \leq b$$

The values of  $c_1$  and  $x_1$  are found by assuming that the formula is exact for the functions of the form  $a_n x^n + a_1 x^2$  polynomial. Then the resulting formula would be exact for integration.

- A.  $\frac{b-a}{2} [f(a) + f(b)]$
- B.  $(b-a) f\left(\frac{a+b}{2}\right)$
- C.  $\frac{b-a}{2} \left[ f\left(\frac{b-a}{2} \left[\frac{-1}{\sqrt{3}}\right] + \frac{b+a}{2}\right) + f\left(\frac{b-a}{2} \left[\frac{1}{\sqrt{3}}\right] + \frac{b+a}{2}\right) \right]$
- D.  $(b-a) f(a)$



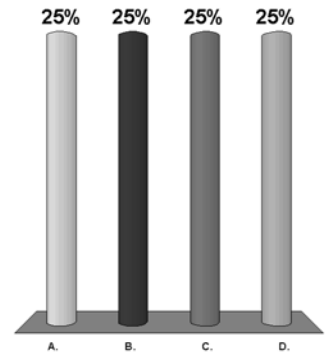
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END

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The exact value of  $\int_{0.2}^{2.2} xe^x dx$  most nearly is

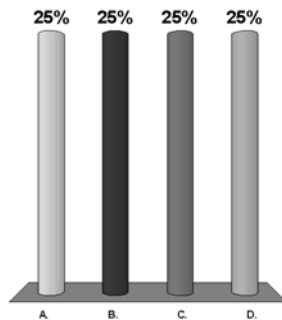
- A. 7.8036
- B. 11.807
- C. 14.034
- D. 19.611



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Velocity distribution of a fluid flow through a pipe varies along the radius, and is given by  $v(r)$ . The flow rate through the pipe of radius  $a$  is given by

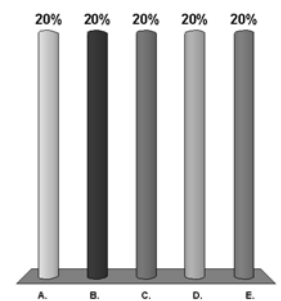
- A.  $\pi v(a)a^2$
- B.  $\pi \frac{v(0) + v(a)}{2} a^2$
- C.  $\int_0^a v(r) dr$
- D.  $2\pi \int_0^a v(r)r dr$



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The value of the integral  $\int x^2 dx$

- A.  $x^3$
- B.  $x^3 + C$
- C.  $x^3/3$
- D.  $x^3/3 + C$
- E.  $2x$



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