Ordinary Differential Equations

Everything is ordinary about them

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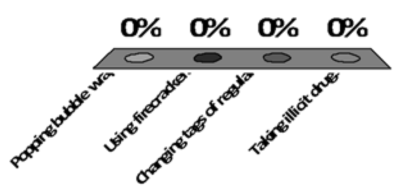
Popping tags means

A. Popping bubble wrap
B. Using firecrackers
C. Changing tags of regular items in a store with tags from clearance items
D. Taking illicit drugs



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Physical Examples

How long will it take to cool the trunnion?

$$mc\frac{d\theta}{dt} = -hA(\theta - \theta_a), \ \theta(0) = \theta_{room}$$

END

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What did I learn in the ODE class?

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In the differential equation

$$\frac{dy}{dx}+3y=e^{-x},\ y(0)=6$$

the variable x is the ______variable

A. Independent

B. Dependent

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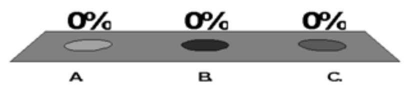
Classify the differential equation

$$2\frac{dy}{dx}+4y=e^{x}+3,\ y(0)=5$$

A. linear

B. nonlinear

C. undeterminable to be linear or nonlinear



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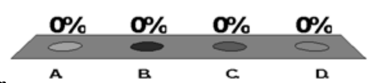
Classify the differential equation

$$2\frac{dy}{dx} + xy = e^x + 3, y(0) = 5$$

A. linear
B. nonlinear
C. linear with fixed constants
D. undeterminable to be linear or nonlinear



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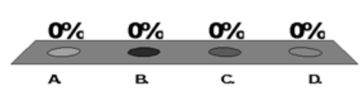
Classify the differential equation

$$2\frac{dy}{dx} + y^2 = e^x + 3, y(0) = 5$$

A. linear
B. nonlinear
C. linear with fixed constants
D. undeterminable to be linear or nonlinear



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The velocity of a body is given by

$$v(t) = e^{2t} + 5, t \geq 0$$

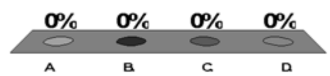
Then the distance covered by the body from *t=0* to *t=10* can be calculated by solving the differential equation for *x*(10) for

A. $\dfrac{dx}{dt} = e^{2t} + 5, x(0) = 0$

B. $\dfrac{dx}{dt} = e^{2t} + 5, x(0) = 5$

C. $\dfrac{dx}{dt} = 2e^{2t}, x(0) = 0$

D. $\dfrac{dx}{dt} = \dfrac{e^{2t}}{2} + 5t, x(0) = 0$



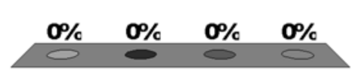
The form of the exact solution to $2\dfrac{dy}{dx} + 3y = e^{-x}, y(0) = 5$ is

A. $Ae^{-1.5x} + Be^{-x}$

B. $Ae^{-1.5x} + Bxe^{-x}$

C. $Ae^{1.5x} + Be^{-x}$

D. $Ae^{1.5x} + Bxe^{-x}$



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In the differential equation

$$\frac{dy}{dx} + 3y = e^{-x}, y(0) = 6$$

the variable *y* is the ______variable

A. Independent
B. Dependent

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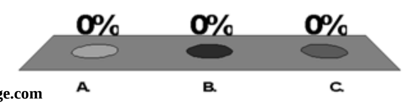
Ordinary differential equations can have these many **dependent** variables.

A. one
B. two
C. any positive integer



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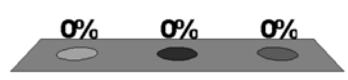
Ordinary differential equations can have these many **independent** variables.

A. one
B. two
C. any positive integer



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8.03
Euler's Method

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Euler's method of solving ordinary differential equations

$\frac{dy}{dx} = f(x,y), y(0) = 0$ states

A. $y_{i+1} = y_i + f(x,y)h$

B. $y_{i+1} = y_i + f(x_i,y_i)h$

C. $y_{i+1} = y_i + f(x_i,y_i)$

D. $y_{i+1} = f(x_i,y_i)h$

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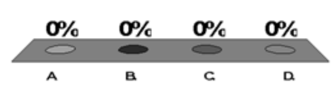


To solve the ordinary differential equation

$$3\frac{dy}{dx}+5y^2=\sin x,\ y(0)=5,$$

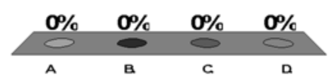
by Euler's method, you need to rewrite the equation as

A. $\dfrac{dy}{dx}=\sin x-5y^2,\ y(0)=5$

B. $\dfrac{dy}{dx}=\dfrac{1}{3}(\sin x-5y^2),\ y(0)=5$

C. $\dfrac{dy}{dx}=\dfrac{1}{3}\left(-\cos x-\dfrac{5y^3}{3}\right),\ y(0)=5$

D. $\dfrac{dy}{dx}=\dfrac{1}{3}\sin x,\ y(0)=5$



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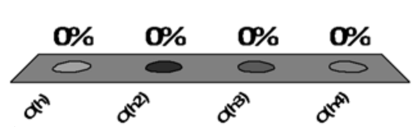
The order of accuracy for a single step in Euler's method is

A. O(h)
B. O(h^2)
C. O(h^3)
D. O(h^4)



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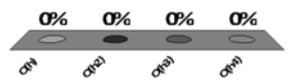
The order of accuracy from initial point to final point while using more than one step in Euler’s method is

A. O(h)
B. O(h^2)
C. O(h^3)
D. O(h^4)



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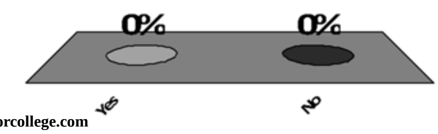
Do you know how Runge- Kutta 4th Order Method works?

A. Yes
B. No
C. Maybe
D. I take the 5th



RUNGE-KUTTA 4TH ORDER METHOD

Runge-Kutta 4th Order Method

$$\frac{dy}{dx} = f(x, y), y(0) = y_0$$

$$y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)h$$

$$k_1 = f(x_i, y_i)$$

$$k_2 = f\left(x_i + \frac{1}{2}h, y_i + \frac{1}{2}k_1 h\right)$$

$$k_3 = f\left(x_i + \frac{1}{2}h, y_i + \frac{1}{2}k_2 h\right)$$

$$k_4 = f(x_i + h, y_i + k_3 h)$$

END

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Physical Examples

Ordinary Differential Equations

Problem:

The trunnion initially at room temperature is put in a bath of dry-ice/alcohol. How long do I need to keep it in the bath to get maximum contraction ("within reason")?

Assumptions

The trunnion is a lumped mass system.

a. What does a lumped system mean? It implies that the internal conduction in the trunnion is large enough that the temperature throughout the ball is uniform.
b. This allows us to make the assumption that the temperature is only a function of time and not of the location in the trunnion.

Energy Conservation

Heat In – Heat Lost = Heat Stored

Heat Lost

Rate of heat lost
due to convection= $hA(T-T_a)$

h = convection coefficient ($W/(m^2.K)$)
A = surface area, m^2
T= temp of trunnion at a given time, K

Heat Stored

Heat stored by mass = mCT

where
m = mass of ball, kg
C = specific heat of the ball, J/(kg-K)

Energy Conservation

Rate at which heat is gained
– Rate at which heat is lost
=Rate at which heat is stored

$$0- hA(T-T_a) = d/dt(mCT)$$
$$0- hA(T-T_a) = m\,C\,dT/dt$$

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Putting in The Numbers

Length of cylinder = 0.625 m
Radius of cylinder = 0.3 m
Density of cylinder material ρ = 7800 kg/m^3
Specific heat, C = 450 J/(kg-C)
Convection coefficient, h= 90 W/(m^2-C)
Initial temperature of the trunnion, T(0)= 27^{o}C
Temperature of dry-ice/alcohol, T_a = -78^{o}C

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The Differential Equation

Surface area of the trunnion

$A = 2\pi rL+2\pi r^2$

$= 2*\pi*0.3*0.625+2*\pi*0.3^2$

$= 1.744\ m^2$

Mass of the trunnion

$M = \rho V$

$= \rho (\pi r^2L)$

$= (7800)*[\pi*(0.3)^2*0.625]$

$= 1378\ kg$

The Differential Equation

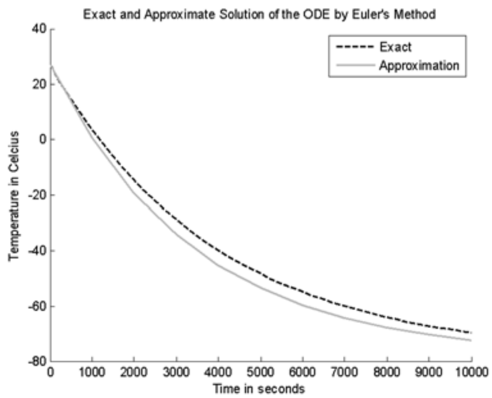
$$-hA(T - T_a) = mC \frac{dT}{dt}$$

$$-90 \times 1.744 \times (T + 78) = 1378 \times 450 \frac{dT}{dt}$$

$$\frac{dT}{dt} = -2.531 \times 10^{-4}(T + 78)),$$

$$T(0) = 27$$

Solution



Time (s)	Temp (^oC)
0	27
1000	0.42
2000	-19.42
3000	-34.25
4000	-45.32
5000	-53.59
6000	-59.77
7000	-64.38
8000	-67.83
9000	-70.40
10000	-72.32

END