Bring it together

Computational Methods

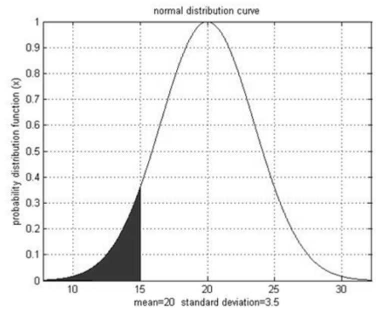
EML3041

Why do we use numerical methods?

Why use Numerical Methods?

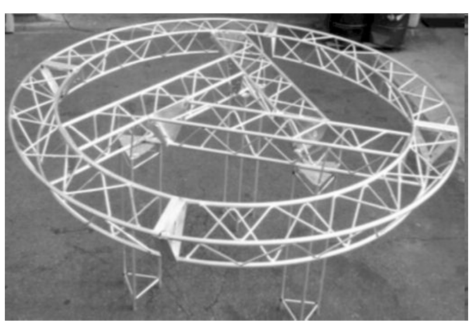
- To solve problems that cannot be solved exactly

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{u^2}{2}} du$$

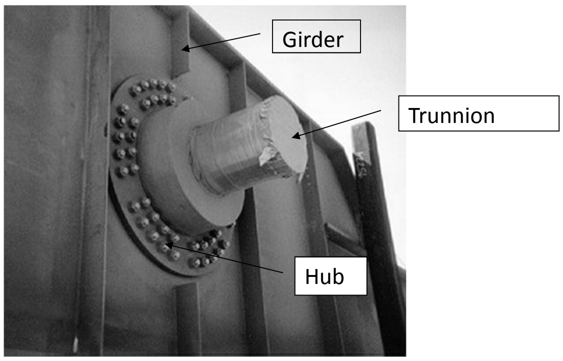


Why use Numerical Methods?

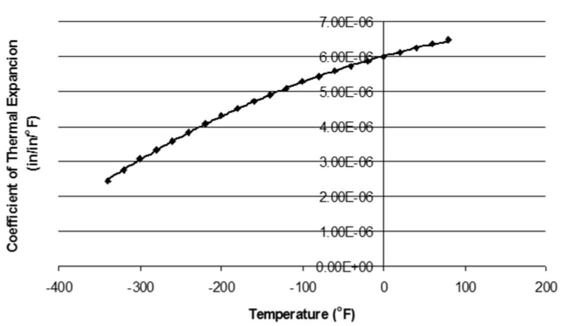
- To solve problems that have exact solutions but are otherwise intractable!



Assembling a fulcrum for a bascule bridge



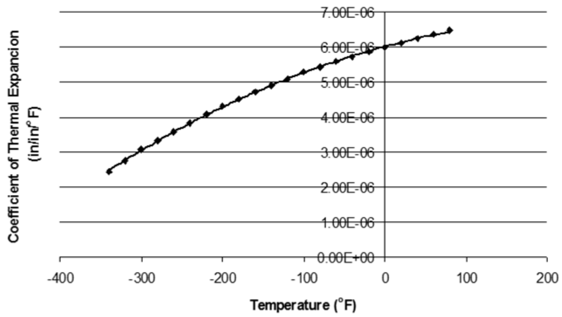
What model should I use to calculate contraction of trunnion?



$$\Delta D = D\alpha\Delta T$$

$$\Delta D = D\int_{T_{room}}^{T_{fluid}} \alpha dT$$

Finding the fluid temperature to get enough contraction



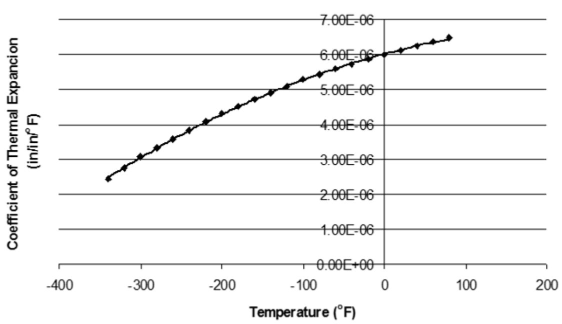
$$\Delta D = D \int_{T_{room}}^{T_{fluid}} \alpha dT$$

$$\alpha = -1.2278 \times 10^{-11} T^2 + 6.1946 \times 10^{-9} T + 6.0150 \times 10^{-6}$$

$$-0.015 = 12.363 \int_{80}^{T_{fluid}} \left(-1.2278 \times 10^{-11} T^2 + 6.1946 \times 10^{-9} T + 6.015 \times 10^{-6}\right) dT$$

$$\mathrm{f}(T_f) = -0.50598 \times 10^{-10} T_f^3 + 0.38292 \times 10^{-7} T_f^2 + 0.74363 \times 10^{-4} T_f + 0.88318 \times 10^{-2} = 0$$

Finding the expression for thermal expansion coefficient



$$\Delta D = D \int_{T_{room}}^{T_{fluid}} \alpha dT$$

$$\alpha = a_0 + a_1 T + a_2 T^2$$

$$S_r = \sum_{i=1}^{n} \left(\alpha_i - \{a_0 + a_1 T_i + a_2 T_i^2\}\right)^2$$

$$\begin{bmatrix} n & \left(\sum_{i=1}^{n} T_i\right) & \left(\sum_{i=1}^{n} T_i^2\right) \\ \left(\sum_{i=1}^{n} T_i\right) & \left(\sum_{i=1}^{n} T_i^2\right) & \left(\sum_{i=1}^{n} T_i^3\right) \\ \left(\sum_{i=1}^{n} T_i^2\right) & \left(\sum_{i=1}^{n} T_i^3\right) & \left(\sum_{i=1}^{n} T_i^4\right) \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{n} \alpha_i \\ \sum_{i=1}^{n} T_i \alpha_i \\ \sum_{i=1}^{n} T_i^2 \alpha_i \end{bmatrix}$$

$$\alpha = -1.2278 \times 10^{-11} T^2 + 6.1946 \times 10^{-9} T + 6.0150 \times 10^{-6}$$

What is the temperature of the trunnion after half an hour?

$$mc\frac{d\theta}{dt} = -hA(\theta - \theta_a),\ \ \theta(0) = \theta_{room}$$

How long does it take a trunnion to cool down?

$$mc\frac{d\theta}{dt} = -hA(\theta - \theta_a),\ \ \theta(0) = \theta_{room}$$

Example

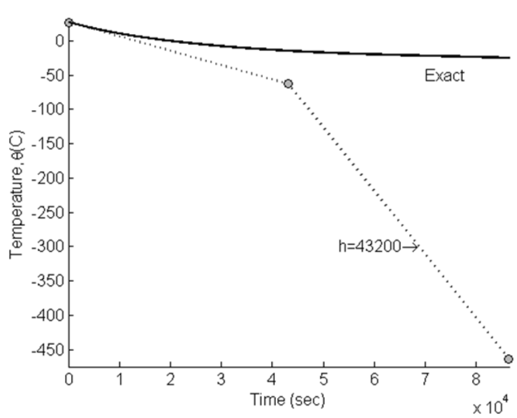
A solid steel shaft at room temperature of 27°C is needed to be contracted so that it can be shrunk-fit into a hollow hub. It is placed in a refrigerated chamber that is maintained at −33°C. The rate of change of temperature of the solid shaft θ is given by

$$\frac{d\theta}{dt} = -5.33 \times 10^{-6}\begin{pmatrix} -3.69 \times 10^{-6}\theta^4 + 2.33 \times 10^{-5}\theta^3 + 1.35 \times 10^{-3}\theta^2 \\ +5.42 \times 10^{-2}\theta + 5.588 \end{pmatrix}(\theta + 33)$$

$$\theta(0) = 27\,°\text{C}$$

Using Euler's method, find the temperature of the steel shaft after 86400 seconds. Take a step size of h = 43200 seconds.

Temperature vs. Time



What is the rate of change of heat stored in the cylinder?

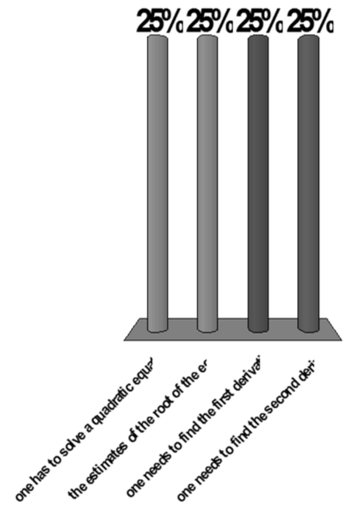
$$mc\frac{d\theta}{dt} = -hA(\theta - \theta_a), \quad \theta(0) = \theta_{room}$$

Can you identify?

- We talked about 7 mathematical procedures in this course. Write them down.
- Now connect the problem just discussed to these 7 mathematical procedures on each of the previous slides.

The Newton-Raphson method of finding real roots of the nonlinear equation $f(x)=0$ can be derived by using the first two terms of the Taylor series. One would then expect that adding one more term of the Taylor series to the Newton-Raphson method would result in higher accuracy of the results for the same number of iterations as the Newton-Raphson method. However, this new approach of adding one more term of Taylor series is **not feasible** because

A. one has to solve a quadratic equation to get the new estimate of the root of the equation.
B. the estimates of the root of the equation may turn out to be complex numbers
C. one needs to find the first derivative of $f(x)$.
D. one needs to find the second derivative of $f(x)$.



It is desired to compute the following matrix product, P×Q×R×S. The sizes of the four matrices are:
P is 1×4, Q is 4×20, R is 20×5, and S is 5×1.
Amongst the choices given below, the most efficient (the least number of floating point operations of add, subtract, multiplication and divide) way to compute the matrix product is

1) (P × Q) × (R × S)
2) ((P × Q) × R) × S
3) P × ((Q × R) × S)
4) (P × (Q × R)) × S