EML3041
Computational Methods

http://nm.MathForCollege.com

Transforming Numerical Methods Education for STEM Undergraduates

1

- **Welcome to EML3041: Computational Methods.**
- **My name is Dr. Kaw.**
- **Have you gotten your textbooks and TI30Xa calculator!**
- **At home, go thru all the CANVAS modules.**
- **All is well that ends well; well-begun is half-done.**
- **No distractions. Daydreamers and sleepers will not be disturbed.**

2

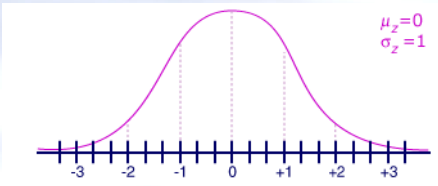
Why use Numerical Methods?

3

Why use Numerical Methods?

- To solve problems that cannot be solved exactly

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{u^2}{2}} du$$



4

Why use Numerical Methods?

- To solve problems that are intractable to solve exactly!

5

Steps in Solving an Engineering Problem

http://nm.MathForCollege.com

6

How do we solve an engineering problem?

Problem Description

↓

Mathematical Model

↓

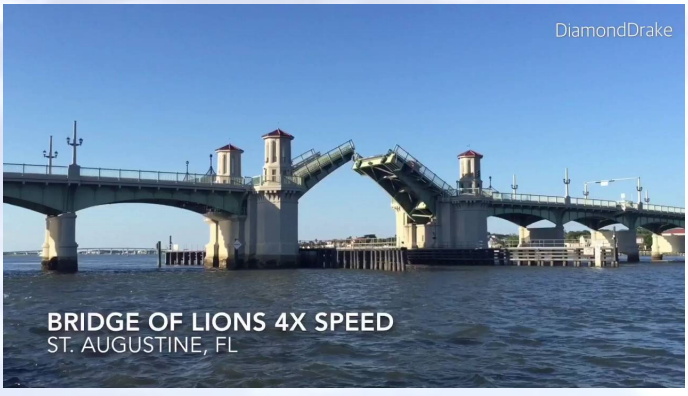
Solution of Mathematical Model

↓

Using the Solution

7

Example of Solving an Engineering Problem

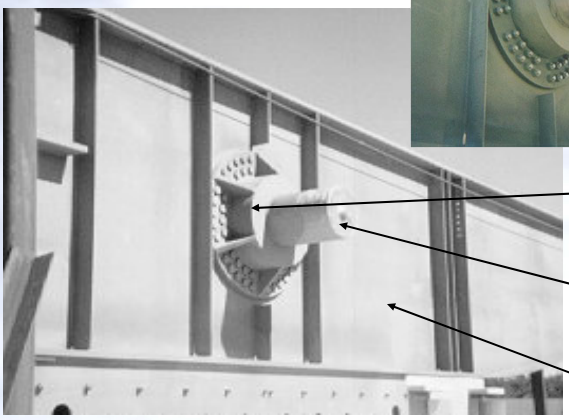


8

Bascule Bridge THG

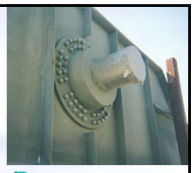
9

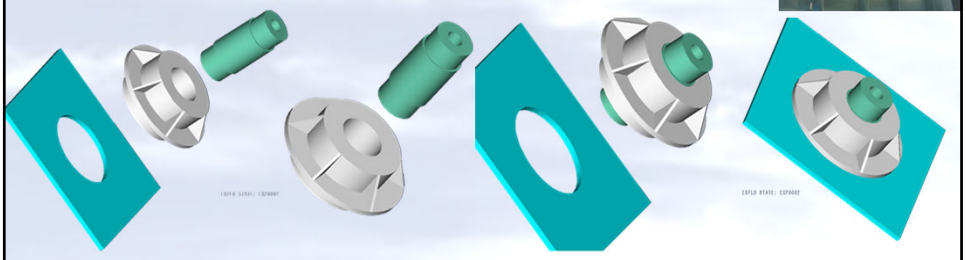
Bascule Bridge THG



10

Trunnion-Hub-Girder Assembly Procedure





Simulation

Step1. Trunnion immersed in dry-ice/alcohol

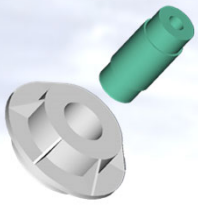
Step2. Trunnion warm-up in hub

Step3. Trunnion-Hub immersed in dry-ice/alcohol

Step4. Trunnion-Hub warm-up into girder

11

Problem

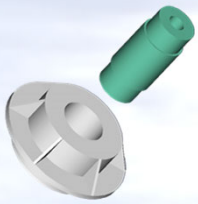


After Cooling, the Trunnion Got Stuck in Hub

12

Why did it get stuck?

Magnitude of contraction needed in the trunnion was 0.015" or more. Did it contract enough?



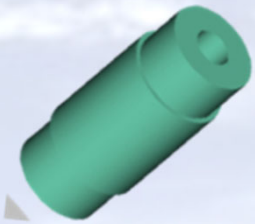
13

Video of Assembly Process

14

Consultant calculations

$$\Delta D = D \times \alpha \times \Delta T$$



$$D = 12.363"$$

$$\alpha = 6.47 \times 10^{-6} in/in/^{o}F$$

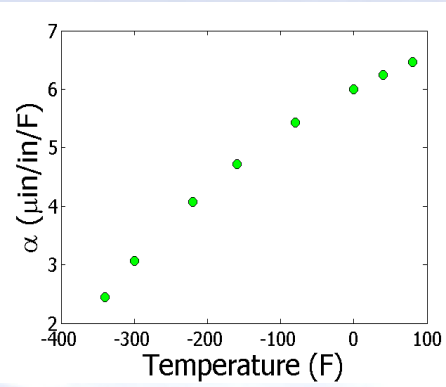
$$\Delta T = -108 - 80 = -188^{o}F$$

$$\Delta D = (12.363)(6.47 \times 10^{-6})(-188)$$
$$= -0.01504"$$

15

Is the formula used correct?

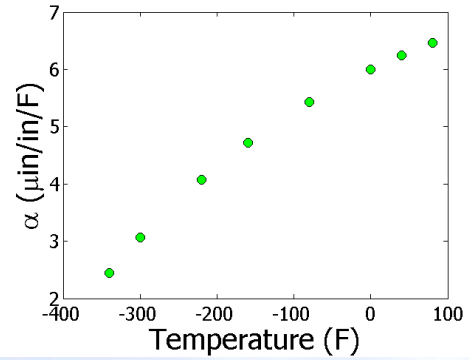
$$\Delta D = D \times \alpha \times \Delta T$$



T(°F)	α (μin/in/°F)
-340	2.45
-300	3.07
-220	4.08
-160	4.72
-80	5.43
0	6.00
40	6.24
80	6.47

16

The Correct Model Would Account for Varying Thermal Expansion Coefficient



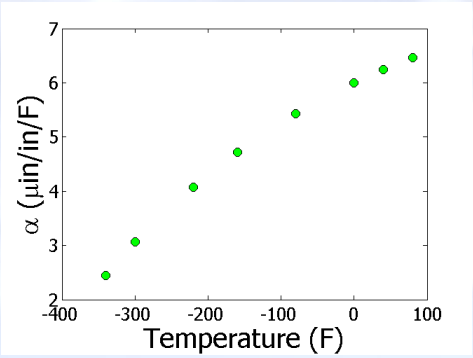
$$\Delta D = D \int_{T_a}^{T_c} \alpha\,(T)dT$$

17

Can You Roughly Estimate the Contraction?

$$\Delta D = D \int_{T_a}^{T_c} \alpha\,(T)dT$$

T_a=80°F; T_c=-108°F; D=12.363”

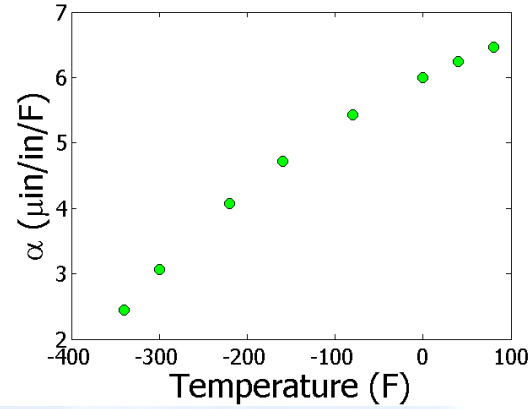


18

Can You Find a Better Estimate for the Contraction?

$$\Delta D = D \int_{T_a}^{T_c} \alpha\,(T)dT$$

$T_a = 80^{o}\text{F}$
$T_c = -108^{o}\text{F}$
$D = 12.363"$

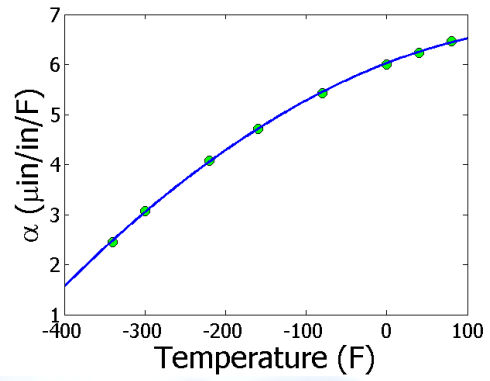


Estimating Contraction Accurately

Change in diameter (ΔD) by cooling it in dry ice/alcohol is given by

$$\Delta D = D \int_{T_a}^{T_c} \alpha\,(T)dT$$

$T_a = 80^{o}\text{F}$
$T_c = -108^{o}\text{F}$
$D = 12.363"$



$$\alpha = -1.2278 \times 10^{-5}T^2 + 6.1946 \times 10^{-3}T + 6.0150$$

$$\Delta D = -0.0137"$$

So what is the solution to the problem?

One solution is to immerse the trunnion in liquid nitrogen which has a boiling point of -321°F as opposed to the dry-ice/alcohol temperature of -108°F.

$$\Delta D = -0.0244"$$

Revisiting steps to solve a problem

1) Problem Statement: Trunnion got stuck in the hub.

2) Modeling: Developed a new model

$$\Delta D = D \int_{T_a}^{T_c} \alpha\,(T) dT$$

3) Solution: 1) Used trapezoidal rule OR b) Used regression and integration.

4) Implementation: Cool the trunnion in liquid nitrogen.

THE END

http://nm.MathForCollege.com

23

Introduction to Numerical Methods

Mathematical Procedures

http://nm.MathForCollege.com

24

Mathematical Procedures

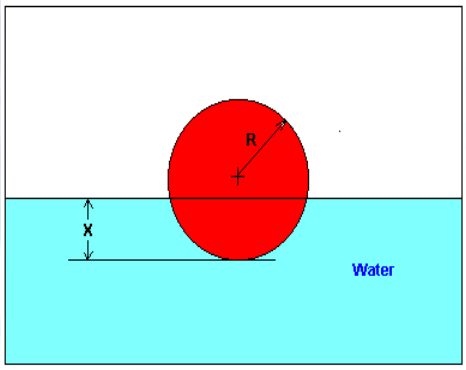
- Nonlinear Equations
- Differentiation
- Simultaneous Linear Equations
- Curve Fitting
  - Interpolation
  - Regression
- Integration
- Ordinary Differential Equations
- Other Advanced Mathematical Procedures:
  - Partial Differential Equations
  - Optimization
  - Fast Fourier Transforms

Nonlinear Equations

How much of the floating ball is under water?

Diameter=0.11m

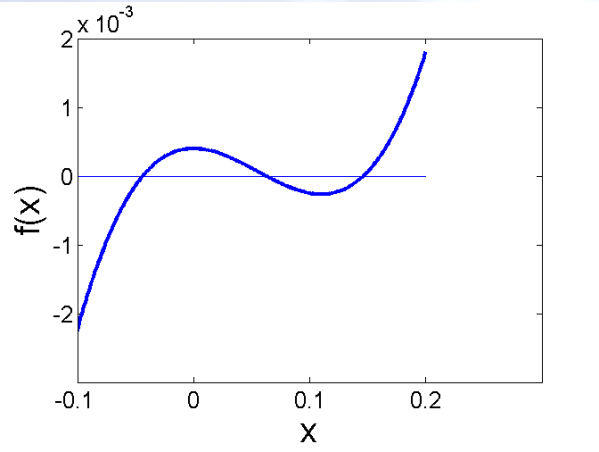
Specific Gravity=0.6



$$x^3 - 0.165x^2 + 3.993 \times 10^{-4} = 0$$

Nonlinear Equations

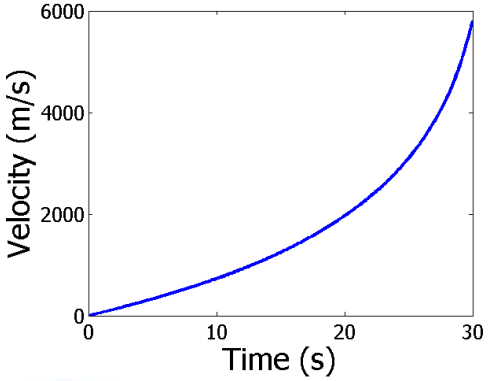
How much of the floating ball is under the water?



$$f(x) = x^3 - 0.165x^2 + 3.993 \times 10^{-4} = 0$$

Differentiation

What is the acceleration at t=7 seconds?



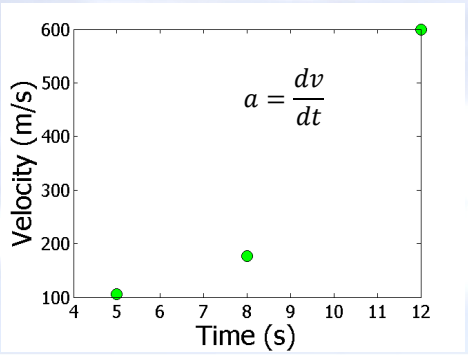
$$v(t) = 2200 \ln\left(\frac{16 \times 10^4}{16 \times 10^4 - 5000t}\right) - 9.8t$$

$$a = \frac{dv}{dt}$$

Differentiation

What is the acceleration at t=7 seconds?

Time (s)	5	8	12
Vel (m/s)	106	177	600



Simultaneous Linear Equations

Find the velocity profile, given

Time (s)	5	8	12
Vel (m/s)	106	177	600

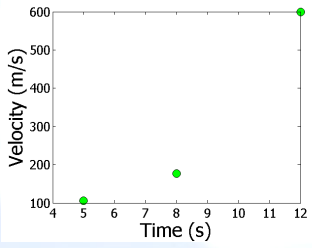
$v(t) = at^2 + bt + c, \quad 5 \le t \le 12$

Three simultaneous linear equations

$$25a + 5b + c = 106$$

$$64a + 8b + c = 177$$

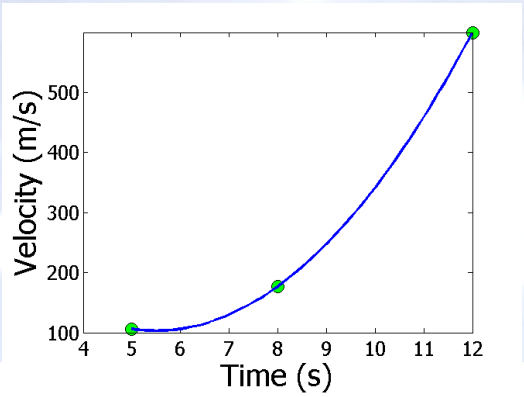
$$144a + 12b + c = 600$$



Interpolation

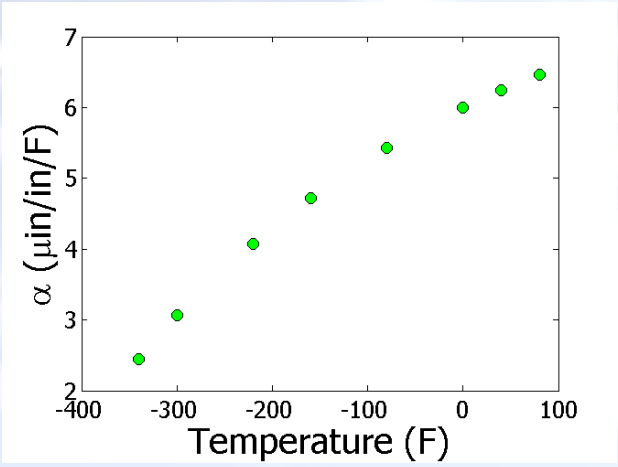
What is the velocity of the rocket at t=7 seconds?

Time (s)	5	8	12
Vel (m/s)	106	177	600

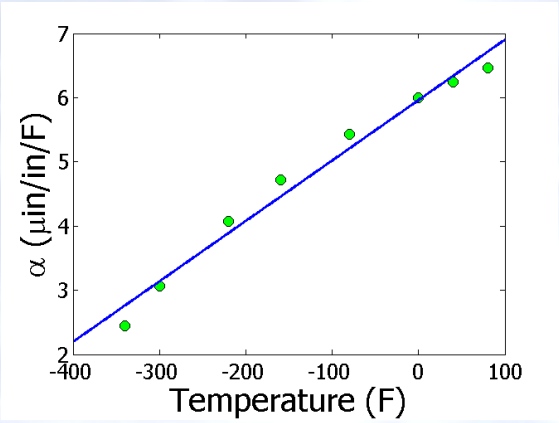


Regression

Thermal expansion coefficient data for cast steel



Regression (cont)

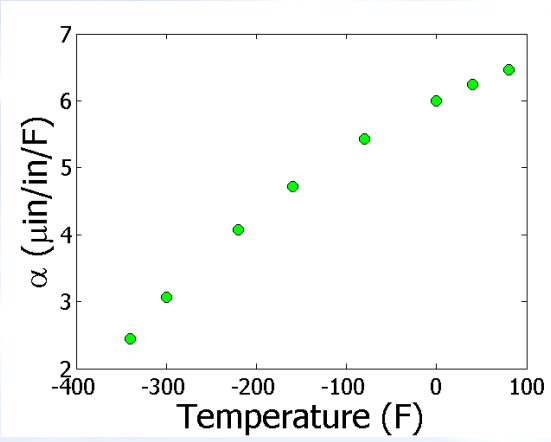


33

Integration

Finding the diametric contraction in a steel shaft when dipped in liquid nitrogen.

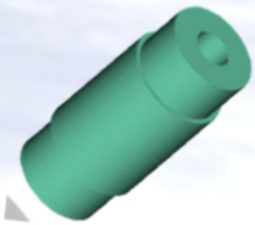
$$\Delta D = D \int_{T_{room}}^{T_{fluid}} \alpha \, dT$$



34

Ordinary Differential Equations

How long does it take a trunnion to cool down?



$$mc\frac{d\theta}{dt} = -hA(\theta - \theta_a),$$

$$\theta(0) = \theta_{room}$$

35

MathForCollege.com

Open Education Resources

36