Computational Methods

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Transforming Numerical Methods Education for STEM Undergraduates

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- **Welcome to EML3041: Computational Methods.**
- **My name is Dr. Kaw.**
- **Have you gotten your textbooks and TI30Xa calculator!**
- **Introduce yourself to the person on your left and right. Say one thing (nothing personal) about yourself to them.**
- **At home, go thru all the CANVAS modules.**
- **All is well that ends well; well-begun is half-done.**
- **Cell phones, laptops or other distractions are discouraged other than for allowed use.**
- **Daydreamers and sleepers will not be disturbed.**

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Why use Numerical Methods?

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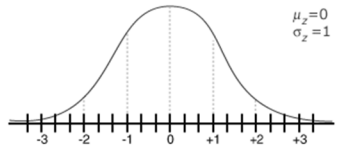
Why use Numerical Methods?

- To solve problems that cannot be solved exactly

$$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-\frac{u^2}{2}}\,du$$



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Why use Numerical Methods?

- To solve problems that are intractable to solve exactly!

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Steps in Solving an Engineering Problem

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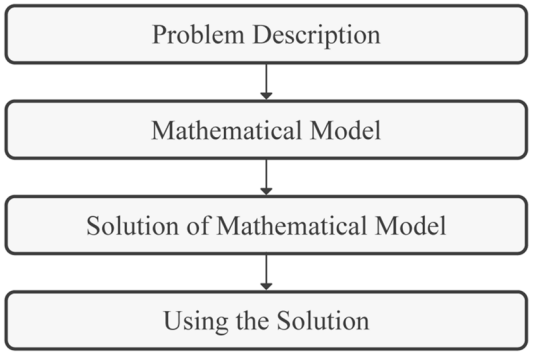
How do we solve an engineering problem?



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PROBLEM DESCRIPTION

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Example of Solving an Engineering Problem

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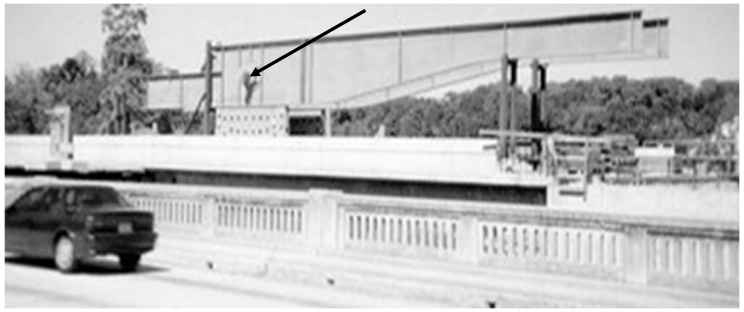
Bascule Bridge THG



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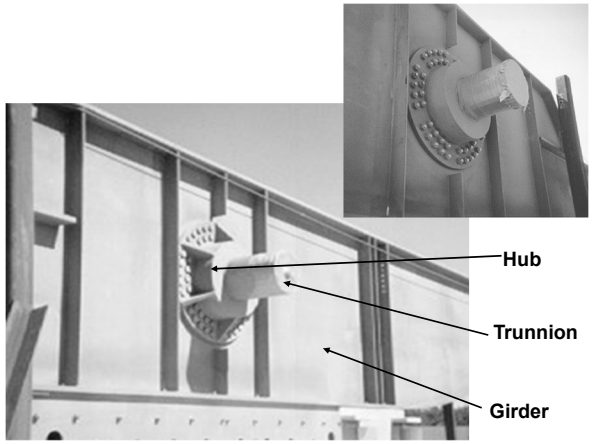
Bascule Bridge THG



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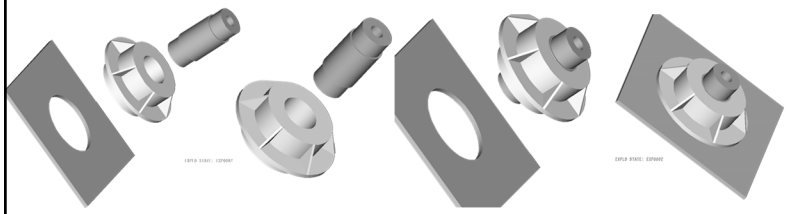
Trunnion-Hub-Girder Assembly Procedure



Step1. Trunnion immersed in dry-ice/alcohol
Step2. Trunnion shrink fit and warm up
Step3. Trunnion-hub immersed in dry-ice/alcohol
Step4. Trunnion-hub shrink fit and warm-up

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Problem

After cooling the trunnion, the trunnion got stuck in hub

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Why did it get stuck?

Magnitude of contraction needed in the trunnion was 0.015" or more. Did it contract enough?

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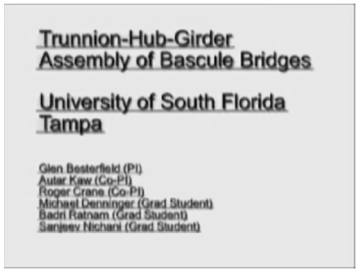
Video of Assembly Process



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MATHEMATICAL MODELING

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Consultant Calculations

$$\Delta D = D\ \alpha\ \Delta T$$

$$D = 12.363"$$

$$\alpha = 6.47 \times 10^{-6}\ \text{in/in/}^{\text{o}}\text{F}$$

$$\Delta T = T_a - T_c$$
$$= -108 - 80$$
$$= -188^{\text{o}}\text{F}$$

$$\Delta D = (12.363)(6.47 \times 10^{-6})(-188)$$
$$= -0.01504"$$

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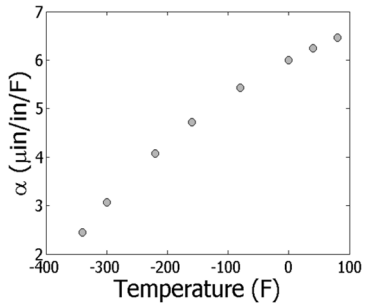
Is the $\Delta D = D\alpha\Delta T$ formula correct?

T(°F)	α (μin/in/°F)
-340	2.45
-300	3.07
-220	4.08
-160	4.72
-80	5.43
0	6.00
40	6.24
80	6.47



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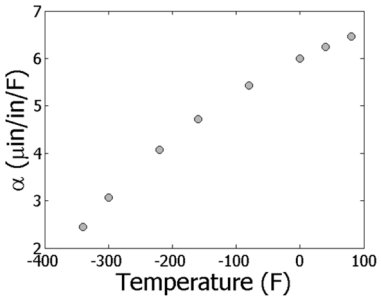
The correct model would account for varying thermal expansion coefficient

$$\Delta D = D \int_{T_a}^{T_c} \alpha\,(T)\,dT$$



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SOLUTION OF MATHEMATICAL MODEL

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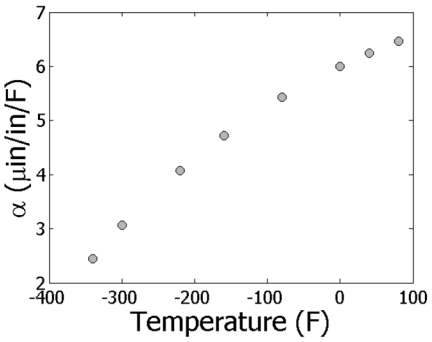
Can you roughly estimate the contraction?

$$\Delta D = D \int_{T_a}^{T_c} \alpha\,(T)dT$$

$T_a = 80°\text{F}$
$T_c = -108°\text{F}$
$D = 12.363"$



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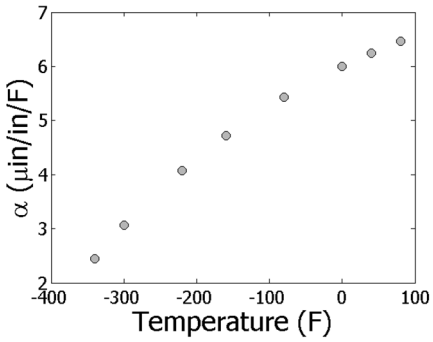
Can You Find a Better Estimate for the Contraction?

$$\Delta D = D \int_{T_a}^{T_c} \alpha(T)dT$$

$T_a = 80°\text{F}$
$T_c = -108°\text{F}$
$D = 12.363"$



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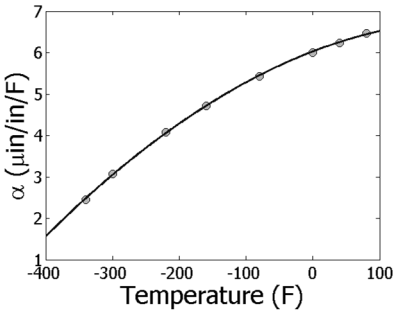
Can you estimate the contraction more accurately?

$$\alpha = -1.2278 \times 10^{-5}T^2 + 6.1946 \times 10^{-3}T + 6.0150$$

$$\Delta D = D \int_{T_a}^{T_c} \alpha\,(T)dT$$

$T_a = 80°\text{F}$
$T_c = -108°\text{F}$
$D = 12.363"$

$\Delta D = -0.0137"$



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USING THE SOLUTION

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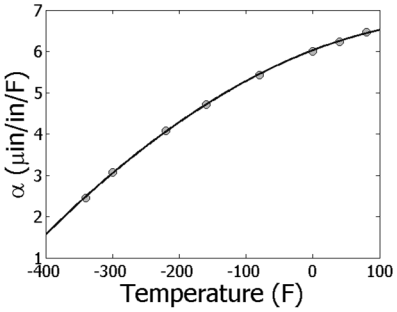
So, what is the solution to the problem?

$$\alpha = -1.2278 \times 10^{-5}T^2 + 6.1946 \times 10^{-3}T + 6.0150$$

$$\Delta D = D \int_{T_a}^{T_c} \alpha\,(T)dT$$

$T_a = 80°\text{F}$
$T_c = -321°\text{F}$
$D = 12.363"$

$\Delta D = -0.0244"$



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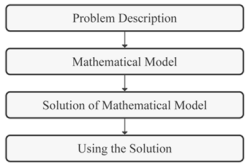
Revisiting steps to solve a problem



1) Problem Statement: Trunnion got stuck in the hub.

2) Modeling: Developed a new model

$$\Delta D = D \int_{T_a}^{T_c} \alpha(T)dT$$

3) Solution: 1) Used trapezoidal rule OR b) Used regression and integration.

4) Implementation: Cool the trunnion in liquid nitrogen.

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To Do List

- Get your textbook and calculator
- Go through first day of class module
- Download MATLAB today for free
- Cell phones or laptops are discouraged in class other than for allowed use
- Please refrain from excessive talking
- Turn on your notifications in CANVAS

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THE END

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Introduction to Numerical Methods

Mathematical Procedures

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Nonlinear Equations

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

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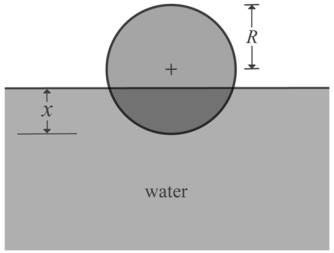
Nonlinear Equations

How much of the floating ball is under water?

Radius=0.055 m

Specific Gravity=0.6

$$x^3 - 0.165x^2 + 3.993 \times 10^{-4} = 0$$



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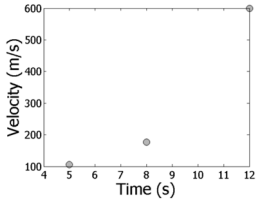
Simultaneous Linear Equations

Find the velocity profile, given

Time (s)	5	8	12
Vel (m/s)	106	177	600

$$v(t) = at^2 + bt + c, 5 \le t \le 12$$



Three simultaneous linear equations

$$a(5^2) + b(5) + c = 106$$
$$a(8^2) + b(8) + c = 177$$
$$a(12^2) + b(12) + c = 600$$

Source: Photo by NASA

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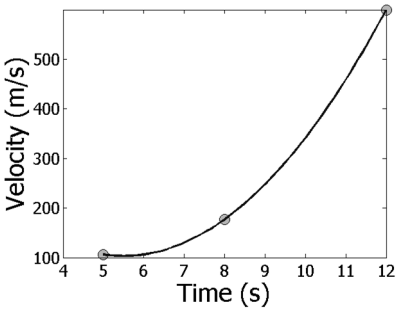
Interpolation

What is the velocity of the rocket at t=7 seconds?

Time (s)	5	8	12
Vel (m/s)	106	177	600



Source: Photo by NASA

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Mathematical Procedures

- Nonlinear Equations
- Differentiation
- Simultaneous Linear Equations
- Curve Fitting
  - Interpolation
  - Regression
- Integration
- Ordinary Differential Equations
- Other Advanced Mathematical Procedures:
  - Partial Differential Equations
  - Optimization
  - Fast Fourier Transforms

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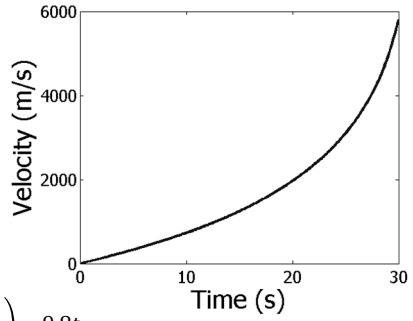
Differentiation

What is the acceleration at t=7 seconds?



$$v(t) = 2200 \ln\left(\frac{16 \times 10^4}{16 \times 10^4 - 5000t}\right) - 9.8t$$

$$a = \frac{dv}{dt}$$

Source: Photo by NASA

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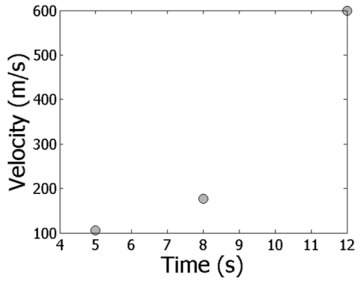
Differentiation

What is the acceleration at t=7 seconds?

Time (s)	5	8	12
Vel (m/s)	106	177	600

$$a = \frac{dv}{dt}$$



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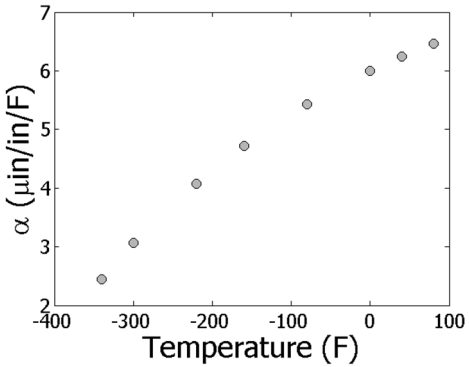
Regression



Thermal expansion coefficient data for cast steel

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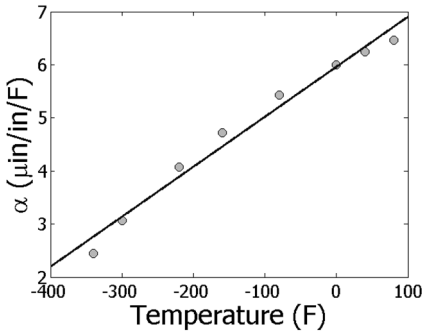
Regression – Linear



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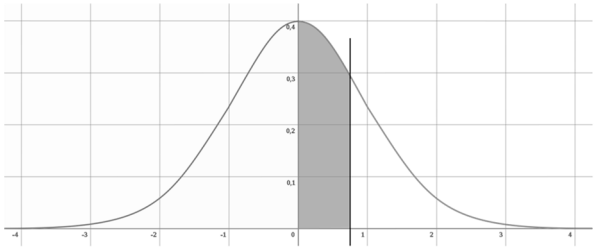
Integration

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-t^2/2}\, dt$$



Source: Wikipedia.org https://upload.wikimedia.org/wikipedia/commons/thumb/f/f9/Z_cumulative_from_mean.svg/1704px-Z_cumulative_from_mean.svg.png

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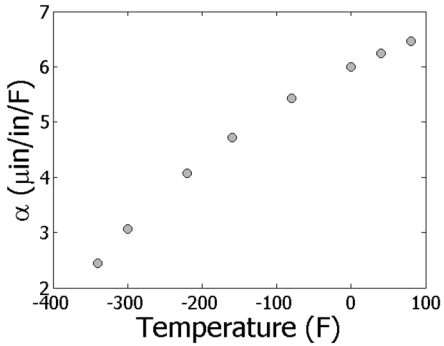
Integration

Finding the diametric contraction in a steel shaft when dipped in liquid nitrogen.

$$\Delta D = D \int_{T_{room}}^{T_{fluid}} \alpha\, dT$$



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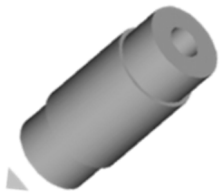
Ordinary Differential Equations

How long does it take a trunnion to cool down?



$$mc\frac{d\theta}{dt} = -hA(\theta - \theta_a), \theta(0) = \theta_{room}$$

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