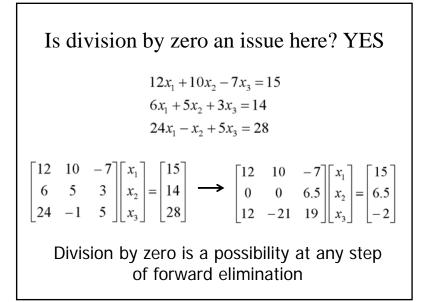
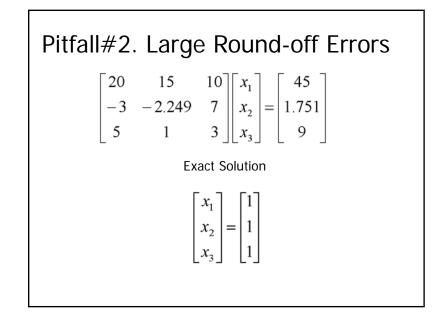


Pitfall#1. Division by zero $10x_2 - 7x_3 = 3$ $6x_1 + 2x_2 + 3x_3 = 11$ $5x_1 - x_2 + 5x_3 = 9$ $\begin{bmatrix} 0 & 10 & -7 \\ 6 & 2 & 3 \\ 5 & -1 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 11 \\ 9 \end{bmatrix}$

Is division by zero an issue here? $12x_{1} + 10x_{2} - 7x_{3} = 15$ $6x_{1} + 5x_{2} + 3x_{3} = 14$ $5x_{1} - x_{2} + 5x_{3} = 9$ $\begin{bmatrix} 12 & 10 & -7 \\ 6 & 5 & 3 \\ 5 & -1 & 5 \end{bmatrix} \begin{bmatrix} x_{1} \\ x_{2} \\ x_{3} \end{bmatrix} = \begin{bmatrix} 15 \\ 14 \\ 9 \end{bmatrix}$





Pitfall#2. Large Round-off Errors			
	20 -3 5	15 -2.249 1	$ \begin{bmatrix} 10\\7\\3 \end{bmatrix} \begin{bmatrix} x_1\\x_2\\x_3 \end{bmatrix} = \begin{bmatrix} 45\\1.751\\9 \end{bmatrix} $
Solve it on a computer using 6 significant digits with chopping			
		$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$	$ = \begin{bmatrix} 0.9625\\ 1.05\\ 0.999995 \end{bmatrix} $

Pitfall#2. Large Round-off Errors $\begin{bmatrix}
20 & 15 & 10 \\
-3 & -2.249 & 7 \\
5 & 1 & 3
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix} = \begin{bmatrix}
45 \\
1.751 \\
9
\end{bmatrix}$ Solve it on a computer using 5 significant digits with chopping $\begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix} = \begin{bmatrix}
0.625 \\
1.5 \\
0.99995
\end{bmatrix}$ Is there a way to reduce the round off error?

Avoiding Pitfalls

Increase the number of significant digits

- Decreases round-off error
- Does not avoid division by zero

Avoiding Pitfalls

Gaussian Elimination with Partial Pivoting

- · Avoids division by zero
- Reduces round off error

Gauss Elimination with Partial Pivoting

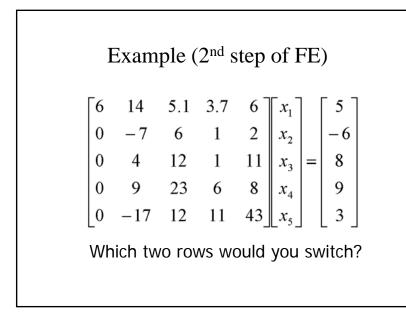
http://nm.MathForCollege.com

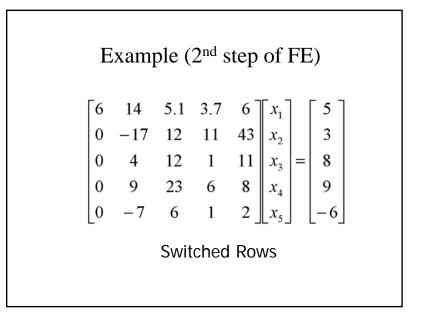
What is Different About Partial Pivoting?

At the beginning of the \textit{k}^{th} step of forward elimination, find the maximum of

 $|a_{kk}|, |a_{k+1,k}|, \dots, |a_{nk}|$

If the maximum of the values is $|a_{pk}|$ in the p^{th} row, $k \le p \le n$, then switch rows p and k.





Gaussian Elimination with Partial Pivoting

A method to solve simultaneous linear equations of the form [A][X]=[C]

Two steps 1. Forward Elimination 2. Back Substitution Gauss Elimination with Partial Pivoting Example Example 2 Solve the following set of equations by Gaussian elimination with partial pivoting

$$\begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 106.8 \\ 177.2 \\ 279.2 \end{bmatrix}$$

Example 2 Cont.

$$\begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 106.8 \\ 177.2 \\ 279.2 \end{bmatrix} \Rightarrow \begin{bmatrix} 25 & 5 & 1 & \vdots & 106.8 \\ 64 & 8 & 1 & \vdots & 177.2 \\ 144 & 12 & 1 & \vdots & 279.2 \end{bmatrix}$$
1. Forward Elimination
2. Back Substitution

Г

Forward Elimination

Number of Steps of Forward Elimination

Number of steps of forward elimination is (n-1)=(3-1)=2

