# Is LU Decomposition better than Gaussian Elimination?

Solve $[A][X] = [B]$

T = clock cycle time and nxn = size of the matrix

**Forward Elimination**

$$CT|_{FE} = T\left(\frac{8n^3}{3} + 8n^2 - \frac{32n}{3}\right)$$

**Back Substitution**

$$CT|_{BS} = T\left(4n^2 + 12n\right)$$

**LU Decomposition**

$$CT|_{DE} = T\left(\frac{8n^3}{3} + 4n^2 - \frac{20n}{3}\right)$$

**Forward Substitution**

$$CT|_{FS} = T\left(4n^2 - 4n\right)$$

**Back Substitution**

$$CT|_{BS} = T\left(4n^2 + 12n\right)$$

# Is LU Decomposition better than Gaussian Elimination?

To solve $[A][X] = [B]$

**Time taken by methods**

| Gaussian Elimination | LU Decomposition |
|---|---|
| $T\left(\frac{8n^3}{3}+12n^2+\frac{4n}{3}\right)$ | $T\left(\frac{8n^3}{3}+12n^2+\frac{4n}{3}\right)$ |

T = clock cycle time and nxn = size of the matrix

So both methods are equally efficient.

# To find inverse of [A]

Time taken by Gaussian Elimination

$$= n\left(CT\,|_{FE} + CT\,|_{BS}\right)$$

$$= T\left(\frac{8n^4}{3} + 12n^3 + \frac{4n^2}{3}\right)$$

Time taken by LU Decomposition

$$= CT\,|_{LU} + n \times CT\,|_{FS} + n \times CT\,|_{BS}$$

$$= T\left(\frac{32n^3}{3} + 12n^2 + \frac{20n}{3}\right)$$

# To find inverse of [A]

Time taken by Gaussian Elimination

$$T\left(\frac{8n^4}{3}+12n^3+\frac{4n^2}{3}\right)$$

Time taken by LU Decomposition

$$T\left(\frac{32n^3}{3}+12n^2+\frac{20n}{3}\right)$$

**Table 1** Comparing computational times of finding inverse of a matrix using LU decomposition and Gaussian elimination.

| $n$ | 10 | 100 | 1000 | 10000 |
|---|---|---|---|---|
| $CT\|_{\text{inverse GE}} / CT\|_{\text{inverse LU}}$ | 3.28 | 25.83 | 250.8 | 2501 |