Title: Length of curve experiment

Background: The motivation behind the experiment is twofold.

- Recreate Runge's numerical experiment that showed that higher-order interpolation is a bad idea. As shown in class, as you use higher order interpolants to approximate $f(x)=1/(1+25x^2)$ in the domain [-1,1], the differences between the original function and the polynomial interpolant became bigger;
- Show the reason why we use spline interpolation rather than polynomial interpolation to find a smooth path to travel through several discrete data points such as in the example of the path of a robot (Refer to Chapter 05.10 for further information).

What to do to set it up: A flexible curve (Figure 1) made of lead-core construction with inch graduations from 0 to 12" is provided. Using the flexible curve, trace a 12" length curve similar (not same) in shape to the Runge's curve on the graphing paper as shown.

Exercises to do: Use the USCS system of units throughout.

1. Write your last name and first name on the graphing paper.
2. Put the graphing paper in landscape mode. Draw the *x*-axis and *y*-axis as you please (putting *x*-axis close to bottom and *y*-axis in middle is an option chosen by most). See Figure 1 only for illustrative purposes for coordinates.
3. Trace a 12" curve in a similar shape as Runge's curve using the flexible curve.
4. Pick one point each at the two ends of the curve.
5. Pick five more points distributed throughout the curve.
6. Find and write the (*x*,*y*) coordinates of the 7 data points chosen in Problem 4 and Problem 5 in inches on the graph paper.
7. On the graph paper, do the following
    a. Using a ruler, draw linear splines (connecting the dots) through the 7 data points.
    b. Use the ruler to measure the length of the linear splines to get an estimate of the length of the curve.
    c. Put the length of each spline on the curve.
    d. Put the length of the curve on your graph paper as "Length of the curve using ruler for linear splines = _____". Show your calculation.

8. Open a new MATLAB mfile. Put your first name, last name as comments.

9. Put %% Problem 9 as a comment. Use the coordinates of the 7 data points and estimate the length of the curve using linear splines and the concept of loops in MATLAB. Display the length of the curve using the disp statement. You can use the length of a straight-line equation to do this. Does it match your answer from Problem 7?

10. Put %% Problem 10 as a comment. Find the polynomial interpolant that passes through the 7 data points using MATLAB commands. **There is no need to display the polynomial expression**. Calculate the length of the polynomial interpolant using MATLAB. You need to use the $\boldsymbol{S = \int_a^b \sqrt{1 + (dy/dx)^2}\, dx}$ for finding the length of the path (not by breaking the curve into small length straight lines). Display the length of the curve using the disp statement.

11. Put %% Problem 11 as a comment Find the cubic spline interpolant that passes through the 7 data points using MATLAB. **There is no need to display the spline expressions**. Calculate the length of the cubic spline interpolant using MATLAB by any scientific method of your choice. Display the length of the curve using the disp statement. How close is your answer to 12″? How far is your answer from that of Problem 10?

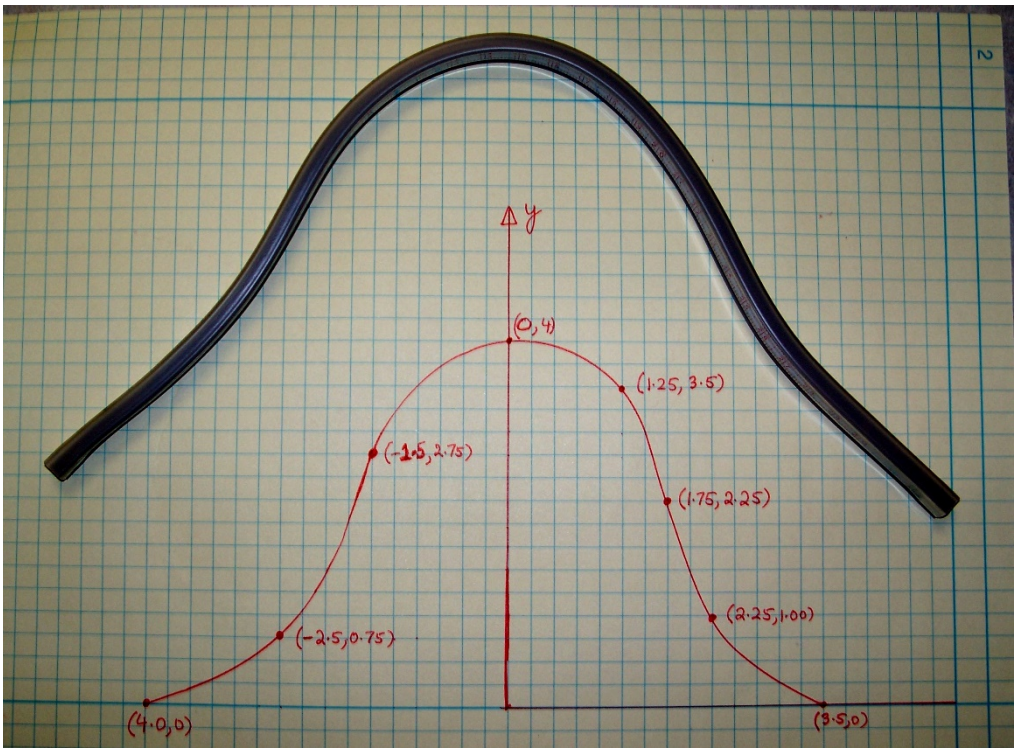

Figure 1. Using a flexible curve to draw a curve of known length. **This is only a sample curve. <u>Your curve should look different</u>, have different coordinates, and have 7 points chosen including the endpoints** (**errata – the first coordinate in the figure is (-4,0)**).