Worksheet Problems from Chapter 6 and 7 for Fall 2019

1)..Two-segment trapezoidal rule of integration is exact for integration of polynomials of order of at most _____

2)..When transforming the data for stress-strain curve $\sigma = k_1 \varepsilon e^{-k_2 \varepsilon}$ for concrete in compression, where $\sigma$is the stress and ε is the strain, the model is rewritten as _____

3).. To choose the best order of the polynomial to use for modeling the behavior of the data, the following data

(200, 40235.8), (250, 62855.7), (300, 90328.3), (350, 123059), (400, 160588), (450, 203263), (500, 250850)

are regressed to different order polynomials. The sum of the square of the residuals is given below for the different order polynomials. Which order of polynomial would you choose and why? **You need to answer this question quantitatively.**

Order of polynomial	1	2	3	4	5	6
S_r	5.24×10^8	5.21×10^4	4.97×10^4	3.36×10^4	1.12×10^4	0

Answer: 2

4)..The true error for the single-segment trapezoidal rule used to calculate the approximate value of the integral $\int_a^b f(x)dx$ is given by

$$E_t = -\frac{(b-a)^3}{12} f''(\alpha), a \le \alpha \le b.$$

For the integral $\int_3^6 3e^{1.1x}\, dx$, find the value of α.

Answer: 4.7386

5)..Water is flowing with a laminar flow through a circular pipe of 0.5 ft radius, and flow velocity (ft/s) measurements are made from the center to the wall of the pipe as follows.

Radial location, r (ft)	0	0.25	0.33	0.50
Velocity, v (ft/s)	10	7.5	5.6	0

Note that the volumetric flow rate, Q is given by

$$Q = \int_0^a 2\pi r v(r) dr$$

where $v(r)$ is the flow velocity of the fluid as a function of radial location r, and a is the radius of the pipe. Estimate the volumetric flow rate in ft^3/s in the pipe by using any scientific method.

Possible Answers:

3.395 ft^3/s – answer from the trapezoidal rule with unequal segments

3.911 ft^3/s – answer using 3rd order polynomial interpolation for v vs r

3.922 ft^3/s – answer using a regression model of $v = k\left(1 - \frac{r^2}{a^2}\right)$, where a=radius of pipe

6)..A scientist finds that regressing *y vs. x* data given below to straight-line $y = a_0 + a_1 x$ results in the coefficient of determination, r^2 for the straight-line model to be **zero.**

x	1	3	11	17
y	2	6	22	?

The missing value for *y* at $x = 17$ most nearly is _____

Answer: -2.444

7)..A scientist develops an approximate formula for integration as

$$\int_a^b f(x)dx \approx c_1 f(x_1), \text{ where } a \le x_1 \le b$$

The values of c_1 and x_1 are found by assuming that the formula is exact for the functions of the form $a_0 x + a_1 x^2$ polynomial. Find c_1 and x_1.

Answer: $x_1 = \frac{2(b^2+a^2+ab)}{3(b+a)}, c_1 = \frac{3(b-a)(b+a)^2}{4(b^2+a^2+ab)}$

8)..You are working for Valdez Spill Proof Oil Company as a petroleum engineer. Your boss is asking you to estimate the life of an oil well. The analysis used in the industry is called the decline curve analysis where the barrels of oil produced per unit time are plotted against time, and the curve is extrapolated. One of the standard curves used is the harmonic decline model, that is

$$q = \frac{b}{1 + at}$$

Where *q* is the rate of production and *t* is the time, *a* and *b* are the constants of the regression model. **You are asked to transform the data.**

Time, *t* (months)	2	6	10	14	20.1
Rate of Production, *q* (barrels per day)	260	189	120	87	75

If 5 barrels per day is considered the production at which the field needs to be abandoned for further production, what is the total predicted life of the oil field?

Answer: 351.0 months