Mechanics of Composite Materials

2nd edition, 2005

CRC Taylor & Francis Group
ISBN 10: 0-8493-1343-0
ISBN 13: 978-0-8943-1343-1

Autar K. Kaw

Answers to Selected Problems

Chapter 2

2.1 21, 13, 9, 5, 2

2.2

$$\mathbf{C} = \begin{bmatrix} 13.675 & 6.39 & 10.846 & 0 & 0 & 0 \\ 6.39 & 6.58 & 6.553 & 0 & 0 & 0 \\ 10.846 & 6.553 & 12.316 & 0 & 0 & 0 \\ 0 & 0 & 0 & 7 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 6 \end{bmatrix} \cdot \text{Msi}$$

$$\mathbf{S} = \begin{bmatrix} 0.25 & -0.05 & -0.1935 & 0 & 0 & 0 \\ -0.05 & 0.3333 & -0.1333 & 0 & 0 & 0 \\ -0.1935 & -0.1333 & 0.3226 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.1429 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.1667 \end{bmatrix} \cdot \frac{1}{\text{Msi}}$$

2.3 1.

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{31} \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} -8.269 \\ -6.731 \\ -4.423 \\ 0 \\ 10 \\ 9 \end{bmatrix} \cdot \text{kPa}$$

$$\mathbf{S} = \begin{bmatrix} 0.5 & -0.5 & -0.4 & 0 & 0 & 0 \\ -0.5 & 0.3 & -0.2 & 0 & 0 & 0 \\ -0.4 & -0.2 & 0.6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.25 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.667 \end{bmatrix} \cdot \frac{1}{\text{GPa}}$$

2.

3.

$$E_1 = 2 \cdot GPa$$

$$E_2 = 3.333 \cdot GPa$$

$$E_3 = 1.667 \cdot GPa$$

$$\nu_{12} = 1$$

$$\nu_{23} = 0.6667$$

$$\nu_{31} = 0.6667$$

$$G_{12} = 4 \cdot GPa$$

$$G_{23} = 2 \cdot GPa$$

$$G_{31} = 1.5 \cdot GPa$$

4. $W = 3.335 \cdot 10^{-2} \cdot Pa$

2.8 $C_{11} = \frac{E \cdot (1 - \nu)}{(1 - 2 \cdot \nu) \cdot (1 + \nu)}$ $C_{12} = \frac{E \cdot \nu}{(1 - 2 \cdot \nu) \cdot (1 + \nu)}$

2.9 No, $\frac{\nu_{12}}{E_1} = \frac{\nu_{21}}{E_2}$

2.10 $\mathbf{S} = \begin{pmatrix} 4.902 \cdot 10^{-3} & -1.127 \cdot 10^{-3} & 0 \\ -1.127 \cdot 10^{-3} & 5.405 \cdot 10^{-2} & 0 \\ 0 & 0 & 1.789 \cdot 10^{-1} \end{pmatrix} \cdot \frac{1}{GPa}$ $\mathbf{Q} = \begin{pmatrix} 204.98 & 4.28 & 0 \\ 4.28 & 18.59 & 0 \\ 0 & 0 & 5.59 \end{pmatrix} \cdot GPa$

2.11 $\begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{pmatrix} = \begin{pmatrix} 17.35 \\ 103.59 \\ -536.70 \end{pmatrix} \cdot \frac{\mu m}{m}$

2.12

$$\mathbf{S}=\begin{bmatrix} \frac{1}{E} & -\frac{\nu}{E} & 0 \\ -\frac{\nu}{E} & \frac{1}{E} & 0 \\ 0 & 0 & \frac{1}{G} \end{bmatrix}, \quad \mathbf{Q}=\begin{bmatrix} \frac{E}{1-\nu^2} & \frac{E\cdot\nu}{1-\nu^2} & 0 \\ \frac{E\cdot\nu}{1-\nu^2} & \frac{E}{1-\nu^2} & 0 \\ 0 & 0 & G \end{bmatrix}$$

2.15

$$E_1 = 5.599\cdot\text{Msi}$$

$$E_2 = 1.199\cdot\text{Msi}$$

$$\nu_{12} = 0.2600$$

$$G_{12} = 0.6006\cdot\text{Ms}$$

2.16

$$\begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{pmatrix} = \begin{pmatrix} 9.019\cdot10^{-1} \\ 5.098 \\ -2.366 \end{pmatrix}\cdot\text{MPa}$$

2.17

$$\begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{pmatrix} = \begin{pmatrix} 2.201 \\ 3.799 \\ -3.232 \end{pmatrix}\cdot\frac{\mu\text{in}}{\text{in}}$$

2.18

$$\underline{\mathbf{Q}} = \begin{pmatrix} 29.06 & 40.40 & 19.50 \\ 40.40 & 122.26 & 61.21 \\ 19.50 & 61.21 & 41.71 \end{pmatrix}\cdot\text{GPa}$$

$$\underline{\mathbf{S}} = \begin{pmatrix} 6.383\cdot10^{-2} & -2.319\cdot10^{-2} & 4.195\cdot10^{-3} \\ -2.319\cdot10^{-2} & 3.925\cdot10^{-2} & -4.676\cdot10^{-2} \\ 4.195\cdot10^{-3} & -4.676\cdot10^{-2} & 9.063\cdot10^{-2} \end{pmatrix}\cdot\frac{1}{\text{GPa}}$$

2.20

1. $\begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{pmatrix} = \begin{pmatrix} 196.4 \\ 126.0 \\ -348.6 \end{pmatrix} \cdot \frac{\mu m}{m}$

2. $\begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{pmatrix} = \begin{pmatrix} -7.36 \\ 329.73 \\ 113.40 \end{pmatrix} \cdot \frac{\mu m}{m}$

3. $\sigma_{max} = 6.162 \cdot MPa$

$\sigma_{min} = -0.1623 \cdot MPa$

$\theta_{p\sigma} = -35.78 \cdot °$

$\varepsilon_{max} = 339.0 \cdot \frac{\mu m}{m}$

$\varepsilon_{min} = -16.64 \cdot \frac{\mu m}{m}$

$\theta_{p\varepsilon} = -39.30 \cdot °$

4. $\tau_{max} = 3.162 \cdot MPa$

$\theta_{s\tau} = 9.22 \cdot °$

$\gamma_{max} = 355.7 \cdot \frac{\mu m}{m}$

$\theta_{s\gamma} = 5.70 \cdot °$

2.21

$\theta = 34.98 \cdot °$

2.22

$E_x = 2.272 \cdot Msi$

$\nu_{xy} = 0.3632$

$m_x = -0.8530$

$E_y = 3.696 \cdot Msi$

$m_y = 9.538$

$G_{xy} = 1.6 \cdot Msi$

2.23

The maximum value of G_{xy} occurs at $\theta = 45°$, $G_{xy}(\theta_3) = 26.54 \cdot GPa$

The minimum value of G_{xy} occurs at $\theta = 0°$, $G_{xy}(\theta_1) = 20.00 \cdot GPa$

2.24

1. Actual $\begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{pmatrix} = \begin{pmatrix} 2.190 \cdot 10^5 \\ -6.021 \cdot 10^4 \\ -4.714 \cdot 10^5 \end{pmatrix} \cdot \frac{\mu in}{in}$

Measured $\begin{pmatrix} \varepsilon'_x \\ \varepsilon'_y \\ \gamma'_{xy} \end{pmatrix} = \begin{pmatrix} 1.936 \cdot 10^5 \\ -3.481 \cdot 10^4 \\ -4.980 \cdot 10^5 \end{pmatrix} \cdot \frac{\mu in}{in}$

$\varepsilon_a = 11.60 \cdot \%$

2. Actual $\begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{pmatrix} = \begin{pmatrix} 4.000 \cdot 10^5 \\ -1.200 \cdot 10^5 \\ 0.000 \end{pmatrix} \cdot \frac{\mu in}{in}$

Measured $\begin{pmatrix} \varepsilon'_x \\ \varepsilon'_y \\ \gamma'_{xy} \end{pmatrix} = \begin{pmatrix} 3.986 \cdot 10^5 \\ -1.186 \cdot 10^5 \\ -5.435 \cdot 10^4 \end{pmatrix} \cdot \frac{\mu in}{in}$

$\varepsilon_b = 0.36 \cdot \%$

2.25

$G_{12} = 10.10 \cdot GPa$

$E_x = 13.4 \cdot GPa$

2.26

1. $E_x(30 \cdot °) = 4.173 \cdot Msi$

Only $\frac{1}{G_{12}} - \frac{2 \cdot \nu_{12}}{E_1}$ can be determined, G_{12} and ν_{12} cannot be determined individually.

2.

2.30

LWR	ζ	$E1_x$ (Msi)
2	1.504E-1	2.629
8	1.803E-2	2.275
16	4.724E-3	2.245
64	2.998E-4	2.235

2.32

1. $G_{12} = 0.0488 \cdot GPa$

2. From 35° ply data, one can find $\underline{S}_{11}$ and $\underline{S}_{12}$ for the lamina. However, there are four unknowns: S_{11}, S_{12}, S_{22}, and S_{66}. Therefore, one cannot find S_{66} nor, in turn, the shear modulus, G_{12}. Although the same is true in the case of the 45° ply, no manipulation of will allow S_{66} to be expressed in terms of $\underline{S}_{11}$ and $\underline{S}_{12}$ for the 35° ply.

2.33

$U_1 = 12.72 \cdot Msi$, $U_2 = 13.52 \cdot Msi$, $U_3 = 3.493 \cdot Msi$, $U_4 = 4.113 \cdot Msi$

$V_1 = 3.046 \cdot 10^{-1} \cdot \frac{1}{Msi}$, $V_2 = -1.695 \cdot 10^{-1} \cdot \frac{1}{Msi}$, $V_3 = -1.014 \cdot 10^{-1} \cdot \frac{1}{Msi}$

$V_4 = -1.091 \cdot 10^{-1} \cdot \frac{1}{Msi}$

2.35

	Positive Shear	Negative Shear
Maximum Stress:	$\tau_{M\sigma Pos} := 134.0 \cdot MPa$	$\tau_{M\sigma Neg} := 70.44 \cdot MPa$
Maximum Strain:	$\tau_{M\varepsilon Pos} := 134.0 \cdot MPa$	$\tau_{M\varepsilon Neg} := 68.99 \cdot MPa$
Tsai-Hill:	$\tau_{TH} := 62.24 \cdot MPa$	
Tsai-Wu (Mises-Hencky criterion):	$\tau_{TWMHPos} := 139.0 \cdot MPa$	$\tau_{TWMHNeg} := 59.66 \cdot MPa$

2.38

Maximum Strain: $\sigma_{M\varepsilon} := -249.9 \cdot MPa$

Tsai-Wu (Mises-Hencky criterion): $\sigma_{TWMH} := -259.9 \cdot MPa$

2.41 Yes

2.43 $$\begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{pmatrix} = \begin{pmatrix} -860 \\ -1010 \\ 0 \end{pmatrix} \cdot \frac{\mu m}{m}$$

$\Delta T_{2\_Offset} = -54.30 \cdot °C$

2.44

$$\begin{pmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{pmatrix} = \begin{pmatrix} 10.403 \\ 6.653 \\ -6.495 \end{pmatrix} \cdot \frac{\frac{\mu in}{in}}{°F}$$

2.46

$$\begin{pmatrix} \beta_x \\ \beta_y \\ \beta_{xy} \end{pmatrix} = \begin{pmatrix} 0.4500 \\ 0.1500 \\ -0.5196 \end{pmatrix} \cdot \frac{\frac{m}{m}}{\frac{kg}{kg}}$$