Mechanics of Composite Materials
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Answers to Selected Problems

Chapter 4 - Macromechanical Analysis of a Laminates

Exercise Set

4.1 Laminate codes

[0/45/-45/90] = [0/±45/90]

[0/45/-45/-45/45/0] = $[0/\pm 45]_S$

[0/90/60/60/90/0] = $[0/90/60]_S$

[0/45/60/45/0] = $[0/45/\overline{60}]_S$

[45/-45/45/-45/-45/45/-45/45] = $[\pm 45_2]_S$

4.2 Laminate codes

$[45/-45]_S$ = [45/-45/-45/45]

$[45/-45_2/90]_S$ = [45/-45/-45/90/90/-45/-45/45]

$[45/0]_{3S}$ = [45/0/45/0/45/0/0/45/0/45/0/45]

$[45/\pm 30]_2$ = [45/30/-30/45/30/-30]

$[45/\mp 30]_2$ = [45/-30/30/45/-30/30]

4.3 Given for a laminate under a complex load

Global strains at top surface of laminate -

$$\varepsilon_0 = \begin{pmatrix} 1.100\cdot 10^{-6} \\ 1.875\cdot 10^{-6} \\ 2.050\cdot 10^{-6} \end{pmatrix} \cdot \frac{in}{in}$$

Global strains at middle surface of laminate -

$$\varepsilon_1 = \begin{pmatrix} 2\cdot 10^{-6} \\ 3\cdot 10^{-6} \\ 4\cdot 10^{-6} \end{pmatrix} \cdot \frac{in}{in}$$

Global strains at bottom surface of laminate -

$$\begin{pmatrix} 2.9\times10^{-6} \\ 4.125\times10^{-6} \\ 5.950\times10^{-6} \end{pmatrix} in/in$$

4.4 Yes, the global strains vary linearly through the thickness of the laminate. Equation (4.16) gives the global laminate strains as a linear function of z, the distance from the mid-plane along the thickness of the laminate.

4.5 No, the global stresses at a point are functions of position, material, and ply angle. The global stresses vary linearly only through the thickness of each lamina making up the laminate. Equation (4.18) shows how there may be discontinuities from lamina to lamina as the ply angle and/or the lamina material change.

4.6 Given for a $[0/45/60]_S$ laminate

$$\begin{pmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{pmatrix} = \begin{pmatrix} 5.581\cdot10^{-4} \\ 3.175\cdot10^{-2} \\ -2.086\cdot10^{-1} \end{pmatrix}\cdot\frac{1}{\text{in}}$$

4.7

Force in x-direction =4.110 lbf
Force in y-direction =4.000 lbf
Shear force in x-y plane = 3.990 lbf
Bending moment around y-axis corresponds to Mx=4.129 lbf-in
Bending moment around x-axis corresponds to My=4.129 lbf-in
Twisting moment =4.129 lbf-in

4.8 For a [0/60/-60] Glass/Epoxy laminate

$$\mathbf{A} = \begin{pmatrix} 4.45\cdot10^4 & 1.2\cdot10^4 & 0 \\ 1.2\cdot10^4 & 4.45\cdot10^4 & 0 \\ 0 & 0 & 1.625\cdot10^4 \end{pmatrix}\cdot\text{psi}\cdot\text{in}$$

$$\mathbf{B} = \begin{pmatrix} -101.84 & 18.13 & -13.70 \\ 18.13 & 65.59 & -34.63 \\ -13.70 & -34.63 & 18.13 \end{pmatrix}\cdot\text{psi}\cdot\text{in}^2$$

$$\mathbf{D} = \begin{pmatrix} 1.0041 & 0.1948 & -0.0685 \\ 0.1948 & 0.7251 & -0.1732 \\ -0.0685 & -0.1732 & 0.2745 \end{pmatrix}\cdot\text{psi}\cdot\text{in}^3$$

$$m = 3.384\cdot10^{-2}\cdot\text{lbm}$$

4.9

$$\mathbf{A}=\begin{bmatrix} \frac{E}{1-\nu^2} & E\cdot\frac{\nu}{1-\nu^2} & 0 \\ E\cdot\frac{\nu}{1-\nu^2} & \frac{E}{1-\nu^2} & 0 \\ 0 & 0 & \frac{1}{2}\cdot\frac{E}{(1+\nu)} \end{bmatrix}\cdot t$$

$$\mathbf{B}=0$$

$$\mathbf{D}=\frac{1}{12}\cdot\begin{bmatrix} \frac{E}{1-\nu^2} & E\cdot\frac{\nu}{1-\nu^2} & 0 \\ E\cdot\frac{\nu}{1-\nu^2} & \frac{E}{1-\nu^2} & 0 \\ 0 & 0 & \frac{1}{2}\cdot\frac{E}{(1+\nu)} \end{bmatrix}\cdot t^3$$

4.11

$$\mathbf{A}=t\cdot\left[\begin{bmatrix} \frac{E_1}{\left[1-(\nu_1)^2\right]} & E_1\cdot\frac{\nu_1}{\left[1-(\nu_1)^2\right]} & 0 \\ E_1\cdot\frac{\nu_1}{\left[1-(\nu_1)^2\right]} & \frac{E_1}{\left[1-(\nu_1)^2\right]} & 0 \\ 0 & 0 & \frac{1}{2}\cdot\frac{E_1}{(1+\nu_1)} \end{bmatrix} \cdots \right.$$

$$\left. + \begin{bmatrix} \frac{E_2}{\left[1-(\nu_2)^2\right]} & E_2\cdot\frac{\nu_2}{\left[1-(\nu_2)^2\right]} & 0 \\ E_2\cdot\frac{\nu_2}{\left[1-(\nu_2)^2\right]} & \frac{E_2}{\left[1-(\nu_2)^2\right]} & 0 \\ 0 & 0 & \frac{1}{2}\cdot\frac{E_2}{(1+\nu_2)} \end{bmatrix}\right]$$

$$\mathbf{B=}\frac{1}{2}\cdot t^{2}\cdot\left[-\begin{bmatrix}\frac{E_1}{\left[1-\left(\nu_1\right)^2\right]} & E_1\cdot\frac{\nu_1}{\left[1-\left(\nu_1\right)^2\right]} & 0\\ E_1\cdot\frac{\nu_1}{\left[1-\left(\nu_1\right)^2\right]} & \frac{E_1}{\left[1-\left(\nu_1\right)^2\right]} & 0\\ 0 & 0 & \frac{1}{2}\cdot\frac{E_1}{\left(1+\nu_1\right)}\end{bmatrix}\cdots\right.$$

$$\left.+\begin{bmatrix}\frac{E_2}{\left[1-\left(\nu_2\right)^2\right]} & E_2\cdot\frac{\nu_2}{\left[1-\left(\nu_2\right)^2\right]} & 0\\ E_2\cdot\frac{\nu_2}{\left[1-\left(\nu_2\right)^2\right]} & \frac{E_2}{\left[1-\left(\nu_2\right)^2\right]} & 0\\ 0 & 0 & \frac{1}{2}\cdot\frac{E_2}{\left(1+\nu_2\right)}\end{bmatrix}\right]$$

$$\mathbf{D=}\frac{1}{3}\cdot t^{3}\cdot\left[\begin{bmatrix}\frac{E_1}{\left[1-\left(\nu_1\right)^2\right]} & E_1\cdot\frac{\nu_1}{\left[1-\left(\nu_1\right)^2\right]} & 0\\ E_1\cdot\frac{\nu_1}{\left[1-\left(\nu_1\right)^2\right]} & \frac{E_1}{\left[1-\left(\nu_1\right)^2\right]} & 0\\ 0 & 0 & \frac{1}{2}\cdot\frac{E_1}{\left(1+\nu_1\right)}\end{bmatrix}\cdots\right.$$

$$\left.+\begin{bmatrix}\frac{E_2}{\left[1-\left(\nu_2\right)^2\right]} & E_2\cdot\frac{\nu_2}{\left[1-\left(\nu_2\right)^2\right]} & 0\\ E_2\cdot\frac{\nu_2}{\left[1-\left(\nu_2\right)^2\right]} & \frac{E_2}{\left[1-\left(\nu_2\right)^2\right]} & 0\\ 0 & 0 & \frac{1}{2}\cdot\frac{E_2}{\left(1+\nu_2\right)}\end{bmatrix}\right]$$

4.12

$$\mathbf{A=}\left[\sum_{k=1}^{n}\left(\underline{\mathbf{Q}}\left(\theta_k\right)\cdot t_k\right)\right]$$

$$\mathbf{B=}\left[\sum_{k=1}^{n}\left(\underline{\mathbf{Q}}\left(\theta_k\right)\cdot t_k\cdot z_{\,mid_k}\right)\right]$$

$$\mathbf{D=}\sum_{k=1}^{n}\underline{\mathbf{Q}}(\theta_k)\cdot t_k\cdot\left[\frac{(t_k)^2}{12}+(z\ mid_k)^2\right]$$

4.13

$$\begin{pmatrix}\sigma_{1\_Top60°}\\ \sigma_{2\_Top60°}\\ \tau_{12\_Top60°}\end{pmatrix}=\begin{pmatrix}-3.988\cdot10^{8}\\ 9.775\cdot10^{7}\\ -9.264\cdot10^{7}\end{pmatrix}\cdot Pa$$

$$M_{x\_0°}=32.95\cdot\frac{N\cdot m}{m}$$

$$M_{x\_60°}=1.185\cdot\frac{N\cdot m}{m}$$

$$M_{x\_Neg60°}=15.864\cdot\frac{N\cdot m}{m}$$

Verifying total load on laminate -

$$M_x := M_{x\_0°}+M_{x\_60°}+M_{x\_Neg60°}$$

$$M_x=50\cdot\frac{N\cdot m}{m}$$

Percentage of bending moment load taken by each ply-

$$M\%_{x\_0°}=65.90\cdot\%$$

$$M\%_{x\_60°}=2.37\cdot\%$$

$$M\%_{x\_Neg60°}=31.73\cdot\%$$

4.14

$N_x := -46.633\cdot\frac{lbf}{in}$ $\quad N_y := 46.633\cdot\frac{lbf}{in}$ $\quad N_{xy} := -26.923\cdot\frac{lbf}{in}$

$M_x := 0.4813\cdot\frac{lbf\cdot in}{in}$ $\quad M_y := 0.3259\cdot\frac{lbf\cdot in}{in}$ $\quad M_{xy} := -0.1346\cdot\frac{lbf\cdot in}{in}$

4.15

$E_x = 25.03\cdot GPa$

$E_y = 25.03\cdot GPa$

$G_{xy} = 46.6\cdot GPa$

$\nu_{xy} = .7471$

$\nu_{yx} = .7471$

$E_{fx} = 22.71\cdot GPa$

$E_{fy} = 22.71\cdot GPa$

$$G_{fxy} = 25.7 \cdot GPa$$

$$\nu_{fxy} = .5838$$

$$\nu_{fyx} = .5838$$

4.16

$$\begin{pmatrix} \sigma_{x\_Top60°} \\ \sigma_{y\_Top60°} \\ \gamma_{xy\_Top60°} \end{pmatrix} = \begin{pmatrix} 0.8727 \\ -2.9246 \\ -2.1745 \end{pmatrix} \cdot ksi$$

4.17

$$\begin{pmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{pmatrix} = \begin{pmatrix} 9.697 \\ 17.811 \\ -8.115 \end{pmatrix} \cdot \frac{\frac{\mu m}{m}}{°C}$$

$$\Delta V = 11.346 \cdot mm^3$$

4.18

$$\begin{pmatrix} \beta_x \\ \beta_y \\ \beta_{xy} \end{pmatrix} = \begin{pmatrix} 1.731 \cdot 10^{-2} \\ 3.586 \cdot 10^{-1} \\ -3.413 \cdot 10^{-1} \end{pmatrix} \cdot \frac{\frac{m}{m}}{\frac{kg}{kg}}$$

4.19

$$\begin{pmatrix} \sigma_{1\_Mid30°} \\ \sigma_{2\_Mid30°} \\ \gamma_{12\_Mid30°} \end{pmatrix} = \begin{pmatrix} 8.550 \cdot 10^{6} \\ 2.563 \cdot 10^{6} \\ -4.371 \cdot 10^{6} \end{pmatrix} \cdot ksi$$

4.20

Since the deflection at center of laminate is assumed to be zero,

$$w(0 \cdot in, 0 \cdot in) = 0 \cdot in$$

and the coordinate definitions are

$x_{Left} := -10 \cdot in$ $\qquad$ $x_{Right} := 10 \cdot in$

$y_{Bottom} := -5 \cdot in$ $\qquad$ $y_{Top} := 5 \cdot in$

Then,

Deflection at upper right hand corner of laminate -

$$w(x_{Right}, y_{Top}) = 0.5054 \cdot in$$

Deflection at lower right hand corner of laminate -

$$w(x_{Right}, y_{Bottom}) = 3.885 \cdot in$$

Deflection at upper left hand corner of laminate -

$$w(x_{Left}, y_{Top}) = 3.885 \cdot in$$

Deflection at lower left hand corner of laminate -

$$w\left(x_{Left}, y_{Bottom}\right) = 0.5054 \cdot in$$

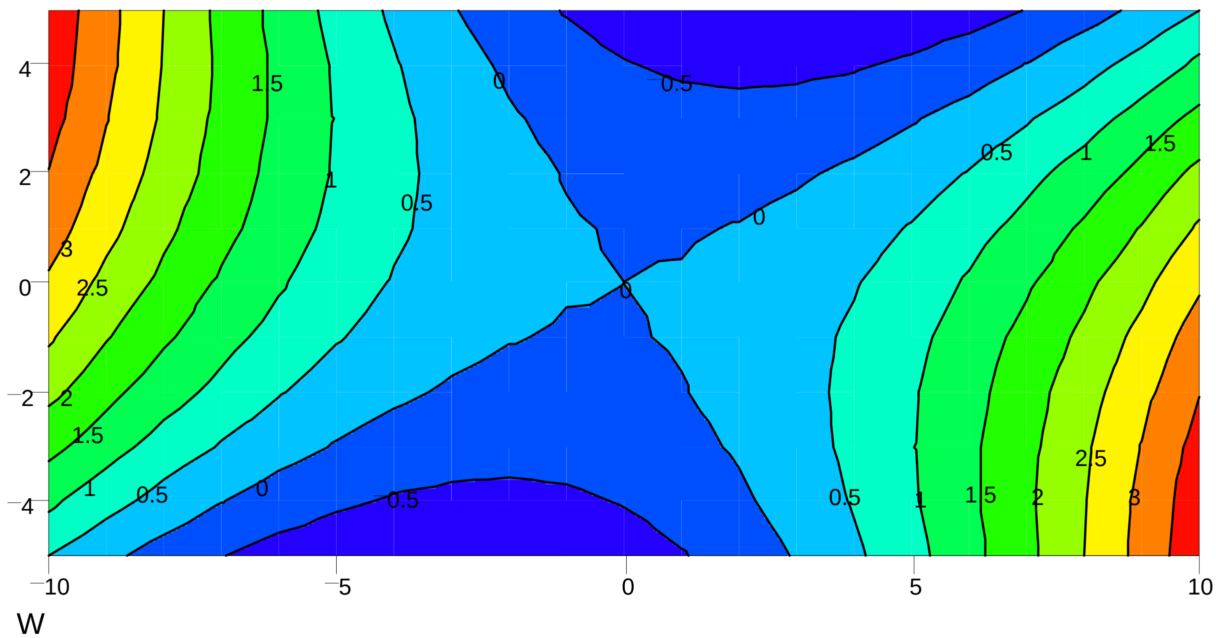

Figure P4.20 (a) - Contour plot of laminate deflection

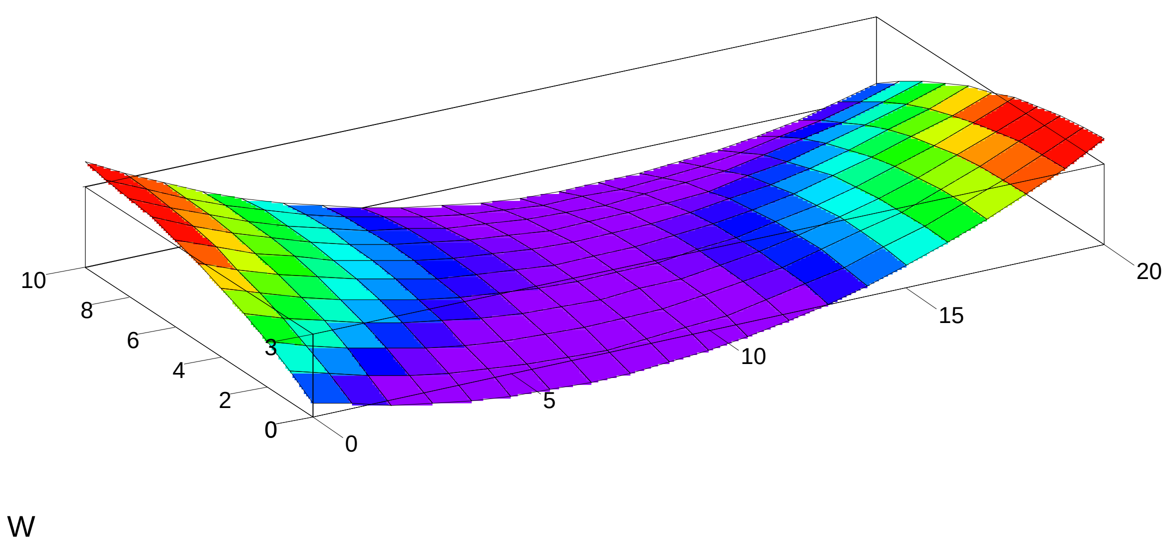

Figure P4.20 (b) - Surface plot of laminate deflection