

Mechanics of Composite Materials
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Answers to Selected Problems

Chapter 4 - Macromechanical Analysis of a Laminates

Exercise Set

4.1 Laminate codes

$$\begin{aligned}
 [0/45/-45/90] &= [0/\pm 45/90] \\
 [0/45/-45/-45/45/0] &= [0/\pm 45]_S \\
 [0/90/60/60/90/0] &= [0/90/60]_S \\
 [0/45/60/45/0] &= [0/45/60]_S \\
 [45/-45/45/-45/-45/45/-45/45] &= [\pm 45]_2
 \end{aligned}$$

4.2 Laminate codes

$$\begin{aligned}
 [45/-45]_S &= [45/-45/-45/45] \\
 [45/-45_2/90]_S &= [45/-45/-45/90/90/-45/-45/45] \\
 [45/0]_{3S} &= [45/0/45/0/45/0/0/45/0/45/0/45] \\
 [45/\pm 30]_2 &= [45/30/-30/45/30/-30] \\
 [45/\pm 30]_2 &= [45/-30/30/45/-30/30]
 \end{aligned}$$

4.3 Given for a laminate under a complex load

Global strains at top surface of laminate -

$$\epsilon_0 = \begin{pmatrix} 1.100 \cdot 10^{-6} \\ 1.875 \cdot 10^{-6} \\ 2.050 \cdot 10^{-6} \end{pmatrix} \cdot \frac{\text{in}}{\text{in}}$$

Global strains at middle surface of laminate -

$$\epsilon_1 = \begin{pmatrix} 2 \cdot 10^{-6} \\ 3 \cdot 10^{-6} \\ 4 \cdot 10^{-6} \end{pmatrix} \cdot \frac{\text{in}}{\text{in}}$$

Global strains at bottom surface of laminate -

$$\varepsilon_2 = \begin{pmatrix} 2.900 \cdot 10^{-6} \\ 4.125 \cdot 10^{-6} \\ 5.950 \cdot 10^{-6} \end{pmatrix} \cdot \frac{\text{in}}{\text{in}}$$

4.4 Yes, the global strains vary linearly through the thickness of the laminate. Equation (4.16) gives the global laminate strains as a linear function of z , the distance from the mid-plane along the thickness of the laminate.

4.5 No, the global stresses at a point are functions of position, material, and ply angle. The global stresses vary linearly only through the thickness of each lamina making up the laminate. Equation (4.18) shows how there may be discontinuities from lamina to lamina as the ply angle and/or the lamina material change.

4.6 Given for a $[0/45/60]_S$ laminate

$$\begin{pmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{pmatrix} = \begin{pmatrix} 5.581 \cdot 10^{-4} \\ 3.175 \cdot 10^{-2} \\ -2.086 \cdot 10^{-1} \end{pmatrix} \cdot \frac{1}{\text{in}}$$

4.7

Force in x-direction = 4.110 lbf

Force in y-direction = 4.000 lbf

Shear force in x-y plane = 3.990 lbf

Bending moment around y-axis corresponds to $M_x = 4.129$ lbf-in

Bending moment around x-axis corresponds to $M_y = 4.129$ lbf-in

Twisting moment = 4.129 lbf-in

4.8 For a $[0/60/-60]$ Glass/Epoxy laminate

$$\mathbf{A} = \begin{pmatrix} 4.45 \cdot 10^4 & 1.2 \cdot 10^4 & 0 \\ 1.2 \cdot 10^4 & 4.45 \cdot 10^4 & 0 \\ 0 & 0 & 1.625 \cdot 10^4 \end{pmatrix} \cdot \text{psi} \cdot \text{in}$$

$$\mathbf{B} = \begin{pmatrix} -101.84 & 18.13 & -13.70 \\ 18.13 & 65.59 & -34.63 \\ -13.70 & -34.63 & 18.13 \end{pmatrix} \cdot \text{psi} \cdot \text{in}^2$$

$$\mathbf{D} = \begin{pmatrix} 1.0041 & 0.1948 & -0.0685 \\ 0.1948 & 0.7251 & -0.1732 \\ -0.0685 & -0.1732 & 0.2745 \end{pmatrix} \cdot \text{psi} \cdot \text{in}^3$$

$$m = 3.384 \cdot 10^{-2} \cdot \text{lbm}$$

4.9

$$\mathbf{A} = \begin{bmatrix} \frac{E}{1-v^2} & E \cdot \frac{v}{1-v^2} & 0 \\ E \cdot \frac{v}{1-v^2} & \frac{E}{1-v^2} & 0 \\ 0 & 0 & \frac{1}{2} \cdot \frac{E}{(1+v)} \end{bmatrix} \cdot t$$

$$\mathbf{B} = 0$$

$$\mathbf{D} = \frac{1}{12} \cdot \begin{bmatrix} \frac{E}{1-v^2} & E \cdot \frac{v}{1-v^2} & 0 \\ E \cdot \frac{v}{1-v^2} & \frac{E}{1-v^2} & 0 \\ 0 & 0 & \frac{1}{2} \cdot \frac{E}{(1+v)} \end{bmatrix} \cdot t^3$$

4.11

$$\mathbf{A} = t \cdot \left[\begin{array}{c} \begin{bmatrix} \frac{E_1}{[1-(v_1)^2]} & E_1 \cdot \frac{v_1}{[1-(v_1)^2]} & 0 \\ E_1 \cdot \frac{v_1}{[1-(v_1)^2]} & \frac{E_1}{[1-(v_1)^2]} & 0 \\ 0 & 0 & \frac{1}{2} \cdot \frac{E_1}{(1+v_1)} \end{bmatrix} \dots \\ + \begin{bmatrix} \frac{E_2}{[1-(v_2)^2]} & E_2 \cdot \frac{v_2}{[1-(v_2)^2]} & 0 \\ E_2 \cdot \frac{v_2}{[1-(v_2)^2]} & \frac{E_2}{[1-(v_2)^2]} & 0 \\ 0 & 0 & \frac{1}{2} \cdot \frac{E_2}{(1+v_2)} \end{bmatrix} \end{array} \right]$$

$$\mathbf{B} = \frac{1}{2} \cdot t^2 \cdot \left[\begin{array}{c} \left[\begin{array}{ccc} \frac{E_1}{[1 - (v_1)^2]} & E_1 \cdot \frac{v_1}{[1 - (v_1)^2]} & 0 \\ E_1 \cdot \frac{v_1}{[1 - (v_1)^2]} & \frac{E_1}{[1 - (v_1)^2]} & 0 \\ 0 & 0 & \frac{1}{2} \cdot \frac{E_1}{(1 + v_1)} \end{array} \right] \dots \\ + \left[\begin{array}{ccc} \frac{E_2}{[1 - (v_2)^2]} & E_2 \cdot \frac{v_2}{[1 - (v_2)^2]} & 0 \\ E_2 \cdot \frac{v_2}{[1 - (v_2)^2]} & \frac{E_2}{[1 - (v_2)^2]} & 0 \\ 0 & 0 & \frac{1}{2} \cdot \frac{E_2}{(1 + v_2)} \end{array} \right] \end{array} \right] \\
 \mathbf{D} = \frac{1}{3} \cdot t^3 \cdot \left[\begin{array}{c} \left[\begin{array}{ccc} \frac{E_1}{[1 - (v_1)^2]} & E_1 \cdot \frac{v_1}{[1 - (v_1)^2]} & 0 \\ E_1 \cdot \frac{v_1}{[1 - (v_1)^2]} & \frac{E_1}{[1 - (v_1)^2]} & 0 \\ 0 & 0 & \frac{1}{2} \cdot \frac{E_1}{(1 + v_1)} \end{array} \right] \dots \\ + \left[\begin{array}{ccc} \frac{E_2}{[1 - (v_2)^2]} & E_2 \cdot \frac{v_2}{[1 - (v_2)^2]} & 0 \\ E_2 \cdot \frac{v_2}{[1 - (v_2)^2]} & \frac{E_2}{[1 - (v_2)^2]} & 0 \\ 0 & 0 & \frac{1}{2} \cdot \frac{E_2}{(1 + v_2)} \end{array} \right] \end{array} \right]$$

4.12

$$\mathbf{A} = \left[\sum_{k=1}^n (\underline{Q}(\theta_k) \cdot t_k) \right] \\
 \mathbf{B} = \left[\sum_{k=1}^n (\underline{Q}(\theta_k) \cdot t_k \cdot z_{mid_k}) \right]$$

$$D = \sum_{k=1}^n \underline{Q}(\theta_k) \cdot t_k \cdot \left[\frac{(t_k)^2}{12} + (z_{\text{mid}_k})^2 \right]$$

4.13

$$\begin{pmatrix} \sigma_{1_Top60^\circ} \\ \sigma_{2_Top60^\circ} \\ \tau_{12_Top60^\circ} \end{pmatrix} = \begin{pmatrix} -3.988 \cdot 10^8 \\ 9.775 \cdot 10^7 \\ -9.264 \cdot 10^7 \end{pmatrix} \cdot \text{Pa}$$

$$M_{x_0^\circ} = 32.95 \cdot \frac{\text{N} \cdot \text{m}}{\text{m}}$$

$$M_{x_60^\circ} = 1.185 \cdot \frac{\text{N} \cdot \text{m}}{\text{m}}$$

$$M_{x_Neg60^\circ} = 15.864 \cdot \frac{\text{N} \cdot \text{m}}{\text{m}}$$

Verifying total load on laminate -

$$M_x := M_{x_0^\circ} + M_{x_60^\circ} + M_{x_Neg60^\circ}$$

$$M_x = 50 \cdot \frac{\text{N} \cdot \text{m}}{\text{m}}$$

Percentage of bending moment load taken by each ply-

$$M\%_{x_0^\circ} = 65.90 \cdot \%$$

$$M\%_{x_60^\circ} = 2.37 \cdot \%$$

$$M\%_{x_Neg60^\circ} = 31.73 \cdot \%$$

4.14

$$N_x := -46.633 \cdot \frac{\text{lbf}}{\text{in}}$$

$$N_y := 46.633 \cdot \frac{\text{lbf}}{\text{in}}$$

$$N_{xy} := -26.923 \cdot \frac{\text{lbf}}{\text{in}}$$

$$M_x := 0.4813 \cdot \frac{\text{lbf} \cdot \text{in}}{\text{in}}$$

$$M_y := 0.3259 \cdot \frac{\text{lbf} \cdot \text{in}}{\text{in}}$$

$$M_{xy} := -0.1346 \cdot \frac{\text{lbf} \cdot \text{in}}{\text{in}}$$

4.15

$$E_x = 25.03 \cdot \text{GPa}$$

$$E_y = 25.03 \cdot \text{GPa}$$

$$G_{xy} = 46.6 \cdot \text{GPa}$$

$$\nu_{xy} = .7471$$

$$\nu_{yx} = .7471$$

$$E_{fx} = 22.71 \cdot \text{GPa}$$

$$E_{fy} = 22.71 \cdot \text{GPa}$$

$$G_{fxy} = 25.7 \cdot \text{GPa}$$

$$\nu_{fxy} = .5838$$

$$\nu_{fyx} = .5838$$

4.16

$$\begin{pmatrix} \sigma_{x_Top60^\circ} \\ \sigma_{y_Top60^\circ} \\ \gamma_{xy_Top60^\circ} \end{pmatrix} = \begin{pmatrix} 0.8727 \\ -2.9246 \\ -2.1745 \end{pmatrix} \cdot \text{ksi}$$

4.17

$$\begin{pmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{pmatrix} = \begin{pmatrix} 9.697 \\ 17.811 \\ -8.115 \end{pmatrix} \cdot \begin{matrix} \frac{\mu\text{m}}{\text{m}} \\ \frac{\text{m}}{^\circ\text{C}} \end{matrix}$$

$$\Delta V = 11.346 \cdot \text{mm}^3$$

4.18

$$\begin{pmatrix} \beta_x \\ \beta_y \\ \beta_{xy} \end{pmatrix} = \begin{pmatrix} 1.731 \cdot 10^{-2} \\ 3.586 \cdot 10^{-1} \\ -3.413 \cdot 10^{-1} \end{pmatrix} \cdot \begin{matrix} \frac{\text{m}}{\text{kg}} \\ \frac{\text{m}}{\text{kg}} \\ \frac{\text{m}}{\text{kg}} \end{matrix}$$

4.19

$$\begin{pmatrix} \sigma_{1_Mid30^\circ} \\ \sigma_{2_Mid30^\circ} \\ \gamma_{12_Mid30^\circ} \end{pmatrix} = \begin{pmatrix} 8.550 \cdot 10^6 \\ 2.563 \cdot 10^6 \\ -4.371 \cdot 10^6 \end{pmatrix} \cdot \text{ksi}$$

4.20

Since the deflection at center of laminate is assumed to be zero,

$$w(0 \cdot \text{in}, 0 \cdot \text{in}) = 0 \cdot \text{in}$$

and the coordinate definitions are

$$x_{\text{Left}} := -10 \cdot \text{in}$$

$$y_{\text{Bottom}} := -5 \cdot \text{in}$$

$$x_{\text{Right}} := 10 \cdot \text{in}$$

$$y_{\text{Top}} := 5 \cdot \text{in}$$

Then,

Deflection at upper right hand corner of laminate -

$$w(x_{\text{Right}}, y_{\text{Top}}) = 0.5054 \cdot \text{in}$$

Deflection at lower right hand corner of laminate -

$$w(x_{\text{Right}}, y_{\text{Bottom}}) = 3.885 \cdot \text{in}$$

Deflection at upper left hand corner of laminate -

$$w(x_{\text{Left}}, y_{\text{Top}}) = 3.885 \cdot \text{in}$$

Deflection at lower left hand corner of laminate -

$$w(x_{\text{Left}}, y_{\text{Bottom}}) = 0.5054 \cdot \text{in}$$

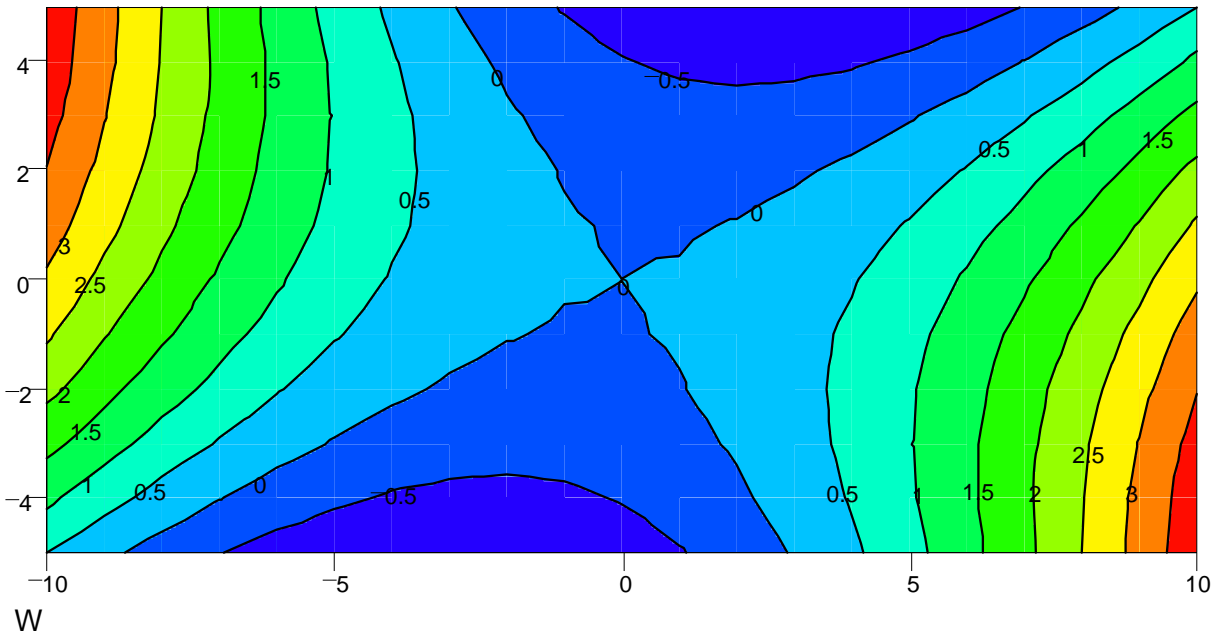


Figure P4.20 (a) - Contour plot of laminate deflection

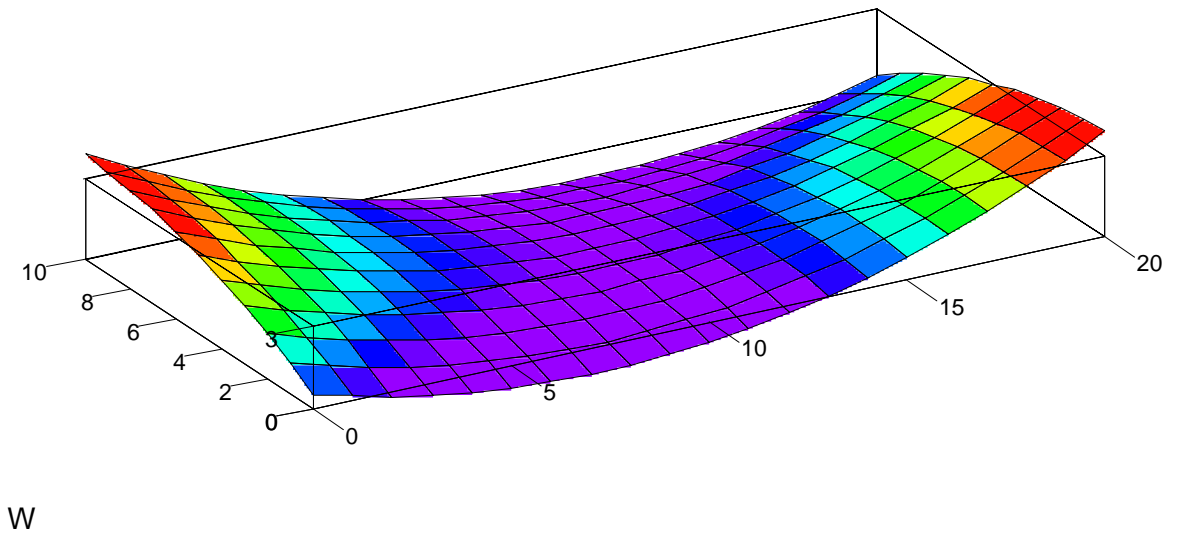


Figure P4.20 (b) - Surface plot of laminate deflection