SURFACE RUNOFF

What factors influence surface runoff?

**Basin characteristics**
- size
- shape
- slope
- land use (cover)
- soil type
- antecedent conditions

**Storm characteristics**
- storm intensity \(i(t)\) and total depth
- storm duration
- spatial variation
- movement

**Hydrography**
- size
- shape
- condition of flow conveyance systems
  (sometimes called the degree of development of the flow system or drainage density)

See Appendix Figure Relationships Between Flow Elements
HYDROGRAPH ANALYSIS

- Definitions
- Losses
- Hydrograph separations
- Unit Hydrographs
- Synthetic Unit Hydrographs

Can we learn anything about the basin from a measured runoff event?

Streamflow = Direct Runoff + Baseflow

Direct Runoff = rainfall excess, $i_e$
   = rainfall - losses

Losses = interception, infiltration, depression storage, etc.
   = sometimes called basin recharge

Rainfall excess or direct runoff = Overland Flow
Interflow (classically) is runoff that infiltrates the top layers of soil and exits to stream prior to reaching zone of saturation.

In Florida, sometimes call interflow water which infiltrates to shallow water table and moves laterally to streams.

Interflow is a rapid phenomenon; baseflow is a slow phenomenon; varies by season.

Baseflow - entry of groundwater into stream

Surface flow:

1) Perennial - all year
2) Ephemeral - seasonal; dry period, wet period
3) Intermittent - streams that go dry along their length that may come back up again downstream

- measure flow in perennial stream
HYDROGRAPH ANALYSIS

- basins with a lot of storage have a large recessional limb
- recessional limb typically has a long tail for Florida
- recession occurs exponentially for baseflow
- hard to say what part is surface runoff and what part is baseflow; have to make assumptions
BASEFLOW SEPARATION

- have to separate baseflow before we can construct hydrograph
- empirical and common practice methods

Empirical Method:

\[ N = A^{0.2} \] - \( N \) is from the point of peak discharge to the point where flow is completely dominated by baseflow
Common Practice Methods:

1) By-Eye Method
2) **Inflection Point Method**

(long hydrographs (TB), large basins)

![Diagram of hydrograph with inflection point](image)

3) **Exponential Point Method** - take advantage of exponential shape

\[ Q = Q_0 e^{-kt} \]
\[ \ln \Theta = \ln \Theta_0 \cdot (-kt) \]

1. Plot on semi-log paper
2. Draw straight lines through recessional data
3. Get slopes (slope is same to tp since relates to physical conditions of basin); slope relates to \( k = \) rise/run
4. Transfer back to arithmetic paper

See Appendix Baseflow Separation Bedient
1) **Regression Analysis**

- works best for long time periods
- works best when there is not appreciable lag (storage) in the system, e.g., more urbanized catchments
  
  (see paper by Diskin, 1970)
- Better if basins haven't experienced significant changes (development)
2) **Coefficient Method**

Runoff volume = \( C(P - DS) \)

Volume → Depth [inches, or cm]
P = Rainfall volume
C = Runoff Coefficient ( 0 ≤ C ≤ 1)
DS = Depression Storage
    = Initial Abstractions = IA

- real, but hard to measure volume filled prior to runoff
- difficult to separate DS from other losses (esp. infiltration)

For areas that are mostly impervious (e.g. urban)

**Chicago Study:**
- DS = 1/16 in for impervious (0.0625 in)
- DS = 1/4 in for pervious (0.25 in)

**Los Angeles:**

<table>
<thead>
<tr>
<th>Soil</th>
<th>DS (in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sand</td>
<td>0.20</td>
</tr>
<tr>
<td>Loam</td>
<td>0.15</td>
</tr>
<tr>
<td>Clay</td>
<td>0.10</td>
</tr>
</tbody>
</table>

**Viessman Data (Chap. 2 or 3):**

\( DS = 0.13 - 0.0301 \cdot S \)

→ linear function

\( S = \) slope in %

\[ d_p = 0.0303 \cdot S^{-0.49} \] curvilinear
3) **Rational Method**

\[ Q_p = C i A \]

- for predicting peak flows
- also called **Lloyd Davis Method** (Ireland, 1850)

\( Q_p = \) peak flow, cfs  
\( C = \) runoff coefficient  
\( i = \) rainfall intensity, in/hr  
\( A = \) area, ac  
\( \text{cfs} \approx \text{in/hr} \cdot \text{ac} (=1.008) \) (handy conversion)

For runoff coefficient, many tables → see Viessman, Table 11-1; Bedient & Huber, Table 6.6, page 396

\[ C \approx 0.95 \text{ for paved areas} \quad C \approx 0.10 \text{ for natural areas} \]

Correct application of Rational Method:

Obtain \( i \) from **IDF** curves with \( t_r \) and \( T \) defined:

Higher \( T \) for more important flood control

\[ T \approx 2-3 \text{ yr for residential} \]
\[ T \approx 10-25 \text{ yr for major highways} \]
Set: $t_r = t_c =$ basin “response time” or time of concentration (time of wave, not particle of water)

$t_r > t_c$ lower peak because i is too low

$t_r < t_c$ also not correct (whole basin not contributing)

Big criticism - no catchment really behaves like this with exception of small metal roof tops because of D.S.

For peak flow estimation - OK

IDF - FDOT Drainage Manual (currently being revised)

New manual - Ch. 6 - see also K.E. Weldon, FDOT - Rainfall IDF "Curve Generation"
4) **Index Flood Method** - Regression Technique  
(Viessman, p. 214, 524)

For Florida, USGS, Barnes & Golden

How does it work?

1) Region is divided into homogeneous sub-areas (for purposes of predicting peak flows)

   a.) Regression

   ![Graph showing the relationship between Mean Annual Flow and Area](image)

   b.) Then take **Flood Flow** (Mean Annual)

   ![Graph showing the relationship between Flood Flow and Return Period](image)

Storage Impact - effect of lakes and swamps
5) **SCS Method**

- prediction of runoff volume (depth)
- very popular
- based upon field measurements
- valid for areas: \( 5 \leq A \leq 2000 \text{ ac} \)
  \( (3 \text{ sq. miles}) \)
- massive database; e.g. hydrologic classification of a soil (A, B, C, D); soil survey interpretation sheets

Define: \( P_e = P - I_a = F + Q \) (like rainfall excess)
Define: \( P_e = \text{Precipitation excess [inches]} \)
\( F = \text{Infiltration} = P_e - Q \text{ [inches]} \)
\( S = \text{Storage (max available soil storage) [inches]} \)
\( Q = \text{Runoff [inches]} \) (normalized by area)

\[
Q = \frac{F}{S} P_e = \frac{F}{S} (P - I_a)
\]

\[
Q = \frac{P_e^2}{P_e + S} = \frac{(P - I_a)^2}{P - I_a + S}
\]

\( I_a = ? \) \( \) Assumed by SCS = 0.2 \( \cdot \) Storage = 0.2 \( \cdot \) S

Therefore, SCS Runoff Equation for Total Volume:

\[
Q = \frac{(P - .2S)^2}{P + .8S}
\]

\( P = \) Precipitation (total) [inches] (may be design storm or IDF value)
\( Q = \) Runoff value [inches]

*Careful, used for larger events only - \( P > 2.5" \)
What is $S$?
- use SCS database
- Assumed $f(\text{soil})$
- $S$ represents soil storage [inches]
- Each soil has a curve number (CN) - a measure of soil storage
- If $S = 0$, $CN = 100$ - implies 100% runoff (only time it represents actual percent of runoff)

$$S = \frac{1000}{CN} - 10$$

CN's? Function of:
- Land use
- Hydrologic condition (good, fair, poor)
- Hydrologic Soil Group (A,B,C,D)

Alternately, $$CN = \frac{1000}{10 + S}$$
Hydrologic Soil Groups

<table>
<thead>
<tr>
<th></th>
<th>Description</th>
<th>Min. Infiltration Rate (in/hr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>well-drained soil - sand (high S)</td>
<td>0.3 - 0.45</td>
</tr>
<tr>
<td>B</td>
<td></td>
<td>0.15 - 0.30</td>
</tr>
<tr>
<td>C</td>
<td></td>
<td>0.05 - 0.15</td>
</tr>
<tr>
<td>D</td>
<td>poorly-drained - clay or muck (low S)</td>
<td>0 - 0.05</td>
</tr>
</tbody>
</table>

Dual classification A/C or B/D

- e.g., A/C: A - artificial drains to lower the water table (urbanization or agric. development)
  
  C - natural conditions (Poorly drained)

Hydrologic Condition? Land Use: poor, average, good

Can Adjust CN for Antecedent Moisture Condition

- AMC 3 over 2 inches in last 5 days*
- AMC 2 normal condition, 1-2 in last 5 days*
- AMC 1 dry conditions, < 1 in last 5 days*

(*may need to look at longer periods than 5 days)
SCS Method Using Triangular Unit Hydrograph

**STEPS:**
1) Find CN for area (use area weighted curve number if different land uses)

\[
CN_{\text{avg}} = \frac{\sum (A_iCN_i)}{\sum A_i}
\]

but better to average storages

\[
S_{\text{avg}} = \frac{\sum (A_iS_i)}{\sum A_i}
\]

\[
CN = \frac{1000}{(S + 10)}
\]

2) Choose rainfall
   - need total depth, \( P (= d = \bar{I} \ast tr) \)
   - usually use IDF curve for specified \( T \)
   - use max (6 hrs, \( t_c \)) = \( t_r \)

*Weak point in methodology: assumes a uniform intensity during storm*

3) Obtain total runoff, with \( P, Q \) (inches) from graph or SCS equation

4) Choose family of dimensionless unit hydrographs (UH) (see figure)

5) Get \( t_p = f(t_c) \) or SCS lag equation

\[
t_p = \frac{l^{0.8} (S + 1)^{0.7}}{1900 y^{0.5}}
\]
Then:

6) \( q_p = (484 \cdot A) / t_p \)
   - unit peak flow
   - comes from triangular UH
   - \( K_s = 484 \) too high for Florida! (commonly use 256)
   - \( q_p \) in cfs/inch of runoff

7) Multiply \( q_p \) x \( Q \) to get storm peak flow [cfs]

8) Use Table 6 of hydrograph to get actual hydrograph

In Viessman, Ch. 4 - see SCS dimensionless UH

Volume of runoff

\[
= \frac{1}{2} \cdot q_p \cdot (t_p + t_r) = 1" \text{ A conv.}
\]

\( Q \) from graph, in inches

\[
\gamma_p = \frac{645.3 \cdot A}{\frac{2}{t_p \left(1 + \frac{t_r}{t_p}\right)}} = \frac{CA}{k}
\]

484 comes from conversion - must multiply by area in sq. miles to obtain \( q_p \) in cfs

\[
Q_p (\text{cfs}) = k \frac{Q}{t_p (\text{hr})} A (\text{mi}^2) \times 645.3
\]

\( Q \) in inches, 645.3 is conversion - What is \( k \)?
Earlier hydrograph has $t_r = 1.67 t_p$

$k = 2/(1 + t_r/t_p)$

$k = 2/(1 + 5/3) = 3/4 = 0.75$

$K_s = k \times 645.3 = 0.75(645.3) = 484$

$Q_p = 484QA/t_p$    Constant (484) depends on shape of triangular hydrograph

BUT in Florida - longer recessions (tails) - so $t_r$ is greater

If: $t_r/t_p$ larger, then, $k$ must be smaller

Study found values of: 645 · $k < 484$  *in fact*, $< 100$ and as low as $k = 12$ in Kiss. River basin

(Capece, Campbell, Baldwin; "Estimating Runoff Peak Rates and Volumes from Flat, High-Water Table Watersheds")

Finally:  SCS does not do very well in Florida unless take into account water table

$S = \text{porosity} \times d$

(upper bound)
ICA October 26, 1999

Due November 4, 1999

Homework:

1) Find recession dimensions of a triangular unit hydrograph with a "shape factor" of 256. \( t_r = \_t_p \) 

1) Plot the storm hydrograph resulting from a 1000 acre basin in Hillsborough County for the 10-yr, 12-hr, Type IIM storm. Use either a dimensionless curvilinear or triangular hydrograph, 1000 ft hydraulic length, 0.3% slope and a CN=75.
RAINFALL - RUNOFF

Basin Urbanization
Hydrograph Comparison:

Result:

1. \( Q_{p,urb} \gg Q_{p,nat} \)
2. \( V_{urb} \gg V_{nat} \)
3. \( t_{p,urb} \ll t_{p,nat} \)
4. \( T_{B,urb} \ll T_{B,nat} \)

Consequence:

Must provide attenuation - reduce peak discharge; delay time to peak
**Stormwater Design Objectives**

1) Provide for reduced flooding potential (design for flood minimization within limitations)

2) Provide increased storage and sufficient attenuation measures to minimize surface water and ground water impacts downstream

3) Provide for water quality treatment and stormwater BMP's to reduce pollutant loading to receiving water systems

How do we do this?

- provide for increased storage, frictional characteristics - i.e., retention, detention ponds, swales, exfiltration pipes, etc.
Retention/Detention Ponds

Typical Retention/Detention Pond:

Retention - long term storage volume removed from the urbanized discharge hydrograph - "permanently" stored on site

Detention - short term storage of hydrograph to be slowly released over time - "temporarily" stored on site

Typical retention volume - SWFWMD/FDER design calls for first 1 inch of rainfall excess for basin $\geq 20$ acre
(0.5 inch for $< 20$ ac)
Factors that Influence Direct Runoff Hydrograph:

1) **Basin characteristics**

- slope, soils, shape, land use, etc.

- Antecedent Moisture Conditions - based on rainfall totals for preceding week

AMC I  - dry, < 0.5"/week
AMC II  - med, 0.5 to 1.5"/week
AMC III - high, > 1.5"/week

In Florida, ≈ 52"/year, 52 weeks → ≈ 1"/week → AMC II
Usually, tables based on AMC II - can adjust tables up or down (to I or III)
2) **Storm characteristics**

a) **Duration**
- need separate UH for each duration
- in practice, maybe have UH's for durations of every 2 hrs or so - 2 hr UH means 2 hr rainfall excess

b) **Time Variation of Intensity**
- can't make UH's for all possible patterns - might try to have separate UH's for convective (constant) and cyclonic storms.
- small basins show larger effect of variations than do larger basins

c) **Spatial Distribution**
- large basins are effected more than small basins
- try to construct UH's for various patterns - practical matter, not usually done
- can try to limit size of basin for application; e.g. A < 2000 miles$^2$
- topographic effects affect all storms in same way

d) **Storm Direction**
- worst possible case: storm and runoff peak in phase

e) **Amount of Runoff**
- assume linearity → time base for all storms of same duration is the same
- but in fact, larger storms have somewhat longer time bases
- better assumption for average storms than extremes
3) **Hydrography**

- condition of flow conveyance systems, basin size and shape

- e.g., effect of basin shape on hydrography
4) **Consistency of Data** (a common check is the $z$ variable)

Double Mass Analysis - $z$ variables, $x$ and $y$

Plot $\Sigma x$ vs. $\Sigma y$

- e.g., $\Sigma$ rain vs. $\Sigma$ runoff;
- or $\Sigma$ rain$_{gage \ 2}$ vs. $\Sigma$ rain$_{gage \ 1}$

How significant are the breaks?

1. Use double mass curve to identify likely data time periods
2. Test for significance using analysis of covariance
UNIT HYDROGRAPHS

ASSUME:
1) Identical (or similar) storms with identical antecedent conditions produce identical (directly related) discharge hydrographs.

2) Time base of all floods caused by rainfall excess of same duration is the same; time base and time to peak for similar storms of the same duration are the same.

\[
\text{Time base} = \text{duration of direct runoff}
\]

3) If rainfall distribution in time and space for several storms is similar, then the ordinates of each hydrograph will be proportional to volume of direct runoff (i.e., 1"/hr, 2"/hr \(\rightarrow\) same shape hydrograph but one twice as big as the other).

**Time Unit (TU)** of the storm = duration of rainfall excess

e.g., If 2.5 hrs of rainfall and first 1/2 hour goes to abstractions (like infiltration), the time unit is 2 hours.
Unit Hydrograph (UH): is the direct runoff hydrograph resulting from a rainfall excess (direct rainfall volume) of *1 inch* over the catchment area.

Strongly implied linearity - different amounts - same ordinates

![Unit Hydrograph](image)

- **Example:** From previous example, **TU = 2 hr**
  - Runoff Volume = \( \int Q(dt) = 2'' \cdot \text{Basin Area} \)
  - UH for this storm based on **1"** → divide by 2 → **1"** · Area of basin (UH is 1/2 the size but TB and TP are the same)
Basin Lag (TP) = time between centers of mass of rainfall excess and direct runoff.

- UH works best if time unit is approximately equal to \( \frac{1}{4} \) of basin lag.

- Often done on rainfall and not rainfall excess for ease.
UNIT HYDROGRAPH PROCEDURES

\[ Q_n(t) = \sum_{i=1}^{n} P_i^* Q_{UH_{n-i+1}} \]

Where: \( Q_n(t) \) is storm hydrograph ordinates \( P_i^* \) is incremental rainfall excess depth (typically hourly). \( Q_{UH_n} \) is unit hydrograph ordinates. \( n \) = number of storm hydrograph ordinates

Mathematical expression for displacement and scaling shown easily in handout.

How to construct UH from isolated storms:

1) Separate base flow and compute **volume** of direct runoff.

2) Scale direct runoff ordinates by dividing by the direct runoff volume (expressed as inches over the drainage area) - this is the UH

3) Study the rainfall records to determine the duration of the rainfall excess - **time unit**

4) Label UH with the time unit (or duration) - e.g. 2 hr storm
Can generate a UH for durations other than those of the original data (in the absence of storms of the desired duration).

e.g., can generate a 6-hr UH by adding displaced ordinates of 3 2-hr UH's and rescaling.

<table>
<thead>
<tr>
<th>t (hr)</th>
<th>UH₁ (1000 cfs)</th>
<th>UH₂ (1000 cfs)</th>
<th>UH₃ (1000 cfs)</th>
<th>Sum (1000 cfs)</th>
<th>Sum/3 = 6-hr UH</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
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</tr>
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<td>2.5</td>
<td>0</td>
<td>8.5</td>
<td>2.8</td>
</tr>
<tr>
<td>6</td>
<td>10.2</td>
<td>6.0</td>
<td>2.5</td>
<td>18.7</td>
<td>6.2</td>
</tr>
<tr>
<td>8</td>
<td>16.4</td>
<td>10.2</td>
<td>6.0</td>
<td>32.6</td>
<td>10.9</td>
</tr>
<tr>
<td>10</td>
<td>25.3</td>
<td>16.4</td>
<td>10.2</td>
<td>51.9</td>
<td>17.3</td>
</tr>
</tbody>
</table>

(but for 3" of rain)

Can get any desired duration from S-curves
S-CURVE (S-Hydrograph) is the hydrograph produced by a continuous series of consecutive rainfalls of equal duration (equal time units); each have 1 inch of rainfall excess per time unit.

i.e., hydrograph produced by continuous rainfall excess of intensity 1/TU

Obtain S-curve by adding UH's:

\[ Q_e = \text{Intensity} \times \text{Drainage Area} \]

\[ Q_e = 1 \text{ in/TU} \times A \]

But "real" S-curves have oscillations - must smooth - can calculate asymptote
How do we construct UH's from S-curves?

1) Have S-curve from UH of time unit $= TU_1$ Want to find UH of $TU_2$

Hypothesize that we could also have an S-curve caused by UH of time unit $\Delta t$

2) Assume that the S-curve for time unit $\Delta t$ ($TU_2$) is geometrically similar to S-curve for time unit $TU_1$

3) If displace S-curve by $\Delta t$, it must include all runoff except that caused by rain during the first $\Delta t$. Hence, difference in S-curves is the hydrograph resulting from rainfall of duration $\Delta t$ and intensity $1/TU$.

4) Take difference in S-curves and rescale by dividing by $1/TU \cdot \Delta t = TU_2/TU_1$ - Be sure and rescale

$$UH |_{\Delta t} = \frac{[Q_s(t) - Q_s(t - \Delta t)]}{\frac{1}{TT}} \cdot \Delta t \quad (Q_s - Q_s\Delta t)_i \cdot \frac{1}{TU_1} \cdot \frac{TU_1}{TU_2}$$
SYNTHETIC UNIT HYDROGRAPHS

Snyder - empirical method (Basins 10 - 10,000 mi²)

Define:
Lag Time, \( t_l \) = time from center of mass of rainfall excess to peak runoff (careful: also called \( t_p \), time to peak [hrs])

\[
t_l = C_t (L \cdot L_{ca})^{0.3}
\]

- \( L \) = main stream length, outlet to divide (miles)
- \( L_{ca} \) = length along channel to a point opposite watershed centroid (miles)

Typically, \( 1.8 \leq C_t \leq 2.2 \)

- the steeper the slope, the lower the value of \( C_t \)
- up to \( C_t \approx 8.0 \) along the Gulf of Mexico
Time Unit, TU (UH duration) = $T_R$ (time of rain)

$$T_R = \frac{t_l}{5.5} \quad \text{TU} = 0.18t_L$$

- if a time unit $T_{Rd}$ is desired then make adjustment to lag

$$t_{IR} = t_p + 0.25(T_{Rd} - T_R)$$

Where: $t_{IR}$ = time lag of new UH [hrs]  
$t_l$ = time lag of original UH [hrs]  
$T_{Rd}$ = specified TU  
$T_R$ = calculated TU = $t_L/5.5$

Please Note: For this course,

$t_p$ = Time to peak discharge from start of rainfall.

$t_l$ = Time to peak discharge from time of center of mass of rainfall excess.
Peak Flow, $Q_p$ (SUH's)

$$Q_p = \frac{640 \cdot C_p \cdot A \cdot Q}{t_L}$$

$A$ = flow producing area (miles$^2$)
$Q_p$ = peak flow (cfs)

$0.4 \leq C_p \leq 0.8$ - large $C_p$ with smaller $C_t$
$Q$ = volume of runoff {inches}

Unit Peak:

$$q_{p(SCS)} = \frac{640 \cdot C_p \cdot A}{t_L} \quad \text{(Actually 645.3)}$$

if use $C_p = 0.75$  
$$q_p = 484 \cdot \frac{A \cdot Q}{t_L}$$

Time Base, $T_B$

$$T_B = 3 + \frac{t_L}{8}$$

$T_B$ (days), $t_L$ (hrs)

For larger basins, $T_B = C \cdot t_L \quad 3 \leq C \leq 5$

Sketch in the Unit Hydrograph
INSTANTANEOUS UNIT HYDROGRAPH (IUH)

IUH = response (runoff) due to an instantaneous (zero duration) rainfall excess of 1 inch over the catchment.

Rainfall

\[ i = \text{pulse, mathematical direct delta function} \]

- height of pulse \( \rightarrow \) infinity
- area of pulse \( \rightarrow \) 1 inch

- break up into a series of direct delta pulses

- volume of a pulse = \( i(t) \, dt \)

Convolution or superposition integration - Linear System Analysis

\[
Q(t) = \int_0^t h(t - \tau) i(\tau) \, d\tau
\]
RUNOFF (and Stream Flow)

Surface Runoff:

1) Overland Flow - thin sheet flow (especially urban areas and sat. soil)
2) Low Order Stream Flow - streams fed by overland flow and smaller rivulets
3) Higher Order Stream Flow - fed primarily by smaller streams

Governing Equations:
Objectives - to predict flow as a function of space and time
Flow = f(x,t) (1 dimension)
Define: Q(x,t) → cfs = vol/time
q → cfs/ft of width = vol/width-time
y(x,t) → depth
- will have momentum and continuity equations
- for derivations of governing eq., see Fluid Mechanics texts (e.g., Eagleson; Henderson; Chow)
Overland Flow

Assume:
1) Hydrostatic Pressure - neg. any "over pressures" (e.g., by raindrops)
2) Wide Rectangular Channel - \( F \gg 1, \frac{y}{b} \ll 1, \, q = vy \)
3) Small Bottom Slope - \( \Theta \approx \sin \Theta \approx \tan \Theta \)
4) No correction in momentum equation for non-uniform velocity distribution
5) Any lateral inflows are perpendicular to flow

Momentum Eqn.

\[
\frac{dv}{dt} + v \frac{dv}{dx} + g \frac{dy}{dx} = -(i - f + \frac{2q_L}{b}) \frac{v}{y} - (1 + \frac{2y}{b}) \frac{\tau_o}{\rho y} + g \sin(\Theta)
\]

where:
- \( \frac{dy}{dx} = \) surface slope
- \( i = \) rainfall
- \( f = \) infiltration
- \( 2q_L/b = \) lateral inflows, both sides; bottom width
- \( 1 + 2y/b = \) bottom and side friction
- \( \frac{\tau_o}{\rho y} = \) wall shear stress
- \( g \sin(\Theta) = \) bottom slope

2 unknowns (v and y)
Continuity Eqn.

\[ \frac{dy}{dt} + v \frac{dy}{dx} + y \frac{dv}{dx} = i - f + \frac{2q_L}{b} \]

Order of magnitude analysis on momentum equation yields first four terms in equation small compared to the 5th and 6th terms. Therefore, the momentum eqn. now becomes:

\[ 0 = -\frac{\tau_o}{\rho y} + g \sin\theta \rightarrow \tau_o = \rho g y \sin\theta = \gamma y \sin\theta \]
How do we get V?

Now, use empirical results to describe \( t_o \)

Empirical results \( \rightarrow \) basic hydraulics

\[
\tau_o = C_f \cdot p \cdot \frac{V^2}{2g} \quad C_f = \text{Friction coefficient}
\]

\[
C_f = \frac{f}{4} \quad \text{where } f = \text{Darcy-Weisbach friction factor}
\]

\[
\nu = \sqrt{\frac{2\tau_o}{C_f \rho}} = \sqrt{\frac{2\gamma y \sin \theta}{C_f \rho}} = C \sqrt{y \sin \theta}
\]

where: \( C = \text{Chezy coefficient} = (2g/C_f)^{\frac{1}{4}} \quad (50 - 150) \)

\( \sin \Theta = \Theta \) or slope for small \( \Theta \)

If assume \( C = \text{constant} \) then \( C \sqrt{\sin \Theta} = \alpha \)

Then: \( V = \alpha y^{\frac{1}{4}} \) and \( g = V y = \alpha y^{\frac{3}{2}} \)
In general, momentum equation (for stage-discharge):

\[ q = \alpha y^m \]  
Simplified Momentum Eq. for steady, uniform flow

\[ \alpha, m = f(\text{friction loss, flow regime}) \]

One example (for turbulent flow → \( m = 3/2 \)):

\[ \alpha = (2 \cdot g \cdot \sin \Theta / C_f)^{\frac{1}{2}} \]  
and \( m = 3/2 \)

For laminar flow,

\[ C_f = 6/R \] for wide rectangular flow (down a wide plain)

where \( R = V y / v \)  
\( v \) = kinematic viscosity

this gives: \( m = 3, \alpha = g \cdot \sin \Theta / 3 \cdot v \) (exact solution)

For turbulent flow:

Manning's Eq. \( V = (1.49/n) \cdot y^{\frac{1}{6}} \cdot \Theta^{\frac{1}{2}} \)  
wide, uniform flow

\[ C_f = 0.9 \cdot g \cdot n^2 \cdot y^{\frac{1}{6}} \]  
\( V = 1.49/n \)

\[ q = \alpha y^{5/3} \]
\[ \alpha = \frac{1.49}{n} \Theta^{1/2}, \quad m = \frac{5}{3} \text{ for turbulent flow} \]
m? What to use?

5/3 (turbulent) \( \leq m \leq 3 \) (laminar)

\[ R \rightarrow \text{turbulence, } f(\text{friction factor}) \quad R = \frac{Vh}{v}, \]

\[ v = 1 \times 10^{-6} \text{ m}^2/\text{s} \]

e.g., \( f = 0.3 \), \( R = 2(10^3) \)

\( m \approx 2 \) is sometimes observed for overland flow

Momentum equation \( \rightarrow \) steady uniform flow

Say \( m = 2 \) from experiments, what is \( \alpha \)?

If have data, get \( \alpha \) from equation of the line

From Manning's Eq.,

\[ \alpha = \frac{1.49}{n} \cdot \Theta^{1/2} \]

\[ q = \frac{1.49}{n} \cdot \Theta^{1/2} \cdot y^{5/3} \cdot (y^{1/6} / y^{1/6}) \]

\[ = \frac{1.49}{n} \cdot \frac{\Theta^{1/6}}{y^{1/6}} y^2 \]

\rightarrow \text{gives } \alpha = f(y) = \frac{1.49}{n} \cdot \frac{\Theta^{1/6}}{y^{1/6}} \]

Then, evaluate \( \alpha \) at an average value of \( y \) (linearization approximation).
Manning's n - some estimates from Stanford Watershed Model (SWM) documentation (obtained from model calibrations):

<table>
<thead>
<tr>
<th>Surface</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>Smooth asphalt</td>
<td>0.012</td>
</tr>
<tr>
<td>Concrete paving</td>
<td>0.014</td>
</tr>
<tr>
<td>Packed clay</td>
<td>0.03</td>
</tr>
<tr>
<td>Light turf</td>
<td>0.20</td>
</tr>
<tr>
<td>Dense turf</td>
<td>0.35</td>
</tr>
<tr>
<td>Dense shrubbery and forest litter</td>
<td>0.40</td>
</tr>
</tbody>
</table>

→ overland flow for FL - higher for vegetated surfaces

Now need to solve for unknowns q and y

\[ q = \alpha y^m \quad \frac{dy}{dt} + \frac{dq}{dx} = i - f + \frac{2q_L}{b} \]

Solve for \( q(x,t) \) \( y(x,t) \)

Momentum: \( q = \alpha y^m \)

Continuity: \( \frac{dy}{dt} + \frac{dy}{dx} = i - f + 2q_L/b \) \( i - f = \) rainfall excess

Solve for \( q(x,t), y(x,t) \)

Analytical Solution → Method of Characteristics

Two governing equations: Kinematic Wave Solution
Kinematic Wave vs. Dynamic Wave

Kinematic Wave - moves only downstream; therefore, no backwater effects; retains only friction and gravity terms in momentum equation

Dynamic Wave - can simulate backwater and flow reversal; that is, can move upstream as well as downstream; retains all terms in momentum equation

Method of Characteristics - follow pathway in time and space of a kinematic wave; if can do this, then the coupled partial differential equations are converted to uncoupled ordinary diff. equations
TIME OF CONCENTRATION, tc

1) The time of concentration is the travel time of a wave to move from the hydraulically most distant point in the catchment to the outlet.

2) The tc is the time to equilibrium of the catchment under a steady rainfall excess, i.e. when \( Q_{\text{catchment}} = i \times A \).

**The tc is not the travel time of a parcel of water to move downstream through the catchment.**

The tc is the time taken for the discharge at the outlet to reflect ("feel") the runoff contribution from all parts of the basin.

\[
t_{\text{wave}} < t_{\text{parcel}}
\]

wave speed >> parcel speed \( \approx V_{\text{avg,basin}} \)

\[
\text{wave speed} = C = \sqrt{gy}
\]
Recall: $t_{c_{SCS, Lag method}}$

Time lag: $L = 1^{0.8} \cdot (S + 1)^{0.7} / (1900 \cdot Y^{0.5})$

and,

$t_c = (5/3) \cdot L$

tcS_{SCS, Velocity method}:

$tc = 1 / V_{avg}$

$V_{avg} =$ average parcel velocity (fps)

tcKinematic wave method:

$t_c = \left( \frac{L}{\alpha \cdot i_e \cdot m} \right)^{1/m}$

$L =$ length of overland flow plane,

$L_{avg} = (Z_{max} - Z_{min}) / \text{Slope}_{avg}$

$i_e =$ average rainfall excess intensity ($f(tc)$)

$\alpha, m =$ hydraulic flow regime parameters

$\alpha = 1.49 \sqrt{S / n}$

$m = 5/3$ for turbulent flow

What about "n" - see FDOT Drainage Manual, Overland Flow Manning's n Values

Separate into: overland flow (sheet flow), channelized flow

Overland flow - maximum of 300 feet nationally, 600 feet in Florida

$q =$ discharge/unit width $\rightarrow$ calculate flow depth of discharge and friction (momentum equation)
Miscellaneous Topics

One more characteristic length - average length of overland flow

\[ L_s = \text{Total length of stream (drainage collection network)} \]
\[ A = \text{Basin area} \]

\[ t_{OF} = A / 2 \cdot L_s \]

Drainage Density - measure of the drainage structure density

\[ D_r = \text{total miles of streams/ sq. mi. of catchment} \]

\[ D_r = \sum L_s / A_r \quad 1 \leq D_r \leq 100 \quad (1/\text{mile}) \]

High in deserts and areas of high relief (low ratios of P/ET)
Low in humid areas and areas of low relief (high ratios P/ET)

In Florida \[ 1 \leq D_r \leq 10 \quad (1/\text{mile}) \]
Particular Methods for Predicting Runoff

**TIME-AREA METHODS**

1) **Select a rainfall hyetograph** - real or synthetic design storm

2) **Compute losses**
   - most methods use Horton infiltration (no special concern about case B)
   - could use SCS S
   - really should consider case B - could use curve shifting method or make $f_i^*$ a func(F)
   - Remember, depression storage (if used) is subtracted first in time

3) **Construct isochrones of equal travel time** (water velocities)
   - would make more physical sense to use wave travel time
   - could adjust by multiplying velocities by m to get c
   - e.g., if $\Delta t = 5$ min (should be same as hyetograph $\Delta t$)
4) (Optional) Construct a Time-Area Curve

![Time-Area Curve Diagram]

5) Construct Time-Area Concentration Curve

\[ W(t) = \frac{dA(t)}{dt} = \frac{\text{derivative of time}}{\text{area}} \]

Fine for continuous functions but for discrete time, use \( \Delta t \) increment

In discrete time, simply plot \( A_i \) vs. \( t_i \) → Time - Area Concentration Curve - actually use:

6) Runoff Computation by Convolution
(linear superposition and addition)

\[ Q_n = \sum_{k=1,n} A_{n-k+1} \cdot i_k \quad (i_k = \text{rainfall excess}) \]

\[ Q_1 = A \cdot i_1 \]
\[ Q_2 = A_2 \cdot i_1 + A_1 \cdot i_2 \]
\[ Q_3 = A_3 \cdot i_1 + A_2 \cdot i_2 + A_1 \cdot i_3 \]
\[ Q_4 = A_4 \cdot i_1 + A_3 \cdot i_2 + A_2 \cdot i_3 + A_1 \cdot i_4 \ldots \text{etc.} \]

RRL - only considers directly connected impervious surfaces
Comments:

If we work in the continuous time domain:

\[ Q(t) = \int_0^t W(t - \tau) i_e(\tau) \, d\tau \]

\[ = \int_0^t \frac{dA(t - \tau)}{dt} i_e(\tau) \, d\tau \]

Since \( W(t) = \frac{dA(t)}{dt} = \) Instantaneous Unit Hydrograph

\[ Q(t) = \int_0^t W(\tau) i_e(t - \tau) \, d\tau \]

or

\[ Q_n = \sum_{k=1}^n A_k i_{n-k+1} \]

No attenuation included in time-area method alone - sometimes include reservoir routing at the lower end of basin to account for attenuation of hydrograph.

Simplest case - use a linear reservoir

Options:
- a time-area curve + linear reservoir
- could use Puls Method - ILLUDAS (Illinois Urban Drainage Simulator)
- considers pervious areas with your choice of Horton or Holton infiltration
- also, pipe routing by translation
- earlier model - RRL - Transport Road Research Lab - Great Britain
- considers only directly connected impervious areas
Santa Barbara Urban Hydrograph Method - Method SBUH

1) Compute runoff from three subareas
   A. Directly Connected Impervious
      
      \[ Q_{dc} = P \cdot A_{dc} \]
      
      \( P \) = precipitation at each time step

   B. Other Impervious
      
      \[ Q_{imp-other} = P \cdot A_{imp-other} \]  (subtract losses)

   C. Pervious
      
      \[ Q_{pervious} = P \cdot A_{perv} \]

   \[ Q_{total} = Q_{dc} + [(Q_{imp-other} + Q_{perv}) - f \cdot A_{perv}] \]

   \( f \) - infiltration by Horton method
   \( Q_{imp-other} \) - in model, some losses from these

2) Route through Linear Reservoir with \( t_e \) (hours) = \( t_e \) (basin)
FLOOD ROUTING - Hydrologic vs. Hydraulic

Simplest methods are based on a form of the lumped continuity eq.

- For large rivers, aren't concerned with rainfall or lateral inflows
  - only change in $Q_i$

  - Assume $Q_i$ known, solve for $q_o$, $s$

Linear Reservoir

$$Q_o = k's$$  \hspace{1cm} k' = 1/time

- outflow linearly related to storage

$$dQ_o/dt + k'Q_o = k'Q_o(t)$$

- can integrate analytically