Analytical expressions for transient specific yield and shallow water table drainage

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[1] New closed-form expressions are introduced to capture the dependence of specific yield on time and depth to water table. The expressions allow the user to convert observations of water table fluctuations to volumes of water released from storage in a shallow water table aquifer. Whereas a linear relationship between water table fluctuations and released volumes holds for a deep water table aquifer, this relationship is nonlinear for shallow water table aquifers. The dependence of specific yield on time stems from the slow drainage of soil water from pores above the water table. The new expressions allow determination of transient specific yield and time to drain the soil water profile for a given water table fluctuation. If the time step in a numerical groundwater model is longer than the time for limiting specific yield, then a constant (time independent) specific yield can be justifiably adopted. The new expressions are easy to use and require knowledge of soil hydraulic properties which are readily available from soil water retention data and surveys. *INDEX TERMS:* 1829 Hydrology: Groundwater hydrology; 1866 Hydrology: Soil moisture; 1875 Hydrology: Unsaturated zone; 1890 Hydrology: Wetlands; 1836 Hydrology: Hydrologic budget (1655); *KEYWORDS:* water table fluctuation, soil moisture storage and release

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1. Introduction

[2] In humid regions, like Florida and the southeastern United States, a dynamic shallow water table controls many hydrologic fluxes across the groundwater-vadose zone-atmosphere continuum. These fluxes may include root water uptake by transpiring vegetation, evaporation from a shallow water table, and runoff by saturation excess following the saturation of a thin unsaturated soil layer [*Meyboom*, 1967; *Abdul and Gilham*, 1989; *Troch et al.*, 1992, *Jayatilaka and Gilham*, 1996; *Bierkens*, 1998]. The importance of modeling the dynamics of shallow water tables has long been recognized and documented in the literature. Recently, concerns about the impact of groundwater withdrawals on ecosystems have increased the interest in this topic. In south and central Florida, many wetlands are supported by shallow water tables which rise and inundate the ground surface during the wet season, then gradually decline during the dry season, creating a cycle of ground surface inundation known as the hydroperiod of wetlands [*Ewel*, 1990]. The continuous drop of water table from excessive groundwater withdrawals by water suppliers in the region continues to have negative impacts on the hydroperiod of sensitive ecosystems and wetlands in Florida [e.g., *Southwest Florida Water Management District*, 1996]. Modeling impacts of groundwater withdrawals on wetlands requires accurate estimation of fluctuation in shallow water tables.

[3] Water table fluctuations are traditionally estimated using a parameter known as specific yield. The specific yield is the volume of water an aquifer releases from or takes into storage per unit aquifer area per unit change in water table depth [*Freeze and Cherry*, 1979]. Mathematically, the specific yield is

$$S_y = \frac{V_w}{A\Delta} \tag{1}$$

where A is an aquifer area and V_w is the volume of water released (drained or stored) resulting from Δ water table fluctuation (draw down or draw up). Application of specific yield has three significant restrictions which are often overlooked by many hydrologists. First, the specific yield in equation (1) is a constant parameter only if the aquifer response is linear; the volume of water released is linearly proportional to the water table fluctuation. Recognizing that the volume released is drained from the unsaturated soil profile above the water table, *Duke* [1972] was among the first to note that this linear behavior will not hold in shallow water table environments. In these environments, the specific yield can be substantially smaller due to the capillary fringe above the water table. Owing to the low specific yield of shallow water table aquifers, *Bouwer and Rice* [1978, 1980] and *Barlow et al.* [2000] suggested that the unconfined water table aquifer could be treated more like a confined aquifer. To account for dependence on water table depth, *Duke* [1972] introduced the simple expression for specific yield, S_y,

$$S_y = (\phi - S_r)\left(1 - \left(\frac{h_a}{d}\right)^{\lambda}\right) \tag{2}$$

where λ and h_a are the pore size distribution index and the soil air-entry (or bubbling) pressure head of the Brooks and

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Corey model of water retention [*Brooks and Corey,* 1964], d is depth to water table, φ is porosity (cm^3 of water/cm^3 of soil) and S_r is soil specific retention (cm^3 of water/cm^3 of soil), the water content at which soil water cannot be drained by gravity. The specific retention is the same as field capacity, a term often used in agriculture an soil hydrology [e.g., *Nachabe et al.*, 2002; *McWhorther and Sunada*, 1977]. In (2), the specific yield approaches the drainable porosity equal to $(\varphi - S_r)$ as d, the depth to water table, increases. The specific yield is zero for $d < h_a$, meaning no water will be released unless the water table depth is larger than the soil air-entry pressure. Duke's expression was used by *Jayatilaka and Gilham* [1996], among others, to simulate variable source runoff emanating from water table mounds adjacent to streams. Later in this study, it will be demonstrated that the expression is valid only when the water table fluctuation is ''small'' compared to the initial water table depth. This condition can be limiting because rapid and sharp fluctuations of the shallow water table are usually observed [e.g., *Gilham*, 1984; *Abdul and Gilham*, 1989].

[4] A second restriction on the application of specific yield lies in the transient nature of water release from the unsaturated profile above a water table. While an observed fluctuation in water table, Δ, can sometimes be instantaneous e.g. during pumping, the volume of soil water, V_w, is released gradually and the time to drain the profile is controlled by water table depth and soil hydraulic properties (see Figure 1). In a field study, *Nwankwor et al.* [1992] demonstrated that delayed drainage of soil water above water table explained the relatively small values of specific yield derived from pumping tests type curves. In regional groundwater models, a constant (i.e. time independent) specific yield can be used, only if the time to drain the unsaturated profile is smaller than the time step used to solve the ground flow equation. Yet, we don't know how to relate this drainage time to soil properties and depth to water table. Finally, the nonlinearity in specific yield can be enhanced by hysteresis in water retention [*Kool and Parker*, 1987]. Air encapsulation during the recharge and rise of a water table results in specific yield values lower than those observed during drainage; thus the same volume V_w results in different observed Δ, depending on whether a water table aquifer is recharged or drained.

[5] The objective of this article is to introduce a new, closed-form, analytical solution for specific yield that (1) captures its dependence on time and (2) applies to any initial water table depth. The specific yield is used by hydrologists to convert shallow diurnal water table fluctuations into ET rates [e.g., *Meyboom*, 1967], to estimate variable source runoff from water table mounds [*Jayatilaka and Gilham*, 1996], or to simulate regional groundwater flow [*McDonald and Harbaugh*, 1988]. Applications of regional groundwater models usually cover large areas and different landscapes, e.g., wetlands with a shallow water table vs. upland landscapes with a deep water table. These models are calibrated to match water table fluctuations in these distinct landscapes so accurate estimation of specific yield is needed for converting observed water table fluctuation into water volumes. A second objective for this article is to provide an approximation of the time to drain an unsaturated soil water profile above a water table. The kinematic wave theory [e.g., *Morel-Seytoux*, 1987; *Nachabe et al.*, 1995; *Charbeneau*, 2000] is adopted to construct the evolution of the water content profile during drainage. A simple, closed-form, solution providing the specific yield as a function of time is introduced.

2. Problem Statement

[6] Consider an observation well recording the water table fluctuation $\Delta = d_2 - d_1$ shown in Figure 1. We are interested in determining the volume of water released, V_w, as the soil water profile evolves from its initial equilibrium position with water table at depth d_1, to a new equilibrium profile with water table at depth d_2. If the *Brooks and Corey* [1964] model is used to describe water retention in soils, then the no-flow equilibrium water content $\theta_i(z_i)$ at elevation z_i above the initial water table is given as:

$$\frac{\theta(z_i) - S_r}{\phi - S_r} = \left(\frac{h_a}{z_i}\right)^{\lambda} \tag{3}$$

for $z_i \geq h_a$ and $\theta(z_i) = \varphi$ for $z_i < h_a$. The Brooks and Corey model of water retention is adopted in this study because representative values of its parameters have been documented for different soils and can also be easily estimated from soil texture [e.g., *Rawls et al.*, 1993]. Because gravity is the main driving force during drainage, the evolution of the water content is governed by the partial differential equation [*Charbeneau*, 2000]:

$$(\phi - S_r)\frac{\partial \Theta}{\partial_t} + \frac{dK}{d\Theta}\frac{\partial \Theta}{\partial z} = 0 \tag{4}$$

where z is elevation above the new water table position, t is time, $\Theta = (\theta - S_r)/(\varphi - S_r)$ is a normalized water content, and $K(\Theta)$ is the unsaturated hydraulic conductivity. The unsaturated hydraulic conductivity is usually a highly nonlinear function of Θ given as:

$$\mathbf{K}(\Theta) = \mathbf{K}_s \Theta^n$$

where K_s is the saturated hydraulic conductivity and the exponent n can be set equal to $(2 + 3\lambda)/\lambda$ if the model of Burdine is adopted [*Burdine*, 1953; *Brooks and Corey*, 1964, 1966].

2.1. Method of Characteristics

[7] Equation (4) is highly nonlinear, but a closed form analytical solution for the spatial and temporal evolution of $\Theta(z,t)$ during drainage can be developed using the method of characteristic. The method of characteristics has a long history of use in soil hydrology [e.g., *Morel-Seytoux*, 1987; *Nachabe et al.*, 1995; *Charbeneau*, 2000] and is well documented in the literature [e.g., *Charbeneau*, 2000]. Essentially, the total variation or differential in Θ is given as:

$$d\Theta = \frac{\partial \Theta}{\partial t}dt + \frac{\partial \Theta}{\partial z}dz \tag{5}$$

Equation (4) could be thought of as vector displacement in the z-t-Θ. The idea behind the method of characteristic is

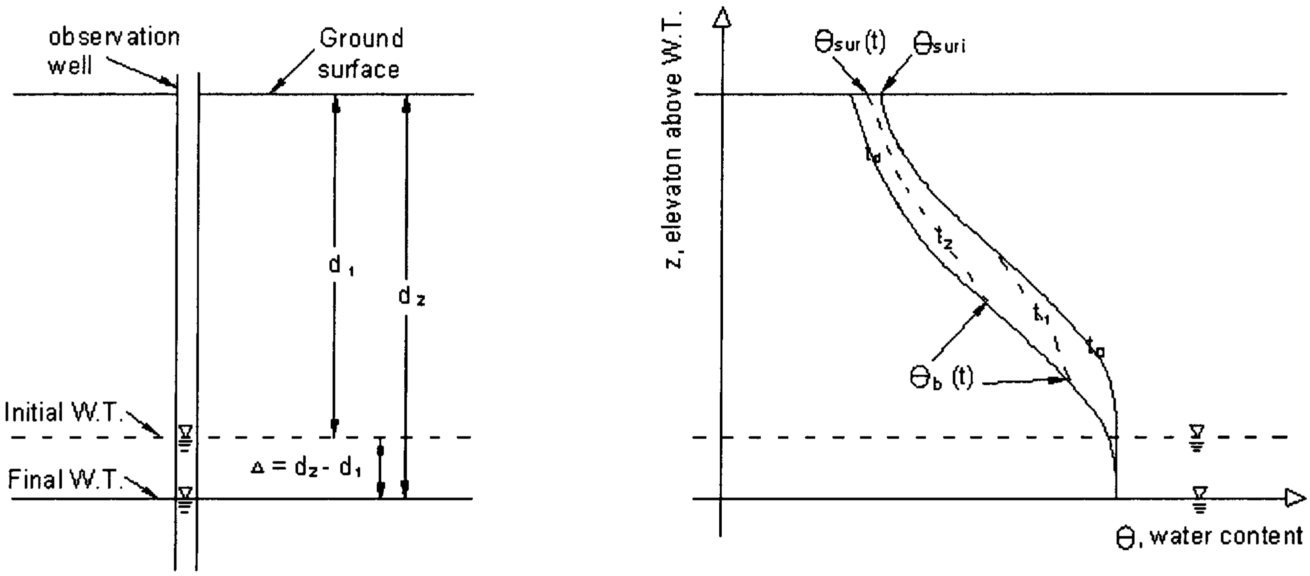

Figure 1. Definition sketch, displaying an initial water table drop Δ and the corresponding transient drainage of the water content profile.

that equation (4) can also be thought of as vector displacement in the z-t-Θ. Thus comparison of (4) and (5) yields [*Charbeneau*, 2000]:

$$\frac{dt}{\phi - S_r} = \frac{dz}{dK/d\Theta} = \frac{d\Theta}{0} \tag{6}$$

Thus partial differential equation (4) is replaced by a set of two ordinary differential equation. According to equation (6) the rate of movement of a constant water content Θ is:

$$\frac{dz}{dt} = \frac{1}{\phi - S_r}\frac{dK}{d\Theta} = \frac{-nK_s}{\phi - S_r}\Theta^{n-1} \tag{7}$$

The negative sign in front of equation (7) results from taking z as positive upward (see Figure 1), thus as time, t, increases, z decreases during drainage. By treating each water content as a wave, integration of (7) yields the elevation z of this water content above the water table:

$$z = \xi - \frac{nK_s}{\phi - S_r}\Theta^{n-1}t \tag{8}$$

where ξ is the beginning elevation of water content above a new water table.

2.2. Evolution of the Water Content Profile During Drainage

[8] At any time t, the water content profile can be made of two segments (see Figure 1). The first segment between φ and $\theta_b(z_b,t)$ is composed of water content waves that have propagated a distance Δ and reached their new equilibrium position above the new water table. With $\xi - z = \Delta$, the temporal evolution of $\theta_b(t)$ is calculated from (8) as:

$$\Theta_b = \frac{\theta_b - S_r}{\phi - S_r} = \left(\frac{\Delta(\phi - S_r)}{nK_s}\right)^{1/(n-1)} t^{1/(1-n)} \tag{9}$$

The second segment is comprised of water content waves propagating from initial elevation $\xi = \Delta + h_a\,\Theta^{-1/\lambda}$ and which have not yet reached their equilibrium position. Replacing ξ in (8) yields:

$$z = \Delta + h_a\Theta^{-1/\lambda} - \frac{nK_s}{\phi - S_r}\Theta^{n-1}t \tag{10}$$

Equations (9) and (10) provide the water content profile evolution $\theta(z,t)$ above the water table. In particular, the water content at ground surface evolves from its initial value $\Theta_{suri} = (h_a/d_1)^{\lambda}$ to $\Theta_{sur}(t)$ determined from (10) by setting $z = d_2$. Mathematically,

$$d_2 = \Delta + h_a\Theta_{sur}^{-1/\lambda} - \frac{nK_s}{\phi - S_r}\Theta_{sur}^{n-1}t \tag{11}$$

2.3. Expressions for Transient Specific Yield

[9] The specific yield is the depth of water per unit area drained between the initial and new profiles of water content divided by the water table fluctuation [e.g., *Bear*, 1979; *Freeze and Cherry*, 1979]. Mathematically this is expressed as:

$$S_y = \frac{V_w}{A\Delta} = \frac{1}{\Delta}\int_{\theta_{sur}}^{\phi}(\xi - z)d\theta$$

where $\theta_{sur}(t)$ is water content at the surface. This integral is broken into three components depending on ξ-z. Mathematically,

$$\begin{aligned} S_y &= \frac{V_w}{A\Delta} \\ &= \frac{\phi - S_r}{\Delta}\left\{\int_{\Theta_{sur}}^{\Theta_{suri}}\left(d_1 - h_a\Theta^{-1/\lambda} + \frac{nK_s}{(\phi - S_r)}\Theta^{n-1}t\right)d\Theta \right. \\ &\quad \left. + \int_{\Theta_{suri}}^{\Theta_b}\frac{nK_s}{(\phi - S_r)}\Theta^{n-1}t \quad d\Theta + \int_{\Theta_b}^{1}\Delta d\Theta\right\} \end{aligned}$$

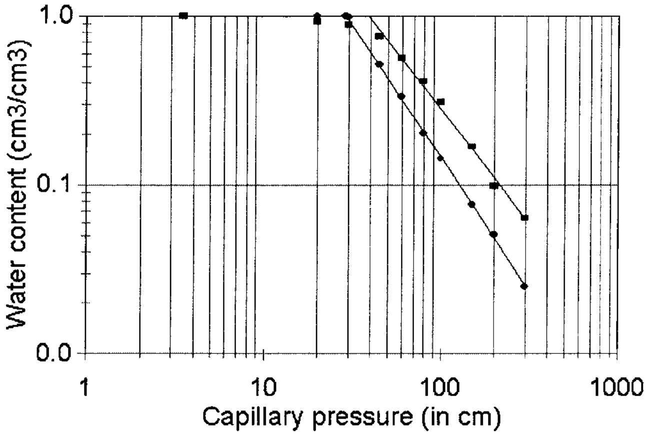

Figure 2. Water retention data and fitted Brooks–Corey Model for Mulat fine sand (circles) and Mulat fine sandy loam (squares).

which yields upon integration

$$S_y(t) = \frac{\phi - S_r}{\Delta}\left[d_1(\Theta_{suri} - \Theta_{sur}) - \frac{\lambda h_a}{\lambda - 1}\left(\Theta_{suri}^{(\lambda-1)/\lambda} - \Theta_{sur}^{(\lambda-1)/\lambda}\right)\right] + \frac{K_s}{\Delta}\left(\Theta_b^n - \Theta_{sur}^n\right)t + (\phi - S_r)(1 - \Theta_b) \tag{12}$$

where $\Theta_{suri} = (h_a/d_1)^\lambda$, $\theta_b(t)$ is provided in (9) and $\theta_{sur}(t)$ is calculated from (11). While (12) may not be difficult to implement with today's computers, further approximations can be sought to simplify calculations and to demonstrate the limitations on Duke's formula.

[10] A first approximation of $S_y(t)$ is to consider that the change in water content at the surface is negligibly small and assume $\Theta_{sur}(t) = \Theta_{suri} = \Theta_{sura}$ = constant = $(h_a/d)^\lambda$ where $d = (d_1 + d_2)/2$, an average depth to water table (see Figure 1). This assumption implies that the water content at ground surface is approximated by Θ_{sura} which does not change with time. This might be justified if Δ is small or if the water table is relatively deep. With this assumption, the first term on the right hand side of (12) is dropped and the resulting closed-form solution for $S_y(t)$ is:

$$S_y(t) = \frac{K_s}{\Delta}\left(\Theta_b^n - \Theta_{sura}^n\right)t + (\phi - S_r)(1 - \Theta_b) \tag{13}$$

In equation (13), only $\Theta_b(t)$ is a function of time as given in equation (9). As time increases, $\Theta_b(t)$ decreases and approaches Θ_{sura} gradually. As the profile continues to drain, the first term on the right hand side of (13) approaches zero, and the specific yield reaches its ultimate value at time $t = t_d$ when drainage stops, i.e. when $\Theta_b(t = t_d) = \Theta_{sura} = (h_a/d)^\lambda$. Replacing this relationship in (13), the ultimate value of specific yield is:

$$S_y = (\phi - S_r)\left(1 - \left(\frac{h_a}{d}\right)^\lambda\right)$$

which is exactly equation (2) provided by Duke. Equation (9) is used to determine the time to drain the profile and reach this specific yield. Mathematically,

$$t_d = \frac{\Delta(\phi - S_r)}{nK_s}\left(\frac{d}{h_a}\right)^{\lambda(n-1)} \tag{14}$$

As expected, the drainage time, t_d, is shorter for large conductivity, K_s, and small fluctuation, Δ. Also equation (14) indicates that a deep water table needs a longer time to drain; however, large air-entry pressure shortens the drainage time.

2.4. Ultimate Specific Yield for Large Water Table Fluctuations

[11] Equation (12) is used to determine the ultimate specific yield when Δ is large. After the profile is drained, $\Theta_{sur}(t = t_d) = \Theta_b(t = t_d) = (h_a/d_2)^\lambda$ which is replaced in (12) to obtain the ultimate specific yield:

$$S_y = \frac{(\phi - S_r)}{\Delta}\left(\Delta + \frac{h_a}{1 - \lambda}\left(\left(\frac{h_a}{d_1}\right)^{\lambda-1} - \left(\frac{h_a}{d_2}\right)^{\lambda-1}\right)\right) \tag{15}$$

Unlike equation (2), equation (15) is valid regardless of the magnitude of Δ.

3. Discussion

[12] Equations for specific yield were tested on two Floridian soils, a fine sand and a fine sandy loam [*Carlisle et al.*, 1989]. For each soil, water retention data were obtained from duplicate undisturbed soil cores placed in Tempe pressure cells, saturated, and then extracted at different pressures. Figure 2 shows the water retention data and the Brooks-Corey empirical model fit. Soil texture was documented in Table 1 along with estimated Brooks-Corey fit parameters. This empirical model provided a good description of water retention, with a coefficient of determination R^2 exceeding 0.98 for pressures larger than the air entry pressure. As expected, the fine sandy loam had a higher air entry pressure than the fine sand due to its higher clay content. Also the fine sandy loam had slightly smaller λ, indicating that this soil had a larger pore size distribution than the fine sand [*Brooks and Corey*, 1964]. The saturated conductivity for both soils is relatively high.

Table 1. Physical Characteristics of Two Floridian Soils

	Soil Texture			Brooks-Corey Fitted Parameters				
	Percent Sand	Percent Silt	Percent Clay	φ, cm³/cm³	S_r, cm³/cm³	h_a, cm	λ	K_s, cm/hr
Mulat fine sand	94.5	3.9	1.6	0.39	0.075	29.2	1.57	3
Mulat fine sandy loam	77.5	4.6	17.9	0.309	0.136	39.1	1.36	0.62

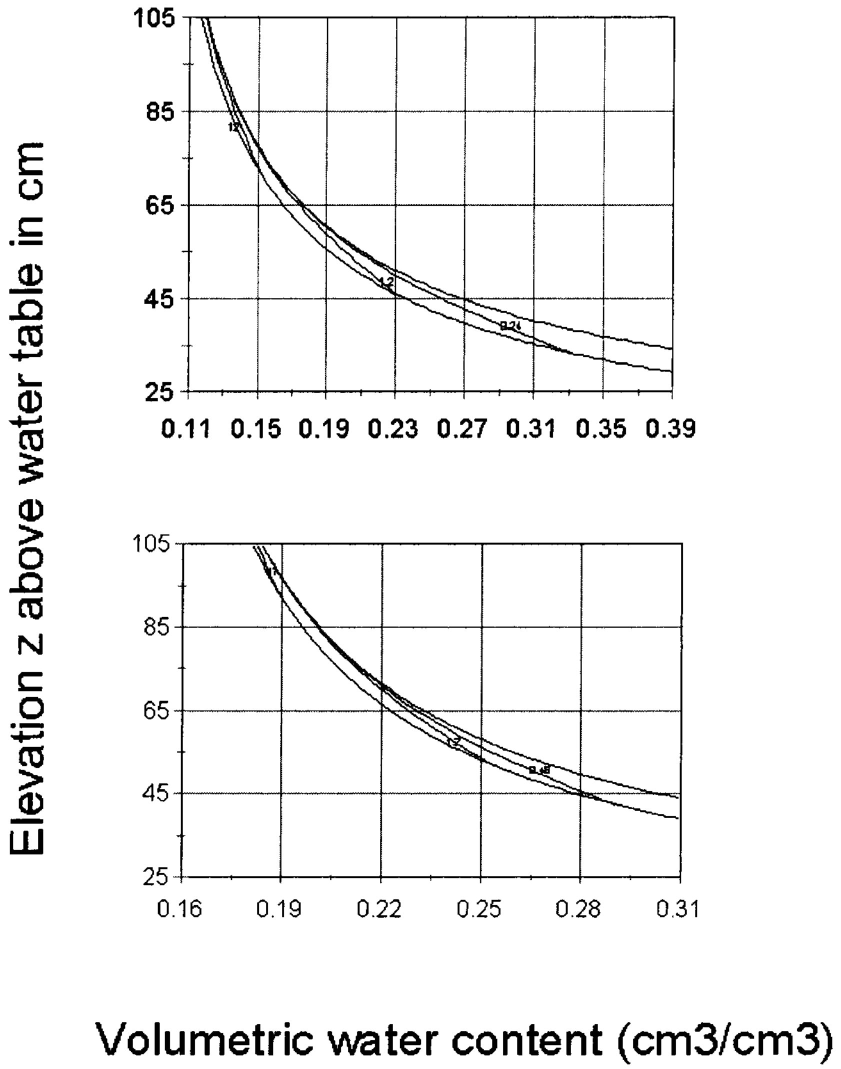

Figure 3. Evolution of water content profiles for (top) fine sand and (bottom) fine loamy sand during drainage.

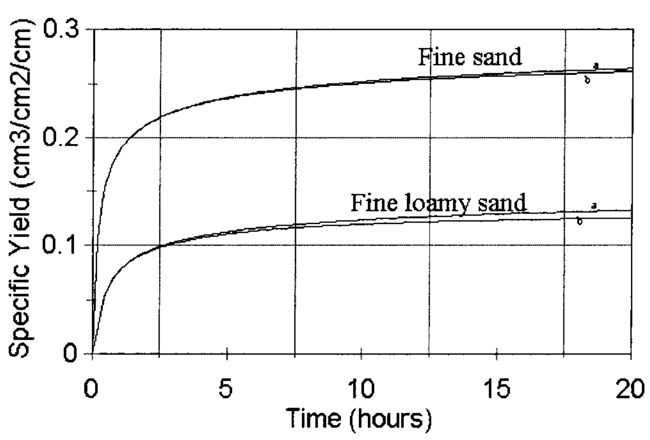

Figure 4. Approximate and accurate solution of transient specific yield for two soils.

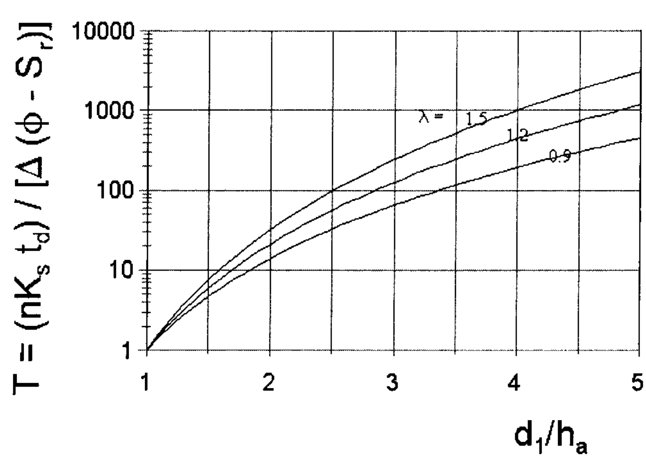

Figure 5. Normalized drainage time versus normalized depth for three values of pore size distribution index.

3.1. Transient Drainage and Specific Yield

[13] Figures 3 (top) and 3 (bottom) show the evolution of the water content profile with time during drainage. In these simulations, the water table was dropped from 100 cm to 105 cm and the transient profiles are calculated at times 0.24, 1.2, and 12 hours for fine sand and 0.48, 1.2, and 17 hours for fine sandy loam. The drainage profiles suggested rapid and nearly complete drainage of both profiles in less than 24 hours. The transient drainage is better described through the specific yield in Figure 4. The specific yield in both soils increased rapidly after the drop in water table, and then gradually progressed toward its ultimate value as the drainage profile approached its new equilibrium. The approximate, but explicit, solution for the transient specific yield (equation (13)) seems to match the more rigorous analytical solution in equation (12). The approximation, however, involved the drainage of the profile close to ground surface, and the differences between the solutions should increase for shallow water tables. A plot of normalized drainage time $T = (n\ K_s\ t_d)/[\Delta\ (\varphi - S_r)]$ versus normalized depth (d/h_a) is shown in Figure 5 for different pore size distribution index λ.

[14] The drainage time, the time at which the entire profile reaches its new equilibrium and drainage stops, increases with the increase in pore distribution index and depth to water table. Knowledge of drainage time or transient specific yield can serve several purposes. First, this knowledge might be used to test whether a constant, time independent specific yield is justified in a numerical groundwater model. If the time step in the groundwater simulation is larger than the time to reach the ultimate specific yield (equation (15)) then groundwater simulation models can use a constant, time independent, specific yield. In simulations where time step is short in comparison with drainage time, a transient specific yield should be used. In addition, curves for water content distribution during drainage can be useful for agricultural and drainage engineers needing to manage the water table or to determine moisture conditions in a shallow root zone.

3.2. Dependence of Ultimate Specific Yield on Depth to Water Table

[15] While the dependence of specific yield on time might be justifiably ignored in certain cases, the dependence of specific yield on water table depth remains significant for many applications of regional groundwater models. Usually, regional groundwater models cover distinct landscapes, e.g., wetlands with typically shallow water tables and upland landscapes with deep water tables. Capturing the variation in specific yield between these landscapes is needed to convert observations of water table fluctuations into water volumes.

[16] To compare (15), which is valid for any Δ, with Duke's expression (equation (2)), we plot the two equations for normalized specific yield = $S_y/(\varphi - S_r)$ versus normalized initial depth to water table (d_1/h_a) for different ratios of Δ/h_a (Figure 6). As expected, for both equations, the specific yield approaches the drainable porosity = $\varphi - S_r$ as depth to water table increases. The differences between the two expressions are significant for a large water table drop (Δ/h_a is large) or a shallow initial water table depth (d_1/h_a is small). This difference, however, becomes negligible for small fluctuation Δ/h_a or deep water table (d_1/h_a is large). Indeed, it is shown in the appendix that equation (15) provides exactly Duke's expression as $\Delta = d_2 - d_1$ approaches zero.

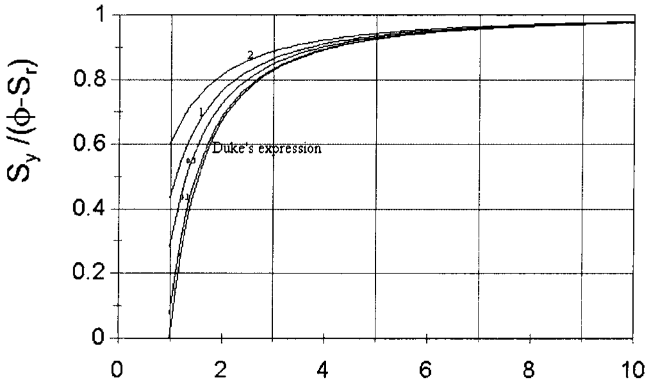

Figure 6. Specific yield for Δ/h_a = 0.1, 0.5, 1, and 2. Note that expressions (2) and (15) are identical in the limit as Δ/h_a approaches zero.

4. Conclusion and Limitations

[17] New expressions are introduced to analyze the dependence of drainage and specific yield on time and depth to water table. For a given soil type and an observed water table fluctuation, the analytical expressions can be used to determine whether a time-independent specific yield is justified in a groundwater model. If the time to reach the ultimate specific yield is long, then a transient expression for specific yield should be used to convert observed water table fluctuations into water volumes. Water content drainage profiles resulting from a water table drop can be easily constructed and might be used in agricultural and drainage engineering. A new expression capturing the dependence of specific yield on depth to water table was also introduced. The expression is valid for any depth or fluctuation of water table and can be used in regional groundwater models intended to simulate distinct landscapes with different depth to water table.

[18] There are two important limitations on the use of the new expressions of specific yield. First, equation (3) implies that the initial water content distribution was at static equilibrium, and there is no downward (recharge) or upward (evapotranspiration) flux in the unsaturated zone. While the assumption of equilibrium profile has been widely adopted for estimating specific yield in groundwater literature [e.g., *Duke*, 1972; *McWhorter and Sunada*, 1977; *Bear*, 1979], it should be known that the actual initial water content profile may deviate from equilibrium if, at time zero, there is a significant steady or transient flux in the unsaturated zone. Secondly, results presented in this article are for water table drainage only. For a rising water table condition, soil properties need to be modified for hysteresis (air entrapment) and air encapsulation below water table. Encapsulated air, which can be as high as 20% of soil porosity [e.g., *Fayer and Hillel*, 1986], is likely to reduce the value of specific yield, i.e. less water volume is needed to provide a positive water table fluctuation (water table rise) than a negative fluctuation (water table drop).

Appendix A

[19] In this appendix, it is shown that equation (15) yields equation (2) as Δ approaches zero. After substituting for $d_2 = d_1 + \Delta$, equation (15) is written as

$$S_y = (\phi - S_r)\left(1 + \frac{1}{\Delta}\left(\frac{h_a^\lambda}{1-\lambda}d_1^{1-\lambda} - \frac{h_a^\lambda}{1-\lambda}(d_1+\Delta)^{1-\lambda}\right)\right)$$

As Δ approaches zero, the second term inside parentheses on the right hand side of this equation tends to 0/0, which is indeterminate. L'Hospital rule is used to determine the limit as Δ approaches zero. Mathematically,

$$\begin{aligned}
&\lim\ as\ \Delta \longrightarrow 0 \\
&\cdot\left\{(\phi - S_r)\left(1 + \frac{1}{\Delta}\left(\frac{h_a^\lambda}{1-\lambda}d_1^{1-\lambda} - \frac{h_a^\lambda}{1-\lambda}(d_1+\Delta)^{1-\lambda}\right)\right)\right\} \\
&= \lim\ as\ \Delta \longrightarrow 0 \\
&\cdot\left\{(\phi - S_r)\left(1 + \frac{1}{1}\left(\frac{h_a^\lambda}{1-\lambda}d_1^{1-\lambda} - \frac{h_a^\lambda}{1-\lambda}(1-\lambda)(d_1+\Delta)^{-\lambda}\right)\right)\right\} \\
&= (\phi - S_r)\left(1 + \frac{1}{1}\left(0 - \frac{h_a^\lambda}{1-\lambda}(1-\lambda)(d_1+0)^{-\lambda}\right)\right) \\
&= (\phi - S_r)\left(1 - \left(\frac{h_a}{d_1}\right)^\lambda\right)
\end{aligned}$$

which is the same as equation (2) introduced by Duke.

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