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MODELING THE DISPLACEMENT OF RESIDENT SOLUBLE SALT DURING INFILTRATION

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We present two analytical models to assess the displacement of the initial saline soil solution during infiltration. The models are extensions of the analytical infiltration solutions of Morel-Seytoux and Khanji (1974) and Broadbridge and White (1988). We then introduce simple experimental methods with the tension infiltrometer to measure the parameters of the two models in the field. A single measurement of sorptivity is needed to determine the free parameter, C, in the Broadbridge and White solution, and the viscous correction factor, β, in the Morel-Seytoux and Khanji solution.

The tension infiltrometer method, in conjunction with the analytical models, provided simple and inexpensive means to predict the leaching of soil solution as required for saline soils. The tension infiltrometer was simple to operate and measurements were performed at the surface to minimize installation time and soil disturbance. The analytical models contained parameters with precise physical meaning and need modest amounts of computation time. The narrow range of 0 to −15 cm of soil-water pressure, in which infiltration rates were measured with the infiltrometer, appeared sufficient for modeling the displacement of soil solution. This was attributed to the rapid decrease of hydraulic conductivity with water pressure, which was manifested by a short macroscopic capillary length for the field soils tested.

Irrigation schemes in arid and semi-arid regions face the problem of salt build-up in the soil. Salt build-up affects the osmotic pressure in the root zone, hindering the absorption of soil moisture by the plants' roots and causing a decline in crop productivity. To maintain productivity, effective leaching strategies are needed to remove the saline soil solution from the root zone. This paper will introduce mathematical techniques and field methods to design leaching strategies using models of water flow and solute transport.

The United States Geological Survey and the Environmental Protection Agency, among others, have introduced numerical models to assist agronomists and soil scientists in designing leaching strategies for soil solution (e.g., Healy 1990; Noziger et al. 1989). These models couple water flow with solute transport. In saline soils, the soil solution, initially retained by capillarity in the soil matrix, is essentially immobile because of very low hydraulic conductivity at this initial volumetric soil solution content. During water application or surface irrigation, the infiltration water displaces the initial solution, resulting in a plane that separates the two bodies of fluids, the fresh water from infiltration and the displaced saline solution (Fig. 1). Dispersion, which dilutes the salt concentration, takes place across this plane of separation in response to local concentration gradient. Other mechanisms that may complicate the transport equation include ionic diffusion, sorption, and degradation. During infiltration, however, movement with bulk water (advection) is the predominant factor affecting salt transport in soils. This has been suggested by numerous experimental studies (e.g., Bresler and Hanks 1969; Smiles and Philip 1978; Kirda et al. 1973). For practical purposes, therefore, we define the leached depth (Zs in Fig. 1) as the depth to the plane of separation of the fresh water and the displaced saline solution.

Models linking water flow to bulk salt transport have offered a physical approach to design leaching strategies for saline soils. Applications of these models, however, can be hindered by two factors: (i) the difficulty of estimating soil hydraulic properties, e.g., hydraulic conductivity and diffusivity, in the field, and (ii) the complexity of the solutions of the coupled water flow and transport equations.

This study has two objectives. The first objective is to use an analytical approach for salt movement as an alternative to numerical models in the case of advection-dominated transport, e.g., determining the leached depth during infiltration. The analytical approach provides simple solu-

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Fresh Water Supply

Soil Solution Content

Water Content Profile

Front

Zs

Water

Depth

Fig. 1. Displacement of soil solution during infiltration. At the front, area "oabc" is equal to area "cde."
into Eq. (4) yields:

$$\theta \frac{\partial \xi}{\partial t} = -q \frac{\partial \xi}{\partial x}$$  \hspace{1cm} (7)$$

Equation (7) applies for transient transport and can be used to determine the leached depth, the depth to the plane of separation between the displaced saline soil solution and the fresh water supplied (Fig. 1).

Following Smiles et al. (1981), Eq. (7) is solved analytically using the characteristics method. For a water supply rate r(t) at the surface, the leached depth, Zs(t), can be determined from the relationship:

$$\int_{0}^{Z_s(t)} \theta(\xi, t) d\xi - \int_{0}^{t} r(w) dw = 0$$  \hspace{1cm} (8)$$

where $\xi$ and $\nu$ are dummy variables of integration with the meaning of time and space. The first term on the right side of Eq. (8) represents the volume of soil solution between the surface and $Z_s$, whereas the second term represents the cumulative supply of water provided at the surface. Mathematically, these two terms should be equal at $Z_s$. The water supply rate at the surface, and the water content profile resulting from the infiltration process, should be introduced in Eq. (8) to determine $Z_s(t)$. In the following sections, we demonstrate the use of this equation with Morel-Seytoux and Khanji (1974) and Broadbridge and White (1988) infiltration models.

**Broadbridge and White infiltration model**

When the water supply rate at the surface is constant, $r(t) = r$, Broadbridge and White (1988) developed the analytical solution for Richards' equation using these particular forms of the conductivity and diffusivity functions:

$$K_r = K - K_n \frac{\Theta^2}{\Delta K}$$  \hspace{1cm} (9)$$

$$D_r = D \frac{C(C - 1)}{(C - \Theta)^2}$$  \hspace{1cm} (10)$$

where $\Theta = (\Theta_s - \Theta_n)/\Delta \Theta$ and C is a free parameter. Broadbridge and White (1988) found that C is related to the sorptivity and macroscopic capillary length scale by the expression:

$$\lambda_s \frac{\pi(C - 1) + B}{4(C - 1) + 2B \Delta \Theta \Delta K}$$  \hspace{1cm} (11)$$

with $B = 1.46147$. The analytical solution of $\theta(x, t)$, given earlier by Broadbridge and White (1988), is shown again here for convenience:

$$\Theta = C \left[ 1 - \left( 2\rho + 1 \right) \left( u^{-1} \frac{\partial u}{\partial \xi} \right)^{-1} \right]$$  \hspace{1cm} (12)$$

$$z_* = \frac{z}{\lambda_s} = C^{-1}$$  \hspace{1cm} (13)$$

$$\left[ \rho^2 \left( 1 + \rho \right) \tau + \rho \left( 2 + \rho \right) \zeta - \ln \nu \right]$$

with

$$u = \frac{1}{2} e^{-\left( t^2 / \tau \right)} \left[ 2e^{\left( \xi - \rho \tau \right)^2 / \tau} + f_1^+ - f_1^- - f_2^- + f_2^+ \right]$$

$$f_1^\pm = f\left( \left( \xi \pm \rho \left( 1 + \rho \right)^{1/2} \right) / \tau^{1/2} \right)$$

$$f_2^\pm = f\left( \left( \xi \pm \rho \tau \right) / \tau^{1/2} \right)$$

$$f(x) = e^{x^2\text{erfc}(x)}$$

$$\zeta = \int (C - \Theta_0) \, dx.$$  

and $m = 4C(C-1)$, $\rho = (r - K_n)/(m \Delta K)$, $\tau = ut$, $\tau = mt$. The solution $\theta(x, t)$ in Eq. (12) is parametric in $\xi$. Substituting this solution into Eq. (8), we find that:

$$A_1 C Z_* = \xi + \left( 1 - \xi / \Delta K \right) t$$  \hspace{1cm} (14)$$

which can be written in terms of the parameters $\xi$ and $\tau$ using the expression for $z_*$ in Eq. (13) as,

$$A_1 \tau + A_2 \zeta = A_3 \ln u(\xi, \tau)$$  \hspace{1cm} (15)$$

In Eqs. (14) and (15), $A_3 = 1 + \Theta_s/(C \Delta \Theta)$, $A_2 = A_3 (2\rho + 1) - 1$, and $A_1 = A_3 (\rho^2 + \rho) - K_n/(m \Delta K)$ are three constants. Hence, to determine $Z_*$ (t), Eq. (15) is first solved to evaluate $\zeta$; then from (15), $Z_*$ is:

$$Z_* = \frac{Z_0}{\lambda_s} = \frac{1}{A_1 C} \left( \zeta + \frac{1}{\Delta K} t \right)$$  \hspace{1cm} (16)$$
Equation (16) can then be used to predict $Z_s(t)$, the leached depth.

**Morel-Seytoux and Khanji infiltration model**

If ponding at the surface occurs, the water supply rate, $r(t)$, is equal to the capacity infiltration rate, $i(t)$. The Broadbridge and White model is not applicable in this case because the supply rate varies with time. By introducing two-phase flow concepts (water and air), Morel-Seytoux and Khanji (1974) derived an analytical solution for the capacity infiltration rate, $i(t)$. This solution takes the form:

$$i(t) = r(t) = K_z \left[ \frac{(\theta_s - \theta_a)(H_c + H) + I(t)}{\beta(t)} \right]$$  \hspace{1cm} (17)

where $\theta_s$ is water content at natural saturation, $I(t)$ is cumulative infiltration, $H$ is ponded depth of water at the surface, $H_c$ is effective capillary drive, and $\beta$ is a correction factor for viscous capillary resistance (Morel-Seytoux and Khanji 1974, 1975). For $\theta_s = \theta_a$ at the surface, the effective capillary drive, $H_c$, is equivalent to the macroscopic capillary length in Eq. (1) (Morel-Seytoux and Khanji 1974; White and Sully 1987). The parameter $\beta$ is a function of soil and fluid properties and is practically independent of the evolution of the shape of the water profile with time. Mathematically,

$$\beta = \frac{2(\theta_s - \theta_a)H_cK_z}{S^2}$$  \hspace{1cm} (18)

If the portion of the water content profile between the surface and $Z_s(t)$ is the natural saturation, then combining Eqs. (17) and (8) yields:

$$t = \frac{\beta}{K_z} [\theta_s Z_s - (H + H_c)(\theta_s - \theta_a)]$$

$$\ln[1 + \frac{\theta_s Z_s}{(H + H_c)(\theta_s - \theta_a)}]$$  \hspace{1cm} (19)

Equation (19) provides $t$ as a function of $Z_s$. The assumption of saturation involving a portion of the soil water profile may underestimate $Z_s(t)$. For ponded infiltration, however, the solution of the flow equation (Brustkern and Morel-Seytoux 1970) indicated that most of the soil profile will be very close to natural saturation. Also, during ponded infiltration, the portion of the profile that will deviate mostly from natural saturation is below $Z_s$.

In the following section, we introduce field methods to estimate the parameters of the Morel-Seytoux and Khanji and the Broadbridge and White models from field infiltration tests.

**FIELD METHODS**

The tension infiltrometer, described originally by Clothier and White (1981), was used at two plots in Boulder, Colorado, to estimate the soil parameters for these models. Transient and quasi-steady state measurements were needed to estimate the free parameter $C$ in the Broadbridge and White model and the parameter $\beta$ in the Morel-Seytoux and Khanji solution. The experimental procedures used were as follows.

The first infiltration test was done at a supply pressure of zero to determine the ponded sorptivity from the early behavior of the transient infiltration (e.g., Perroux and White 1988). Symbolically,

$$I(t)_{t=0} = St^{1/2}$$  \hspace{1cm} (20)

where $I(t)$, cumulative infiltration, was measured by recording the falling water level in the reservoir of the infiltrometer. The ponded sorptivity, $S$, is the slope of the plot of $I(t)$ vs. $t^{1/2}$.

At later times, the infiltration beneath the infiltrometer disc reached a quasi-steady state (Wooding 1968). The unconfined quasi-steady infiltration rate, $q_s(\psi_a)$, was measured at supply pressures, $\psi_a = 0, -3, -6$, and $-15$ cm (Ankeny et al. 1991). For a three dimensional unconfined infiltration from a circular disc into a dry soil, a simple solution for $q_s$ can be written as (Wooding 1968):

$$q_s(\psi_a) = \frac{K'(\psi_a)(1 + \frac{4\lambda_s}{\pi R})}{\pi R}$$  \hspace{1cm} (21)

where $R$ is the radius of the infiltrometer and $K'(\psi_a)$ is the quasi-linear hydraulic conductivity function, written as (Philip 1966, 1969):

$$K'(\psi)K_z \exp(\psi / \lambda_s)$$  \hspace{1cm} (22)

In Eqs. (21) and (22), $\lambda_s$ is equivalent to the inverse of $\alpha$ because this ensures that quasi-linear solutions, developed with Eq. (22), provide optimal fit for arbitrary hydraulic conductivity functions (Philip 1985). The substitution of (22) in
(21) yields, after taking the natural logarithm on both sides of the resulting equation:

$$\ln q_s(\psi_0) = \psi_0 / \lambda_s + \ln \left( K_s \left( 1 + \frac{4 \lambda_s}{\pi R} \right) \right)$$

(23)

where $\lambda_s$ is the inverse of the slope of the linear regression of $\ln q_s$ on $\psi_0$. Knowing $\lambda_s$, the saturated hydraulic conductivity was determined from (21) at supply pressure $\psi_0 = 0$, mathematically,

$$K_s = \frac{q_s(0)}{1 + \frac{4 \lambda_s}{\pi R}}$$

(24)

Finally, the free parameter $C$ in the Broadbridge and White infiltration model was evaluated by solving the algebraic Eq. (11), after substituting for the measured $K_s$, $\lambda_s$, and S. Similarly, the parameter $\beta$ in Morel-Seytoux and Khanji solution was determined from Eq. (18).

RESULTS AND DISCUSSIONS

Infiltration tests with the tension infiltrometer were completed in approximately 2.5 h at each plot. Starting with a zero supply pressure (ponded condition) reduced the time needed to reach steady state. The time to steady state was longer for -15-cm water supply pressure. As shown in Table 1, the parameters $C$ and $\beta$ were within the range found by Broadbridge and White and Morel-Seytoux and Khanji. The macroscopic capillary length scale, which is a measure of the sensitivity of unsaturated hydraulic conductivity to water pressure (Eq. 1), was considered short for these plots (Table 1). This short $\lambda_s$ indicates a rather rapid decrease of hydraulic conductivity with water pressure, as expected for undisturbed field soils that tend to be dominated by macropores (Nachabe 1995).

The measured parameters can be used in the analytical models to design leaching strategies under ponded and unsaturated water supply conditions. Ideally, the saline solution should be displaced to drains below the root zone to ensure effective leaching. For unsaturated surface conditions, the design variables for a leaching strategy are the duration of water supply and the water supply rate, $r$. During flood irrigation, the design variable is the ponded depth of water, $H$, instead of $r$.

For the sake of illustration, the Broadbridge and White model (Eq. 16) was used to assess the effectiveness of two leaching strategies at plot 1. In the first strategy, a constant supply rate of 1 cm.h$^{-1}$ was used. In the second strategy, the water supply rate was doubled. An initial uniform volumetric solution content, $\theta_n$, of 0.05 cm$^3$/cm$^3$ was assumed in both cases. As shown in Fig. 2, the first strategy was more effective because it required less cumulative water supply to achieve a leaching depth. For example, to leach the top 15 cm of soil, 3.5 cm of water should be supplied at the surface in the first strategy compared with 4.5 cm for the second strategy. The first strategy, however, needed more operation time because of the smaller water supply rate. To demonstrate the influence of the initial volumetric solution content on the mechanism of displacement, a simulation with $\theta_n = 0.1$ and $r = 1$ cm/h was done. As shown in Fig. 3, the leached depth, $Z_s$, decreased when $\theta_n$ was increased. This decrease suggested that the saline solution was easier to displace at

<table>
<thead>
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<th>TABLE 1</th>
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<td>Parameters estimated with the tension infiltrometer</td>
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<tr>
<th>Plot</th>
<th>$S$ (cm.h$^{-1}$)</th>
<th>$K_s$ (cm.h$^{-1}$)</th>
<th>$\lambda_s$ (cm)</th>
<th>$t_s$ (h)</th>
<th>$C$</th>
<th>$\beta$</th>
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<tr>
<td>Plot 1</td>
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<td>1.43</td>
<td>2.1</td>
</tr>
<tr>
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<td>5.90</td>
<td>4.18</td>
<td>0.27</td>
<td>1.16</td>
<td>1.10</td>
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Fig. 3. Leached depth of soil when $\theta_a$ is increased from 0.05 to 0.1. The supply rate is 1 cm/h in both simulations.

Fig. 4. Water content profile and leached depth for $\theta_a$ equal 0.05 (solid line) and 0.1 (dashed line) after 2 h of water supply.

lower initial volumetric water content. This behavior was anticipated intuitively because it is easier to displace air than to displace soil-solution when the initial volumetric solution content is low. Figure 4 shows the wetting front, solution of Eqs. (12) and (13), and Zs. The leached depth lagged behind the wetting front regardless of $\theta_a$. This lag was also observed in the experiments by Kirda et al. (1973).

These results emphasize the need to distinguish between the mechanism of water flow and solute transport. Water flow is governed by capillarity and gravity, whereas transport represents actual displacement of soil solution through the pore space. The distinction between these mechanisms is essential to designing leaching strategies during infiltration.

For ponded infiltration, Eq. (19) was used to predict Zs at plot 2. Figure 5 compares the duration of water application and the ponded depth of water, H, required to achieve a leaching depth of 10, 20, and 40 cm at this plot. Increasing H served to reduce the time of water application. The graphs in Fig. 5 can be used to design H and duration of water application to compare leaching strategies. For example, a ponded depth of 5 cm should be maintained for 0.79 h to displace the solution from the top 20 cm of soil. The duration of application can be reduced to 0.41 h if H is increased to 26 cm. Obviously, the assessment of leaching strategies, as presented here, is based on the knowledge of soil hydraulic properties and the dynamics of fluid flow in soils.

Fig. 5. Duration of ponded depth needed to leach the soil.

CONCLUSION

We demonstrated the use of two analytical models to determine the leaching of soil solution during infiltration. The models, intended to assess designs of leaching strategies, are also applicable for advection-dominated solute transport where the source of contamination is from the applied water. The analytical models, in conjunction with the field method with the infiltrometer, provided means to design and compare leaching strategies. The analytical models were simple to use and required trivial computations as compared with the numerical schemes. Yet, the analytical models were based on the dynamics of fluid flow in porous media and contained measurable parameters. Simple methods with the tension infiltrometer were introduced to determine these parameters in the field. The infiltrometer was simple to operate, and the tests were performed at the soil surface to minimize soil disturbance and installation time.

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REFERENCES


