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Macroscopic Capillary Length, Sorptivity, and Shape Factor in Modeling the Infiltration Rate

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ABSTRACT

Infiltration tests in the field involve measurements of sorptivity and macroscopic capillary length. These two parameters are strongly related through the shape factor, which is a measure of the nonlinearity of the soil hydraulic diffusivity. In this study, relationships were developed between the macroscopic capillary length, the sorptivity, and the shape factor and the parameters of the Brooks and Corey and van Genuchten expressions of hydraulic conductivity and diffusivity. These relationships are important for users of numerical models who need to estimate the parameters of these expressions to predict water flow and contaminant transport in soils. Numerical simulations with a dimensionless form of Richards' equation show that the predicted infiltration rate will not be very sensitive to small variations in the shape factor, provided the macroscopic capillary length is the same. This result is encouraging because the shape factor is difficult to determine accurately in the field. The similarity of the dimensional infiltration solutions implies that the macroscopic capillary length is a scale factor, because (i) it renders predictions of infiltration rates fairly insensitive to the expressions of hydraulic conductivity and diffusivity used, and (ii) it reduces the number of parameters needed to characterize infiltration. Therefore, the infiltration curve into a particular soil can be deduced by choosing units of length and time (i.e., scaling) of a generalized infiltration solution.

MODELING THE INFILTRATION RATE at the soil surface is important in numerous applications of soil science including contaminant fate and transport in the vadose zone, irrigation management, and rainfall–runoff prediction. Computer models (e.g., U.S. Environmental Protection Agency, 1989; U.S. Geological Survey, 1987) are used routinely to assist regulators, agronomists, and hydrologists in predicting infiltration rates. Successful applications of these models can be achieved only if reliable estimates of analytical expressions of hydraulic conductivity and diffusivity are available. This study introduces methods to estimate suitable parameters for hydraulic conductivity and diffusivity using the information from field infiltration tests.

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The disk permeameter and the tension infiltrometer provide simple and inexpensive means to conduct field infiltration tests. The sorptivity and the macroscopic capillary length are two parameters that emerge from the analysis of an infiltration test (e.g., White and Sully, 1987; Perroux and White, 1988; Nachabe and Illangasekare, 1994). The macroscopic capillary length, λ_s, is related to the conductivity and diffusivity of a soil by the expression (White and Sully, 1987)

$$\lambda_s = \frac{1}{\Delta K}\int_{\psi_n}^{\psi_o} K(\psi)\,d\psi = \frac{1}{\Delta K}\int_{\theta_n}^{\theta_o} D(\theta)d\theta \qquad [1]$$

where $K(\psi)$ is the hydraulic conductivity, ψ is the water pressure ($\psi < 0$), ψ_o is the water pressure at the supply surface, ψ_n is the antecedent water pressure, $\Delta K = K(\psi_o) - K(\psi_n)$, $D(\theta)$ is the diffusivity, θ is the volumetric water content, $\theta_o = \theta(\psi_o)$ is the wet (or supply) water content applied at the surface, and $\theta_n = \theta(\psi_n)$ is the dry (or antecedent) water content. Similarly, the sorptivity, S, is well approximated by the expression (Parlange, 1975)

$$S^2 = \Delta\theta\int_{\theta_n}^{\theta_o} D(\theta)d\theta + \int_{\theta_n}^{\theta_o} (\theta - \theta_n)D(\theta)d\theta \qquad [2]$$

where $\Delta\theta = \theta_o - \theta_n$. The significance of the macroscopic capillary length and sorptivity is well documented in the literature on soil physics; the macroscopic length scale is equivalent to the wetting front suction or the effective capillary drive in the Green and Ampt model of infiltration (Morel-Seytoux and Khanji, 1974). The macroscopic capillary length is also equivalent to the inverse of the sorptive number, the single parameter in the exponential (or the quasi-linear) hydraulic conductivity function, which is used to linearize and solve the multidimensional, transient Richards' equation (e.g., Wooding, 1968; Raats, 1976; Philip, 1986). The sorptivity, which can be determined from the early stage of an infiltration test, is a parameter in the popular two-term infiltration solution of Philip (1969). White and Sully (1987) have shown that λ_s and S are strongly related by the expression determine, approximately, the value of the other parameter, provided that ΔK and $\Delta\theta$ are measured in the infiltration test.

There were three objectives for this study. The first objective was to address an issue that faces users of computer models of water flow in the unsaturated zone. The issue is how to relate the macroscopic capillary length and sorptivity, parameters that are easily measured in infiltration tests, to the parameters in the Brooks and Corey (1964) (B–C from hereafter) or van Genuchten (1980) (vG from hereafter) analytical expressions of hydraulic conductivity and diffusivity. These analytical expressions are essential for predicting infiltration with existing numerical models of Richards' equation (e.g., U.S. Environmental Protection Agency, 1989; U.S. Geological Survey, 1987). For this purpose, this study developed convenient relations that allow users of numerical models to determine parameters of the vG and B–C expressions by conserving the macroscopic capillary length and the sorptivity (or the shape factor). Conserving the macroscopic capillary length and sorptivity means that the diffusivity and conductivity expressions of vG and B–C yield the same λ_s and S when substituted in Eq. [1] and [2]. Philip (1985) claimed that any analytical expression of hydraulic conductivity should conserve at least the macroscopic capillary length for accurate prediction of the steady infiltration rate. This study will examine this claim for transient infiltration by performing numerical simulations of a scaled form of Richards' equation.

Using physical similarity of the dimensional infiltration solutions, the second objective was to develop a generalized infiltration solution. The infiltration into any particular soil can then be determined from the generalized infiltration solution by simply changing the units of length and time (i.e., scaling) of the generalized solution. This approach is particularly attractive to describe spatial variability because repeated simulations are performed with few computations (Warrick et al., 1985). An example is provided to demonstrate this scaling procedure. Warrick et al. (1985) developed a generalized infiltration solution that requires matching the first terms of Philip's (1969) semianalytical series solution for water flow. It applies for the early stage of infiltration when the influence of gravity is relatively small. The generalized infiltration numerical solution of Richards' equation in this study applies for all times.

The shape factor, b, cannot be measured directly in the field. Also, b is difficult to estimate accurately because (i) it varies within a narrow range, and (ii) it involves many other parameters (see Eq. [3]). White and Sully (1987) suggested that using $b = 0.55$ is practical for most field soils. Warrick and Broadbridge (1992) proposed to refine the value of b depending on a nonlinearity index. Warrick (1995) showed that b can be expressed in terms of the moments of the diffusivity function. Yet, the influence of b in models that predict the transient infiltration at the surface is not well known. Thus, the third objective for this study is to examine the sensitivity of the infiltration rate predictions to small variations in b.

METHODS

Macroscopic Capillary Length, Sorptivity, and Shape Factor for Brooks and Corey Analytic Expressions

Using the Burdine (1953) model, B–C expressions of hydraulic conductivity and soil-water retention are:

$$\frac{K}{K_s} = \psi_*^{-(2+3\lambda)}$$

$$\Theta = (\psi_*)^{-\lambda} \quad [4]$$

In Eq. [4], Θ is the dimensionless water content,

$$\Theta = \frac{\theta - \theta_r}{\theta_s - \theta_r} \quad [5]$$

ψ_* is the dimensionless water pressure,

$$\psi_* = \frac{\psi}{\psi_d} \quad [6]$$

K_s is the saturated hydraulic conductivity, θ_s is the saturated water content, θ_r is the residual water content, and ψ_d and λ are two fitting parameters that characterize the pore-size distribution of the soil. Traditionally, ψ_d is known as the displacement or air entry pressure, and λ is a dimensionless index of pore-size distribution (λ must not be confused with λ_s, the macroscopic capillary length). Brooks and Corey found that, for typical porous media, λ is about 2. Undisturbed uniform sands have higher values of λ, say 4 or 5. Media with a wide range of pore sizes have values of $\lambda < 2$. For $\psi_* > 1$, the diffusivity $D(\theta) = K(\theta)\, d\psi/d\theta$ of the B–C expression is

$$D(\Theta) = \frac{-\psi_d K_s}{\lambda(\theta_s - \theta_r)} \Theta^{2\lambda+1/\lambda} \quad [7]$$

Relationship between ψ_d and λ, parameters of the B–C expression, and λ_s is developed by substituting Eq. [7] in Eq. [1] to obtain

$$\frac{\lambda_s}{-\psi_d} = \frac{1}{3\lambda + 1} \frac{K_s}{K(\Theta_o) - K(\Theta_n)} \left(\Theta_0^{3+1/\lambda} - \Theta_n^{3+1/\lambda}\right) \quad [8]$$

where $\Theta_0 = (\theta_0 - \theta_r)/(\theta_s - \theta_r)$ and $\Theta_n = (\theta_n - \theta_r)/(\theta_s - \theta_r)$. For infiltration tests in which the initial soil-water is relatively immobile, $K(\Theta_n) \approx 0$ in Eq.[8], the term $\Theta_n^{3+1/\lambda}$ is one to two orders of magnitudes smaller then $\Theta_o^{3+1/\lambda}$, Eq. [8] can be written as

$$\frac{\lambda_s}{-\psi_d} = \frac{1}{3\lambda + 1}\left(\Theta_o^{-1/\lambda}\right) \quad [9]$$

Similarly, an expression for sorptivity in terms of B–C parameters is obtained by substituting for Eq. [7] in [2] to obtain

$$S^2 = -\psi_d\, \Delta\theta K_s \frac{(7\lambda + 2)}{(3\lambda + 1)(4\lambda + 1)} (\Theta_o)^{-1/\lambda} \quad [10]$$

The substitution of S and λ_s into Eq. [3] yields the expression for the shape factor

$$b = \frac{4\lambda + 1}{7\lambda + 2} \quad [11]$$

The value of b for the B–C expression ranges between 0.5 and 4/7. For $\lambda = 0$, b is ½, which corresponds to a dirac

$$b = \frac{4\lambda + 1}{7\lambda + 2} \quad [11]$$

The value of b for the B-C expression ranges between 0.5 and 4/7. For $\lambda = 0$, b is ½, which corresponds to a dirac delta diffusivity function. Figure 1 is a plot of $(\lambda_s/-\psi_d)$ vs. λ (Eq. [9]) for $\Theta_o = 0.4$, 0.6, 0.8, and 1. The ratio $(\lambda_s/-\psi_d)$ decreases as λ increases.

Equation [4] is valid for $\psi_* > 1$ only. For $\psi_* = 0$ at the surface, λ_s is given as (Nachabe and Illangasekare, 1994)

$$\frac{\lambda_s}{-\psi_d} = \frac{(3\lambda + 2)}{(3\lambda + 1)} \quad [12]$$

Equation [12] includes the integration of hydraulic conductivity (see Eq. [1]) over the capillary fringe zone where $K(0 < \psi_* < 1) = K_s$.

Macroscopic Capillary Length, Sorptivity, and Shape Factor for van Genuchten Analytic Expression

Using the Mualem model (Mualem, 1976), the diffusivity of the vG analytic expression is (van Genuchten, 1980)

$$D(\theta) = \frac{(1-m)K_s}{\alpha m(\theta_s - \theta_r)} \Theta^{1/2-1/m} [(1 - \Theta^{1/m})^{-m} + (1 - \Theta^{1/m})^m - 2] \quad [13]$$

where α and m are two fitting parameters that are considered measures of the pore-size distribution of the soil. In practice, m ranges between 0.1 and 0.9. Small m values are associated with soils that have a wide pore-size distribution.

Relationship between λ_s and m and α is obtained by substituting Eq. [13] in [1]. The result is

$$\lambda_s = \frac{1}{\alpha}\left(\frac{1-m}{m}\right)\frac{K_s}{\Delta K}\mathrm{INT}1 \quad [14]$$

Similarly an expression for b is

$$b = \frac{\mathrm{INT}1\,(\Theta_o - \Theta_n)}{\mathrm{INT}1(\Theta_o - \Theta_n) + \mathrm{INT}2} \quad [15]$$

where

$$\mathrm{INT}1 = \int_{\Theta_n}^{\Theta_o} \Theta^{1/2-1/m} [(1 - \Theta^{1/m})^{-m} + (1 - \Theta^{1/m})^m - 2]\, d\Theta \quad [16]$$

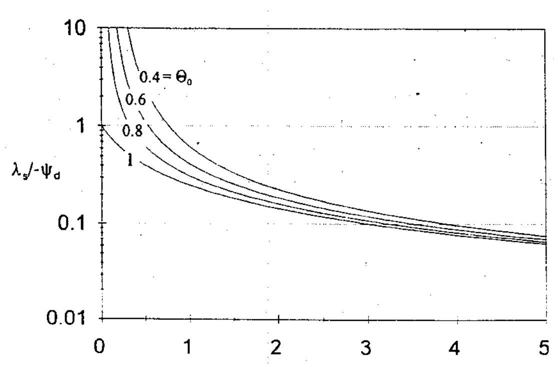

Fig. 1. Relationships between the macroscopic capillary length and the parameters of the Brooks and Corey expression. Numbers on curves refer to supply water content, Θ_o.

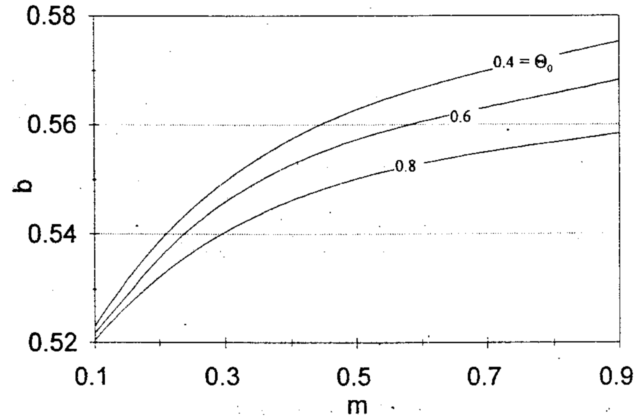

Fig. 2. Shape factor b as a function of the parameter m of the van Genuchten expression. Numbers on curves refer to supply water content, Θ_o.

and

$$\mathrm{INT}2 = \int_{\Theta_n}^{\Theta_o} (\Theta - \Theta_n)\,\Theta^{1/2-1/m} [(1 - \Theta^{1/m})^{-m} + (1 - \Theta^{1/m})^m - 2]\, d\Theta \quad [17]$$

The integrals in Eq. [16] and [17] are evaluated numerically for $\Theta_o = 1$, 0.8, 0.6, and 0.4 and $\Theta_n = 0$. For $\Theta_o = 1$, excellent polynomial approximations of λ_s and b are

$$\lambda_s = \left(\frac{1}{\alpha}\frac{0.0128m + 0.66m^2 - 0.73m^3}{0.229 + 0.077m - 0.231m^2 - 0.12m^3}\right) \quad [18]$$

$$b = \frac{0.085 + 0.077m + 0.056m^2 + 0.056m^3}{0.17 + 0.118m + 0.125m^2 + 0.125m^3} \quad [19]$$

The boundary condition $\Theta_o = 1$ implies the soil surface is at maximum saturation (ponding condition). For this condition, the infiltration rate is known as the infiltration capacity of the soil. Similar polynomial approximations were obtained for other values of Θ_o. The results are presented graphically in Fig. 2 and 3.

NUMERICAL PROCEDURES AND DISCUSSION

Hydraulic conductivity and diffusivity expressions are needed in soil-water flow models that solve Richards' equation

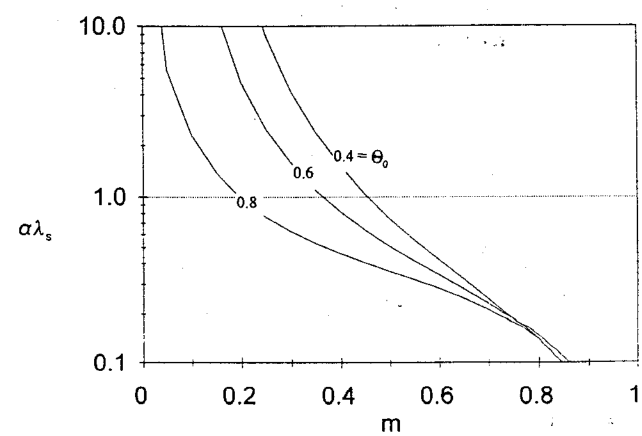

Fig. 3. Relationships between the macroscopic capillary length and the parameters of the van Genuchten expression. Numbers on curves refer to supply water content, Θ_o.

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z}\left[D(\theta)\frac{\partial \theta}{\partial z}\right] - \frac{\partial K(\theta)}{\partial z} \quad [20]$$

In Eq. [20], z is depth taken zero at the soil surface and positive downward, and t is time from the beginning of a simulation. By introducing the macroscopic capillary length, Eq. [20] is written in the dimensionless form (e.g., Broadbridge and White, 1988; Nachabe et al., 1995):

$$\frac{\partial \theta_*}{\partial t_*} = \frac{\partial}{\partial z_*}\left[D_*\frac{\partial \theta_*}{\partial z_*}\right] - \frac{\partial K_*}{\partial z_*} \quad [21]$$

where $z_* = z/\lambda_s$, $t_* = t/t_s$ where $t_s = \Delta\theta\ \lambda_s/\Delta K$, the sorptive time, $\theta_* = (\theta - \theta_n)/\Delta\theta$, $D_* = Dt_s/\lambda_s^2$, a normalized diffusivity, and $K_* = (K - K_n)/\Delta K$, a normalized hydraulic conductivity. The procedure to determine m and α parameters of the vG or λ and ψ_d parameters of the B–C expressions of conductivity and diffusivity are as follows. If, in addition to the hydraulic conductivity, the macroscopic capillary length and sorptivity are measured from an infiltration test, Eq. [3] is used to estimate b. Then, for a selected θ_0 the parameter m in vG expression is determined from the graphs in Fig. 2. If one desires to solve Eq. [20] using the B–C expression instead, Eq. [11] is solved for λ, mathematically,

$$\lambda = \frac{2b - 1}{4 - 7b} \quad [22]$$

The parameters λ or m are then used as abscissa in the graphs of Fig. 1 and 3 to determine ψ_d of B–C or α of vG expression. This procedure guarantees that B–C and vG analytical expressions of diffusivity and conductivity conserve the λ_s and S measured in the field. In reverse, if the parameters λ and ψ_d in the B–C expression or α and m in the vG expression are measured from a laboratory experiment (e.g., by curve fitting of water retention data), the macroscopic capillary length can be determined using Fig. 1 and 3.

To study the influence of b in predicting the infiltration rate, and to justify conserving the macroscopic capillary length/sorptivity, the dimensionless Richards' Eq. [21] is solved numerically using B–C and vG expressions for different b values and constant λ_s and t_s. In all the numerical simulations with Eq. [21], the value of λ_s is 100 units of length, that of t_s is 100 units of time. Saturated hydraulic conductivity is one unit of length per one unit of time. Obviously, in the dimensionless Eq. [21], the unit of length (one unit of length = L) and the unit of time (one unit of time = T) can be specified to suit any particular soil. The parameters of the vG and B–C expressions of conductivity and diffusivity are determined by the procedures proposed above. For example for $b = 0.5555$ (see Soil 1 in Table 1), λ in the BC expression is 1 as predicted by Eq. [22]. For $\Theta_o = 0.80$ at the surface, the value of $\lambda = 1$ is used in Fig. 1 to yield a ratio $\lambda_s/(-\psi_d)$ of 0.3125. Therefore, the value of $(-\psi_d)$ is 320 L for $\lambda_s = 100\ L$.

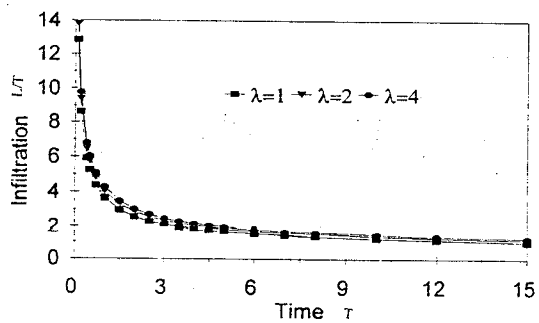

Fig. 4. Predicted infiltration rate at the surface for the three soils in Table 1 (case $\Theta_o = 0.80$).

Table 1. Brooks and Corey parameters for three hypothetical soils (supply water content $\Theta = 0.8$).

	λ	b	$-\psi_d$	λ_s	t_s
			L	L	T
Soil 1	1	0.5555	320	100	100
Soil 2	2	0.5625	626	100	100
Soil 3	4	0.5667	1229	100	100

For $\Theta_o = 0.80$, Fig. 4 shows the calculated infiltration rate using Eq. [21] and the B–C parameters of the three hypothetical soils in Table 1. The values of λ in Table 1 are representative of the range of λ for a variety of soils (Brooks and Corey, 1964). Clearly, the predicted infiltration rate at the surface was practically the same, regardless of λ or shape factor. Apparently, the variation in shape factor, which was within a very narrow range for these soils (Table 1), had no significant impact on the predicted infiltration rate. For $\Theta_o = 1$, three additional simulations were conducted using the vG expression and the parameters in Table 2. The values of m in Table 2 are typical for soils as demonstrated by van Genuchten (1980). Again, the predicted infiltration rates for these soils were very close regardless of m or shape factor (Fig. 5). Considering the experimental errors in estimating hydraulic conductivity and diffusivity expressions in the field, the match between the infiltration curves is considered very good. These results suggest that small variations in the shape factor play a minor role in predicting the infiltration rate, provided the capillary length is the same. Also these results confirm the need to conserve the capillary length in fitting hydraulic conductivity and diffusivity functions for modeling infiltration.

The physical similarity of the infiltration curves across a variety of soils with different pore-size distributions, i.e., different m and α in vG or λ and ψ_d in B–C, have broader physical implications. It suggests that the capillary length can be used as a scaling parameter to generalize infiltration calculations. The infiltration curve into a particular soil type can then be deduced by changing the units of length and time (i.e., scaling) of the generalized infiltration curve solution. This scaling procedure is demonstrated for Guelph loam.

Table 2. Van Genuchten parameters for three hypothetical soils (supply water content $\Theta_o = 1$).

	m	b	α	λ_s	t_s
			L^{-1}	L	T
Soil 1	0.1	0.509	0.00282	100	100
Soil 2	0.2	0.516	0.000969	100	100
Soil 3	0.667	0.52	0.006	100	100

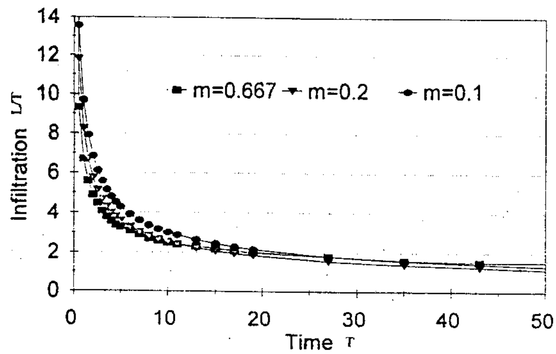

Fig. 5. Predicted infiltration rate at the surface for the three soils in Table 2 (case $\Theta_o = 1$).

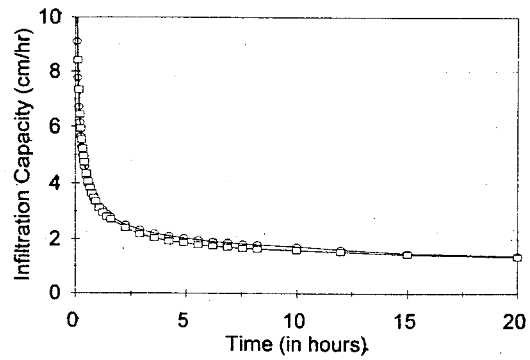

Fig. 6. Infiltration capacity for Guelph loam as predicted by solving Richards' equation (squares) and by scaling a generalized infiltration curve solution (circles).

Scaling of Infiltration Rate Using λ_s and t_s

The parameters of the vG expression for Guelph loam were determined by van Genuchten (1980). In addition to these parameters, Table 3 shows the corresponding macroscopic capillary length and sorptive time for Guelph loam. Initially, the soil is assumed dry [$\Theta_n(z,t) = 0$] and a boundary condition $\Theta_o = 1$ (ponding condition) is applied at the surface. For this boundary, infiltration is at capacity, and the infiltration rate at the surface is the infiltration capacity of the soil. This boundary condition is important for subdividing rainfall into runoff and infiltration in watershed models.

The infiltration capacity is determined for Guelph loam using two methods. In the first method, the infiltration capacity is determined by solving Richards' equation using the vG expression and the parameters m and α in Table 3. In the second method, the infiltration capacity is determined by scaling the infiltration rate solution of Soil 2 (see Table 2 and Fig. 5) by changing units of time, T, and length, L, to match the macroscopic capillary length and sorptive time for Guelph loam. Therefore, $\lambda_s = 36.15$ cm for Guelph loam is set at 100 L, the macroscopic capillary length for Soil 2 of Table 2 resulting in $L = 0.3615$ cm. Similarly, the sorptive time scale is $t_s = 8.29$ h $= 100\ T$, which results in $T = 0.0829$ h. To determine the scaled infiltration rate for Guelph loam, the time axis for the generalized solution (Soil 2, Fig. 5) is multiplied by $T = 0.0829$ h, and the infiltration rate is multiplied by $\Delta\theta(L/T) = 0.302(0.3615/0.0829) = 1.317$ cm h^{-1}.

Figure 6 shows the infiltration rate for Guelph loam by the two methods for 20 h. The time t_g at which gravity begins to dominate the flow (Philip, 1969) for this simulation is 15.8 h. Clearly, the scaled infiltration rate simulates well the infiltration rate by the Richards' equation. Similar results (not shown) were obtained for other soils.

Table 3. Physical properties of Guelph loam (after van Genuchten, 1980).

θ_s	θ_r	K_s	α	m	λ_s	t_s
— cm^3 cm^{-3} —		cm h^{-1}	cm^{-1}		cm	h
0.52	0.218	1.317	0.0115	0.507	36.2	8.29

CONCLUSION

Simple procedures were introduced to guarantee that vG and B–C expressions of hydraulic properties conserve the macroscopic capillary length and sorptivity (or shape factor). These relationships are important for users of computer models who need to estimate the parameters of these analytical expressions. The shape factor varied within a narrow range for the B–C and vG expressions. Apparently, small variations in b do not affect the infiltration rate significantly. This result is encouraging because the macroscopic capillary length and sorptivity can be measured directly, whereas the shape factor is difficult to estimate accurately from an infiltration test.

The physical similarity of the dimensional infiltration curves suggested that the capillary length can be used as a scale factor. Infiltration into a particular soil can then be determined by changing the units of length and time (i.e., scaling) of a generalized infiltration curve solution. This approach is particularly attractive for analysis of spatial variability because repeated infiltration curves are developed with minimal computations.

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