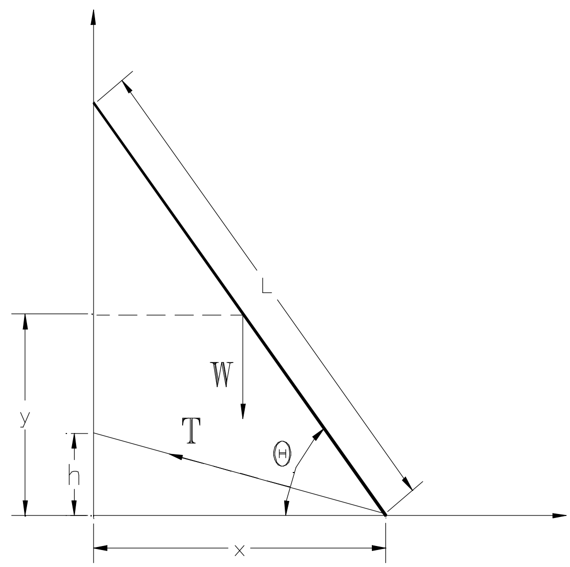
EML 6273 ADVANCED DYNAMICS OF MACHINERY 10/3/94

TEST I

1. A uniform ladder of length L and weight W is in equilibrium at an angle θ with the floor. If the wall and floor are smooth, the ladder must be held in place by a rope fastened at the bottom of the ladder and to a point of the wall at a height h above the floor. Find the tension T in the rope.

Problem 1.



2. A uniform rod of mass M lies on the z-axis between z=0 and z=L. Find the gravitational force on a point mass m placed on the axis at distance z from the origin (z>L).

3. A head-on collision of two balls takes place on a smooth plane. The balls are of same size and mass but because of manufacturing defect the center of mass of one ball is off center by a distance of δ. Find equations that will describe this collision and determine the velocities after the collision. Make some additional assumptions.

4. Consider a particle of mass m moving a plane. It subjects only to the gravitational force with the center of gravity at the origin of the xy coordinates. Derive the equations of motion with the use of Lagrangian equations.

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Test II

1. A hoop with mass m and radius a is rolling down along an inclined plane without slipping. Find the necessary friction between the hoop and the surface of the inclined plane.

2. Consider a vehicle that is driven up a spiral parking ramp, modeled as a particle that moves along a spiral. With the use of Lagrange's equation with constraints, derive the equation of motion. Assume some necessary conditions.

3. Find y=y(x) such that the variation of the integral of

$$f = y'^2 + z'^2 - ay$$

is zero, where z=ay and a is constant.

4. A comet follow a parabolic trajectory, which brings it to a distance d from the sun at its closest point. The orbit of the comet is in the plane of the Earth's orbit, taken as a circle of radius R. Find the fraction of a year which the comet spends inside the orbit of Earth as a function of the ratio d/R.

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FINAL EXAMINATION

1. Consider a heavy symmetrical top. The symmetry axis of the top is chosen as the z-axis; the bottom (support) point of the top is the origin of coordinates. Use the Euler angles θ, ϕ, and ψ, as the angular coordinates. Find the equations of motion through the use of Lagrangian equations.

2. Suppose that a car is just started and is to be driven on a horizontal ground with ice. Find the equations to describe the motion and find the required frictional coefficient between the tires and the ice. Explain why the driver should not push the throttle pad very hard. Assume that the mass of the car is M, the moment of inertia of wheels is I and the torque exerting on the driving wheels is T. Make some necessary assumptions to simplify the problem.

3. A satellite is launched from the surface of Earth. At the time of burnout the satellite is located at altitude of 1,000 km with the radial velocity of v_r = 500 m/s. Determine the required tangential velocity such that the minimum radius of the orbit is 7,000 km.

4. A satellite is launched from the surface of Earth. At the time of burnout the satellite is located at altitude of 700 km with velocity of $\mathbf{v} = 1000\mathbf{e}_r + 5000\mathbf{e}_\theta$ m/s. Determine the impulse required to increase the velocity in the tangential direction when v_r is zero so that the orbit of satellite is circular around the Earth.

5. Suppose that a two stage rocket is to be designed. The total initial mass is 480 kg and the payload is 30 kg. Assume that the structure coefficients, $\in_1$=0.14, $\in_2$=0.15. For the best design find the payload ratios, λ_1 and λ_2 ,for the two stages.

6. Construct the Brehme diagram for x'y'z' system moving with **v** = -**i**v toward xyz system. Taking any point in the diagram, show that the Lorentz transformation is true.

7. (a) Prove that $\mathbf{A}\bullet(\mathbf{n}\times\overleftrightarrow{1}) = \mathbf{A}\times\mathbf{n}$.

(b) Prove that $(\mathbf{n}\times\overleftrightarrow{1})\bullet(\mathbf{n}\times\overleftrightarrow{1}) = \mathbf{nn} - \overleftrightarrow{1}$.