Delay Spread Estimation for Wireless Communication Systems

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Abstract

In this paper, average frequency correlation and root-mean-squared (rms) delay spread estimation for wireless communication systems are described. A practical algorithm for averaged channel frequency correlation estimation, which can be used for measuring frequency selectivity, is given. Average rms delay spread of the channel is obtained directly from the channel frequency correlation estimate without obtaining the channel power delay profile (PDP). Analytical relations between channel frequency correlation and rms delay spread value is derived by assuming an exponentially decaying PDP. Robustness of the proposed rms delay spread estimation method for other PDPs is evaluated. An orthogonal frequency division multiplexing (OFDM) based wireless communication system is considered for evaluating the performance of the proposed algorithm. It is observed that proposed channel frequency correlation and rms delay spread estimation algorithms work very well in various environments with different PDPs.

1. Introduction

In digital wireless communication systems, transmitted information reaches the receiver after passing through a radio channel, which can be represented as an unknown, time-varying filter. Transmitted signals are typically reflected and scattered, arriving at the receiver through multiple paths. When the relative path delays are on the order of a symbol period or more, images of *different symbols* arrive at the same time, causing intersymbol interference (ISI). Traditionally, ISI due to time dispersion is handled with equalization techniques. As the wireless communication systems making transition from voice centric communication to interactive Internet data and multi-media type of applications, the desire for higher data rate transmission is increasing tremendously. The higher data rates, with narrower symbol durations experiences significant dispersion, requiring highly complex equalizers.

New generations of wireless mobile radio systems aim to provide higher data rates to the mobile users while serving as many users as possible. Adaptation methods are becoming popular for optimizing mobile radio system transmission and reception at the physical layer as well as at the higher layers of the protocol stack. These adaptive algorithms allow improved performance, better radio coverage, and higher data rates with low battery power consumption. Many adaptation schemes require a form of measurement (or estimation) of one or more variable(s) that might change over time. The information about the frequency selectivity of the channel, and the corresponding time domain root-mean-squared (rms) delay spread can be very useful for improving the performance of the wireless radio receivers through transmitter and receiver adaptation. For example, in a time division multiple access (TDMA) based GSM system, the number of channel taps needed for equalization might vary depending on channel dispersion. Instead of fixing the number of channel taps for the worst case channel condition, it can be changed adaptively [1], allowing simpler receivers with reduced battery consumption and improved performance. Similarly, in [3] a TDMA receiver with adaptive demodulator is proposed using the measurement about the dispersiveness of the channel. Dispersion estimation can also be used for other parts of transmitters and receivers. For example, in channel estimation using channel interpolators, instead of fixing the interpolation parameters for the worst expected channel dispersion as commonly done in practice, the parameters can be changed adaptively depending on the dispersion information [7].

Although dispersion estimation can be very useful for many wireless communication systems, we believe that it is particularly crucial for orthogonal frequency division multiplexing (OFDM) based wireless communication systems. OFDM, which is a multi-carrier modulation technique, handles the ISI problem due to high bit rate communication by splitting the high rate symbol stream into several lower rate streams and transmitting them on different orthogonal carriers. The OFDM symbols with increased duration might still be effected by the previous OFDM symbols

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due to multipath dispersion. Cyclic prefix extension of the OFDM symbol avoids ISI from the previous OFDM symbols if the cyclic prefix length is greater than the maximum excess delay of the channel. Since the maximum excess delay depends on the radio environment, the cyclic prefix length needs to be designed for the worst case channel condition which makes cyclic prefix as a significant portion of the transmitted data, reducing spectral efficiency. One way to increase spectral efficiency is to adapt the length of the cyclic prefix depending on the radio environment [5]. The adaptation requires estimation of maximum excess delay of the radio channel, which is also related to the frequency selectivity of the channel. Other OFDM parameters that could be changed adaptively using the knowledge of the dispersion are OFDM symbol duration and OFDM subcarrier bandwidth. In summary, adaptation through dispersion estimation provides better overall system performance, improved radio coverage, and higher data rates.

Characterization of the frequency selectivity of the radio channel is studied in [9, 10, 8] using level crossing rate (LCR) of the channel in frequency domain. Frequency domain LCR gives the average number of crossings per Hz at which the measured amplitude crosses a threshold level. Analytical expression between LCR and the time domain parameters corresponding to a specific multipath power delay profile (PDP) is given. LCR is very sensitive to noise, which increases the number of level crossing and severely deteriorates the performance of the LCR measurement [8]. Filtering the channel frequency response reduces the noise effect, but finding the appropriate filter parameters is an issue. If the filter is not designed properly, one might end up smoothing the actual variation of frequency domain channel response. In [6], instantaneous rms delay spread, which provides information about local (small-scale) channel dispersion, is obtained by estimating the channel impulse response (CIR) in time domain. The detected symbols in frequency domain are used to re-generate the time domain signal through inverse fast Fourier transform (IFFT). Then, this signal is used to correlate the actual received signal to obtain CIR, which is then used for delay spread estimation. Since, the detected symbols are random, they might not have good autocorrelation properties, which can be a problem especially when the number of carriers is low. In addition, the use of detected symbols for correlating the received samples to obtain CIR provides poor results for low signal-to-noise-ratio (SNR) values. In [5], the delay spread is also calculated from the instantaneous time domain CIR, where in this case the CIR is obtained by taking IFFT of the frequency domain channel estimate.

In this paper, estimation of frequency selectivity using channel frequency correlation estimation will be discussed. A practical algorithm for average channel frequency correlation estimation is given. Channel frequency correlation estimates will then be used to calculate rms delay spread of the channel. Analytical expression for calculating rms delay spread from channel frequency correlation is derived. The robustness of the proposed rms delay spread estimation for various PDPs is evaluated through computer simulation. It is observed that the proposed channel frequency correlation and rms delay spread estimation algorithms work very well in various environments with different PDPs.

The paper is organized as follows. First, a generic OFDM system description is given. Time and frequency domain channel models are discussed briefly. Then, estimation of frequency correlation from the channel frequency response is given in Section 3. A mathematical relation between the channel frequency correlation and rms delay spread is also derived in this Section. Performance results of the proposed algorithms are presented in Section 4, followed with the concluding remarks.

2 System Model

An OFDM based system model is used. Time domain samples of an OFDM symbol can be obtained from frequency domain symbols as

$$\begin{aligned} x_m(n) &= IDFT\{S_m(k)\} \\ &= \textstyle\sum_{k=0}^{N-1} S_m(k) e^{j2\pi nk/N} \quad 0 \leq n \leq N-1, \end{aligned} \tag{1}$$

where $S_m(k)$ is the k-th subcarrier of the m-th OFDM symbol, and N is the number of subcarriers. After the addition of cyclic prefix and D/A conversion, the signal is passed through the mobile radio channel. Assuming a wide-sense stationary and uncorrelated scattering (WSSUS) channel, the channel $H(f,t)$ can be characterized for all time and all frequencies by the two-dimensional spaced-frequency, spaced-time correlation function

$$\phi(\Delta f, \Delta t) = E\{H^*(f,t)H(f+\Delta f, t+\Delta t)\}. \tag{2}$$

In this paper, we assume the channel to be constant over an OFDM symbol, but time-varying across OFDM symbols, which is a reasonable assumption for low and medium mobility.

At the receiver, the signal is received along with noise. After synchronization, down sampling, and removing the cyclic prefix, the simplified received baseband model of the samples can be formulated as

$$y_m(n) = \sum_{l=0}^{L-1} x_m(n-l)h_m(l) + z_m(n), \tag{3}$$

where L is the number of channel taps, $z_m(n)$ is the additive white Gaussian noise (AWGN) sample with zero mean and variance of σ_z^2, and the time domain CIR, $h_m(l)$, over an

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OFDM symbol is given as time-invariant linear filter. After taking DFT of the OFDM symbols, the received samples in frequency domain can be shown as

$$\begin{aligned} Y_m(k) &= DFT\{y_m(n)\} \\ Y_m(k) &= S_m(k)H_m(k) + Z_m(k), \end{aligned} \tag{4}$$

where $H_m(k)$ and $Z_m(k)$ are DFT of $h_m(l)$ and $z_m(n)$, respectively.

3 Frequency Selectivity and Delay Spread Estimation

In this section, first a practical method for frequency correlation estimation from channel frequency response is introduced. Then, an analytical expression for the correlation as a function of rms delay spread is derived. Finally, calculation of coherence bandwidth and the corresponding rms delay spread from frequency correlation for a given correlation value is explained.

3.1 Frequency correlation estimation

The channel estimates in frequency domain can be obtained using OFDM training symbols, or by transmitting regularly spaced pilot symbols in between the data symbols and by employing frequency domain interpolation. In this paper, we assume transmission of training OFDM symbols. Using the knowledge of the training symbols, channel frequency response can be estimated as

$$\begin{aligned} \hat{H}_m(k) &= \frac{Y_m(k)}{S_m(k)} \\ \hat{H}_m(k) &= H_m(k) + w_m(k), \end{aligned} \tag{5}$$

where $w_m(k)$ is the channel estimation error which is modeled as AWGN with zero mean and variance of σ_w^2. From the channel estimates, the instantaneous channel frequency correlation values can be calculated as

$$\hat{\phi}_H(\Delta) = E_k\{\hat{H}^*(k)\hat{H}(k+\Delta)\}, \tag{6}$$

where E_k is the mean with respect to k (averaging within an OFDM symbol). These instantaneous correlation estimates are noisy and needs to be averaged over several OFDM symbols. An *alpha tracker* is used for averaging the instantaneous values, the nth average can be found as

$$\tilde{\phi}_H^n(\Delta) = \alpha\tilde{\phi}_H^{n-1}(\Delta) + (1-\alpha)\hat{\phi}_H^{n-1}(\Delta). \tag{7}$$

After going through the mathematical details, the averaged correlation estimates can be derived as

$$\tilde{\phi}_H(\Delta) = \begin{cases} \phi_H(\Delta) & \text{if } \Delta \neq 0 \\ \phi_H(0) + \sigma_w^2 & \text{if } \Delta = 0. \end{cases} \tag{8}$$

3.2 Delay spread estimation

The PDP estimate $\tilde{\phi}_h(\tau)$ can be obtained by taking IDFT of the averaged frequency correlation estimates

$$\begin{aligned} \tilde{\phi}_h(\tau) &= IDFT\{\tilde{\phi}_H(\Delta)\} \\ \tilde{\phi}_h(\tau) &= \phi_h(\tau) + \sigma_w^2. \end{aligned} \tag{9}$$

The statistics like rms delay spread and maximum excess delay spread can be calculated from PDP. However, this requires IDFT operation which increases computational complexity. Instead, the desired parameters can be calculated directly from the averaged frequency correlation estimates. In the rest of this section, the direct relation between rms delay spread and frequency correlation is derived.

Equation 6 can be re-written as

$$\begin{aligned} &\hat{\phi}_{H_m}(\Delta) = E_k\{H_m^*(k)H_m(k+\Delta)\} \\ &= E_k\{R_m(k)R_m(k+\Delta)\} + E_k\{Q_m(k)Q_m(k+\Delta)\} \\ &\quad + jE_k\{R_m(k)Q_m(k+\Delta)\} \\ &\quad - jE_k\{Q_m(k)R_m(k+\Delta)\}, \end{aligned} \tag{10}$$

where $R_m(k)$ and $Q_m(k)$ are the *real* and *imaginary* parts of $H_m(k)$. Going through intensive mathematical details, the terms in (9) can be calculated as

$$\begin{aligned} &E_k\{(R_m(k)R_m(k+\Delta)\} = E_k\{Q_m(k)Q_m(k+\Delta)\} \\ &= \frac{1}{2}\sum_{l=0}^{N-1}\cos\frac{2\pi\Delta l}{N}\left(r_m^2(l) + q_m^2(l)\right) \end{aligned} \tag{11}$$

and

$$\begin{aligned} &E_k\{(R_m(k)Q_m(k+\Delta)\} = -E_k\{R_m(k+\Delta)Q_m(k)\} \\ &= \frac{1}{2}\sum_{l=0}^{N-1}\sin\frac{2\pi\Delta l}{N}\left(r_m^2(l) + q_m^2(l)\right), \end{aligned} \tag{12}$$

where $r_m(l)$ and $q_m(l)$ are the real and imaginary parts of the lth tap of CIR, $h_m(l)$.

In order to derive an analytical expression between frequency correlation and rms delay spread, we need to assume an appropriate and generic model for PDP. Exponential PDP is the most commonly accepted model for indoor channels. It has been shown theoretically and with experimental verification as the most accurate model [4]. In this Section, rms delay spread values are derived assuming an exponentially decaying PDP. Later, in Section 4 the robustness of this assumption against other PDPs is tested through computer simulation.

Exponential delay profile can be expressed as,

$$\phi_h(l) = Ce^{-\frac{\tau_0}{\tau_{rms}}l}, \tag{13}$$

where τ_0 is time duration between two consecutive discrete taps, and τ_{rms} is the rms delay spread value, and C is a constant term that normalizes the power.

Instantaneous correlation estimates can be obtained using (10) as

$$\begin{aligned}\hat{\phi}_{H_m}(\Delta) &= 2E_k\{(R_m(k)R_m(k+\Delta)\} \\ &\quad +j2E_k\{(R_m(k)Q_m(k+\Delta)\} \\ &= \sum_{l=0}^{N-1}\cos\frac{2\pi\Delta l}{N}\left(r_m^2(l)+q_m^2(l)\right) \\ &\quad +j\sum_{l=0}^{N-1}\sin\frac{2\pi\Delta l}{N}\left(r_m^2(l)+q_m^2(l)\right) \quad (14)\end{aligned}$$

The real and imaginary parts of the taps in CIR, $r_m(l)$ and $q_m(l)$, can be written as $r_m(l) = a_m(l)e^{-\frac{\tau_0}{2\tau_{rms}}l}$ and $q_m(l) = b_m(l)e^{-\frac{\tau_0}{2\tau_{rms}}l}$, where $a_m(l)$ and $b_m(l)$ are Gaussian distributed independent random variables with zero mean and identical variances.

Replacing $r_m(l)$ and $q_m(l)$ with the above defined values, (14) can be re-written as

$$\begin{aligned}\hat{\phi}_{H_m}(\Delta) = \textstyle\sum_{l=0}^{N-1}\cos\frac{2\pi\Delta l}{N}e^{-\frac{\tau_0}{\tau_{rms}}l}\left(a_m^2(l)+b_m^2(l)\right) \\ +j\textstyle\sum_{l=0}^{N-1}\sin\frac{2\pi\Delta l}{N}e^{-\frac{\tau_0}{\tau_{rms}}l}\left(a_m^2(l)+b_m^2(l)\right). \quad (15)\end{aligned}$$

As in (7) channel frequency correlation is obtained from the instantaneous correlation estimates through averaging, which can also be formalized as

$$\begin{aligned}\tilde{\phi}_{H_m}(\Delta) &= E_m\{\tilde{\phi}_{H_m}(\Delta)\} \\ &= A\sum_{l=0}^{N-1}\cos\frac{2\pi\Delta l}{N}e^{-\frac{\tau_0}{\tau_{rms}}l} \\ &\quad +jA\sum_{l=0}^{N-1}\sin\frac{2\pi\Delta l}{N}e^{-\frac{\tau_0}{\tau_{rms}}l}, \quad (16)\end{aligned}$$

where A is $E_m\{a_m^2+b_m^2\}$. Absolute value of frequency correlation is obtained from (16) as

$$|\tilde{\phi}_H(\Delta)| = \sqrt{A^2\sum_{l=0}^{N-1}\sum_{u=0}^{N-1}\cos\frac{2\pi\Delta(l-u)}{N}e^{-\frac{\tau_0}{\tau_{rms}}(l+u)}}. \quad (17)$$

We will assume $N\to\infty$ to simplify the results. This is a reasonable assumption since maximum excess delay of PDP is much less then OFDM symbol duration $L\leq N$. After expanding the cosine term into exponentials, Geometric series can be used to simplify the equation. The frequency correlation values should be normalized to 1 to get $|\tilde{\phi}_H(0)|=1$. After going through these steps the absolute value of the channel frequency correlation can be obtained as

$$|\tilde{\phi}_H(\Delta)| = \sqrt{\frac{1-2e^{-\tau_0/\tau_{rms}}+e^{-2\tau_0/\tau_{rms}}}{1-2e^{-\tau_0/\tau_{rms}}\cos\frac{2\pi\Delta}{N}+e^{-2\tau_0/\tau_{rms}}}}. \quad (18)$$

3.3 Calculation of rms delay spread and coherence bandwidth

Coherence bandwidth (B_c), which is a statistical measure of the range of frequencies over which the two frequency components have a strong correlation, can be calculated from the averaged frequency correlation estimate. Coherence bandwidth B_c of level K is defined as $|\phi_H(B_c)| < K$, [2]. Popularly used values for K are 0.9 and 0.5. In our simulations we have used $K=0.9$, since the estimated correlation for small Δ values are more reliable, as more data points are used to obtain these values. Fig. 2 shows how to calculate Δ for a given K.

For given K and the corresponding Δ value, rms delay spread can be derived from (18) as

$$\tau_{rms} = \frac{\tau_0}{\log\frac{2-2K^2\cos\frac{2\pi\Delta}{N}+\sqrt{(2K^2\cos\frac{2\pi\Delta}{N}-2)^2-4(1-K^2)^2}}{2(1-K^2)}}. \quad (19)$$

The above equation is very complex since it requires cosine, square root and logarithm operations. However, we can approximate this equation. A good function to approximate is $\tau_{rms} = \frac{c\tau_0}{\Delta}$ as Δ and τ_{rms} are known to be inversely proportional. The constant c can be calculated by minimizing mean squared error between exact values and the approximation. Fig. 1 shows the comparison of approximation and exact values obtained using (19). We can see that the approximation is nearly perfect and gives accurate results.

4 Performance Results

Simulation results are obtained in an OFDM based wireless communication system with 64 subcarriers. Fig. 3 shows the difference between the frequency correlation estimates and ideal correlation values for different rms delay spread values. An exponentially decaying PDP is assumed in this figure. As can be seen, the correlation estimates are very close to the ideal correlation values. As described in previous section, correlation estimate is used to get the coherence bandwidth for a given correlation value of K. This is shown in Fig. 2 for ideal correlation values. Observe that as rms delay spread increases, coherence bandwidth decreases. Three different coherence bandwidth values and rms delay spread estimates for $K=0.9$ are shown in the figure.

Fig. 4 shows the PDPs that are used in the simulations in order to test the robustness of the proposed method. Smulders' PDP is included as it has been considered by many authors as an alternative to exponential decaying PDP in indoor channels [10]. The rectangular and triangular PDPs are also used for measuring the robustness of the proposed algorithms. Although rectangular PDP is not a commonly used model for wireless channels, it provides a worst case scenario for measuring the robustness of the proposed PDP. Fig. 5 shows the performance of the proposed rms delay spread estimator. Normalized mean-squared-error performance is obtained for different rms delay spread values. As expected, exponentially decaying PDP works the best, since the algorithm is designed by assuming this type of PDP. As can be seen, the algorithm works well for other PDPs also. Notice that even for the rectangular PDP, the rms delay spread estimates are reasonably well.

5 Conclusion

In this paper, frequency selectivity and rms delay spread estimation for wireless communications systems are described. A practical algorithm for estimating averaged frequency correlation is given. Coherence bandwidth and rms delay spread, which are commonly used measures for frequency selectivity, are obtained from the correlation estimates. The proposed algorithm is tested for a variety of channel PDPs. It is observed that the algorithm is robust and works well in various environments.

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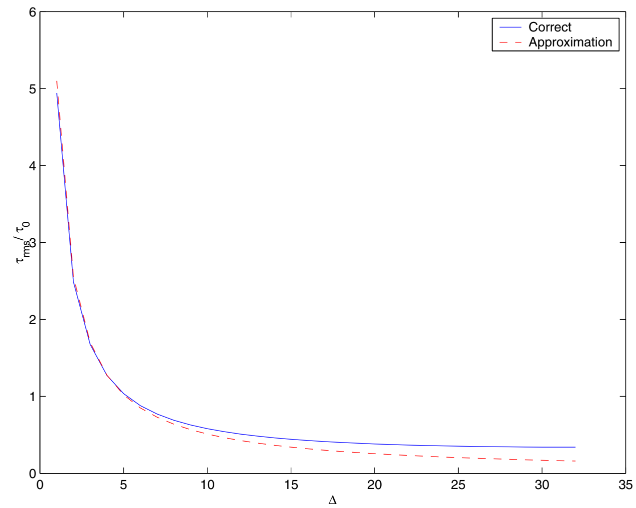

Figure 1. Comparison of the approximation and exact values obtained using (19).

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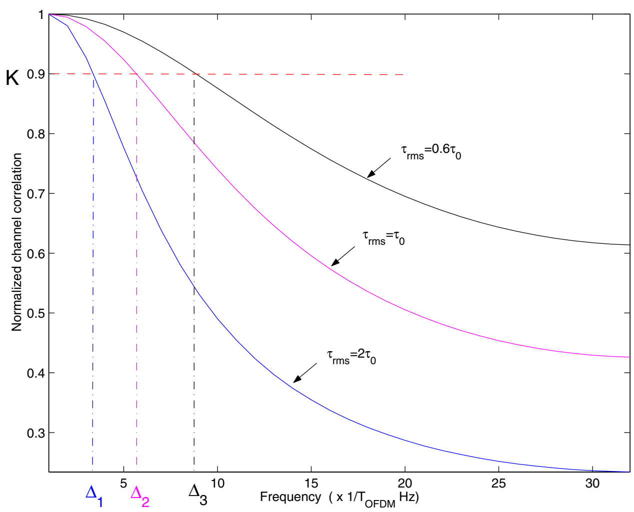

Figure 2. Estimation of coherence bandwidth B_c of level K from absolute correlation estimates, corresponding to different rms delay spread values.

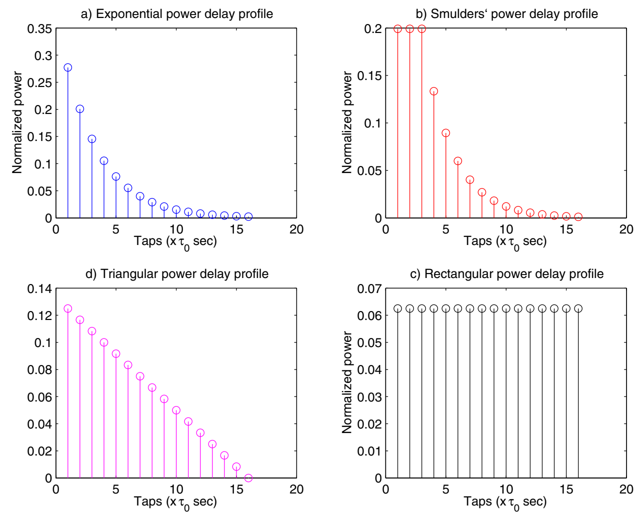

Figure 4. Different power delay profiles that are used in the simulation.

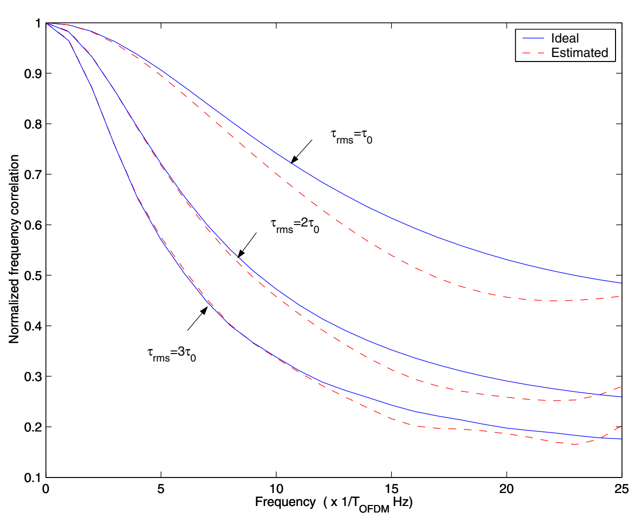

Figure 3. Comparison of the estimated frequency correlation with the ideal correlation for different rms delay spread values.

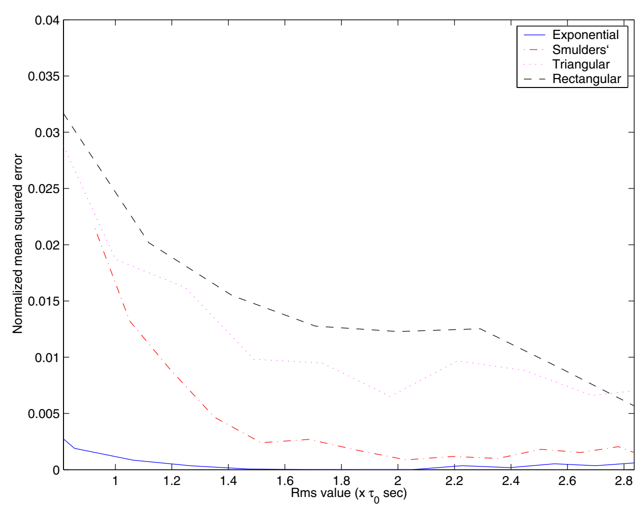

Figure 5. Normalized mean-squared-error performance of rms delay spread estimation in different PDPs.

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