JOINT CHANNEL AND FREQUENCY OFFSET ESTIMATION FOR OFDM SYSTEMS

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ABSTRACT

In this paper, a joint channel and frequency offset estimation algorithm is proposed. The time variation of the effective channel impulse response (CIR) due to frequency offset is tracked using a channel tracker. This variation is used to estimate the frequency offset as well as the actual time-invariant channel coefficients. The proposed method does not require transmission of repetitive pattern, thus allowing a covert communication for applications where security is important. It can be used to estimate not only the fractional part but also the integer part of the frequency offset. The proposed algorithm is tested using computer simulations. It is observed that it has performance close to the Cramér-Rao lower bound.

INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) provides effective solution to high data-rate transmission by its robustness against multi-path fading. However, OFDM systems are more sensitive to the frequency errors than single carrier systems and the sensitivity increases with the number of sub-carriers and with the constellation size [1]. Frequency offset results in the loss of orthogonality among sub-carriers and causes inter-carrier interference (ICI) which has Gaussian statistics. ICI degrades the performance of both the channel estimation [2] and symbol detection.

Traditionally, frequency offset is estimated using the periodic nature of the cyclic prefix [3], or training sequences are designed to have periodic parts [4,5]. There are two main categories for training data based timing and frequency estimation: autocorrelation or cross-correlation based algorithms. In auto-correlation based algorithms, repeated training data is transmitted and frequency offset is estimated by finding the phase difference between repeated parts. An example of this group of methods is Schmidl's algorithm which is given in [4]. In cross-correlation algorithms, the received signal is correlated with the known training data which is usually a pseudo noise (PN) sequence with good autocorrelation properties [6].

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COVERT TRANSMISSION

Although, transmission of repeated training information eases the frequency offset estimation problem, it creates security problems since the repetitive structure of the preamble may be used by undesired users for synchronizing to the transmitted signal (both time and frequency synchronization). For example, method given in [4] can be used for estimating the timing position and frequency offset without the knowledge of transmitted preamble if the period of the repetition is known, which is not difficult to estimate. Therefore, secure synchronization techniques for preamble based transmission need to be developed which use a preamble without periodic parts.

In order to estimate the frequency offset of time distorted signals without periodic preambles, the channel response should be known; or frequency offset and channel response should be estimated jointly. Joint frequency and channel estimation algorithms are suitable for applications which require covertness since repetitive patterns are not desired as discussed, and frequency errors degrade the performance of channel estimation. In [7] and [8], joint estimation is achieved by iteratively estimating the channel impulse response (CIR) and frequency. In [9], variation of the CIR and frequency offset is tracked using extended Kalman filtering. Channel frequency response and frequency offset is estimated jointly by searching for the frequency offset hypothesis that maximizes the auto-correlation of channel frequency response (CFR) in [10]. In [8], ML estimator and Cramér-Rao bound for joint channel and frequency offset estimation is derived.

In this paper, a new joint channel and frequency offset estimation algorithm is proposed. The proposed algorithm uses a channel tracker to find the effective CIR which is a combination of the CIR and frequency offset. The frequency offset and channel coefficients are, then, calculated from the effective CIR. With this method, covert transmission is possible since no periodicity is introduced in the transmitted signal. The performance of the proposed algorithm is tested using Monte-Carlo simulations and mean-squared-error (MSE) performance of the frequency offset and channel estimation results are compared with theoretical limits.

SYSTEM MODEL

A burst mode OFDM transmission is assumed. Each transmitted burst contains a known preamble followed by regular OFDM symbols which use N sub-carriers. The symbol period is T. After passing through the wireless channel, assuming the channel is time-invariant during the duration of preamble, the received sequence in the presence of frequency offset can be expressed as

$$y[n] = e^{j\phi} e^{j\frac{2\pi\epsilon n}{N}} \sum_{l=0}^{L-1} x[n-l] h_l + v[n] \quad (1)$$

$$= e^{j\phi} \sum_{l=0}^{L-1} x[n-l] \underbrace{h_l e^{j\frac{2\pi\epsilon n}{N}}}_{w_l[n]} + v[n] \quad (2)$$

where ϵ is the frequency offset normalized to subcarrier spacing $1/NT$, $x[n]$ is the training sequence of length M, $v[n]$ is zero-mean complex white Gaussian noise with variance of σ^2, and h_l is the lth tap of the CIR with length L. We define $w_l[n] = h_l e^{j\frac{2\pi\epsilon n}{N}}$ as the *effective* time-varying CIR where the time variation is only due to frequency offset. The constant phase ϕ can be folded into the channel taps and it will be ignored in the rest of this paper.

Eqn. (2) can be re-written in the matrix form as

$$y[n] = \mathbf{w}^T[n]\mathbf{x}[n] + v[n] \quad (3)$$

where $\mathbf{w}[n]$ and $\mathbf{x}[n]$ are the tap-weight and tap-input vectors respectively and they are defined as

$$\mathbf{w}[n] = [w_0[n]\ w_1[n] \cdots w_{L-1}[n]]^T \quad (4)$$

$$\mathbf{x}[n] = [x[n]\ x[n-1] \cdots x[n-L+1]]^T \ . \quad (5)$$

Our goal is to estimate h_l and ϵ by using the knowledge of $y[n]$ and $x[n]$.

ALGORITHM DESCRIPTION

Frequency offset causes time variation in the effective CIR, $w_l[n]$. This variation is tracked using a simple least-mean-square (LMS) tracker in order to obtain the combined effect of time-invariant channel h_l and frequency offset ϵ. When the frequency offset is large, the variation of channel taps will be significant and LMS algorithm may not track all the variation. Therefore, a multi-pass LMS is used to solve this problem. The frequency offset and channel states are initialized to zero first. After first pass using LMS algorithm, an initial estimate of the frequency offset and channel estimate is obtained. In the next pass, the frequency offset from the previous pass is used to remove the effect of frequency offset on the received signal, and channel estimate from the previous pass is used as initial state. This iterative process is stopped when the frequency offset estimate at the last pass is smaller than a threshold value.

CHANNEL TRACKING

Time-variation of wireless channels can be tracked using adaptive filters such as LMS, recursive least-squares (RLS) and Kalman filters. In this paper, a simple LMS algorithm is used because of its simplicity and stability, although any other methods would work.

The LMS algorithm has been widely used in adaptive filtering because of its stability and simplicity of implementation [11, 12]. It uses a rough approximation of the gradient instead of its true value as done in steepest descent algorithm. The recursive relation for updating the channel estimates is given as

$$\hat{\mathbf{w}}[n+1] = \hat{\mathbf{w}}[n] + \mu\mathbf{x}[n]e^*[n] \quad (6)$$

where $e[n] = y[n] - \hat{\mathbf{w}}^T[n]\mathbf{x}[n]$ is the estimation error and μ is the step size parameter. For stable operation, the step-size should satisfy the following inequality:

$$0 < \mu < \frac{2}{L} \ . \quad (7)$$

Fig. 1 shows the MSE of the channel estimation (only one pass is used) as a function of frequency offset and step-size. Although not shown in the figure, the tracker becomes unstable if the step size is bigger than the upper bound given in (7). On the other hand, very small step-sizes can not track the variation due to frequency offset and yields large errors.

By updating the channel estimates with (6), the variation of the channel due to the frequency offset is

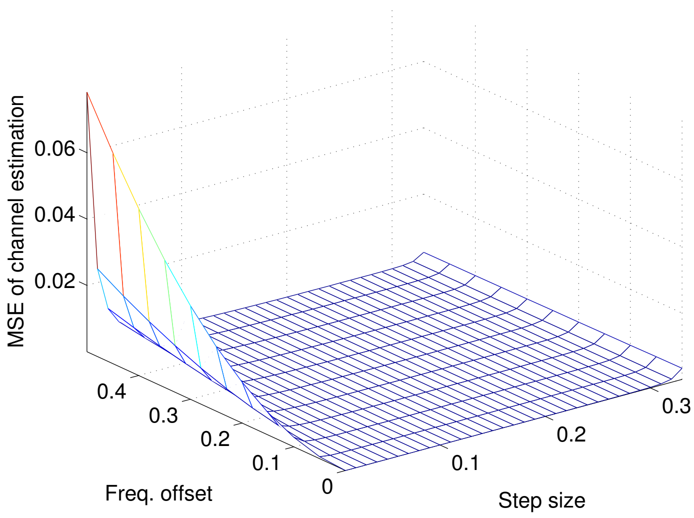

Figure 1: Mean square error of channel estimation as a function of frequency offset and step-size.

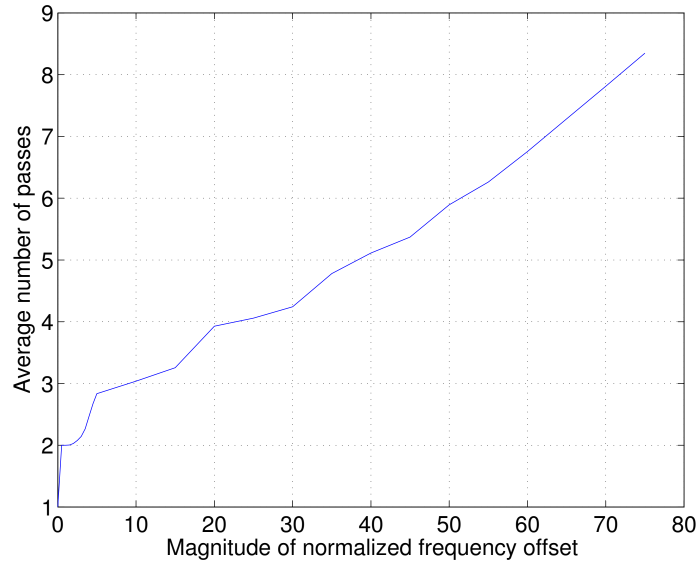

Figure 2: Average number of passes for different frequency offsets when SNR is 10dB.

tracked. In fact, the true relation between the effective channels is given by

$$\mathbf{w}[n+1] = \mathbf{w}[n]e^{j\frac{2\pi\epsilon}{N}} \ . \tag{8}$$

The larger the frequency offset, the larger the variation in effective channel response. Therefore, the number of passes depends on the frequency offset. Fig. 2 shows the average number of passes for different frequency offset values at a signal-to-noise ratio (SNR) of 10dB. The number of passes increases linearly with the frequency offset. To ensure stability, the maximum number of passes can be limited to a threshold.

In the next sections, the methods that are used to estimate the frequency offset and channel coefficients from the estimated effective CIR is given.

FREQUENCY OFFSET ESTIMATION

Once the estimates of the effective channel taps are obtained using the channel tracker, the frequency offset can be estimated and corrected in each pass. The estimates obtained by using the channel tracker can be approximated as[1]

$$\hat{w}_l[n] = h_l e^{j\frac{2\pi\epsilon_r n}{N}} + V_h[n] \ , \tag{9}$$

where $V_h[n]$ is the error term.

Estimation of the frequency (ϵ_r) from (9) is studied in the literature. Methods based on Periodogram [13] or based on the phase information of the autocorrelations [14, 15] are proposed. In this paper, we use a simple differential estimator. The estimates are multiplied with the conjugate of the successive samples and the phase information is used to estimate the frequency offset. The information from different taps are combined using maximum ratio combining. In this case, the frequency offset can be estimated as

$$\hat{\epsilon} = -\frac{N}{2\pi}\angle\left(\sum_{l=0}^{L-1}\sum_{n=1}^{M-1}\hat{w}_l[n]\hat{w}_l^*[n+1]\right) \ . \tag{10}$$

After the last pass, the frequency offset is estimated using a larger correlation lag. This estimation will have a smaller range ($|\epsilon| < 1$), but the accuracy is better. If we choose the correlation lag equal to $M/2$, this estimator can be written as

$$\hat{\epsilon} = -\frac{N}{M\pi}\angle\left(\sum_{l=0}^{L-1}\sum_{n=1}^{M/2}\hat{w}_l[n]\hat{w}_l^*[n+\frac{M}{2}]\right) \ . \tag{11}$$

CHANNEL ESTIMATION

Once the frequency offset is estimated, time-invariant CIR, h_l, can be estimated from the effective CIR by removing the effect of frequency offset and averaging. This operation can be formulated as

$$\hat{h}_l = \sum_{n=1}^{M}\hat{w}_l[n]e^{-j\frac{2\pi\hat{\epsilon}n}{N}} \tag{12}$$

[1]The multiplicative terms are ignored for simplicity.

Table 1: Power delay profile used in the simulations

Path	**1**	**2**	**3**	**4**	**5**	**6**
Strength (dB)	-3	0	-2	-6	-8	-10

where $\hat{\epsilon}$ is the frequency offset estimate obtained form w_l as explained in the previous section.

SIMULATION RESULTS

The proposed algorithm is tested using an OFDM system which operates in burst mode. The number of subcarriers is chosen to be 1024. A maximum-length sequence (m-sequence) of length 1023 is used as preamble. M-sequences are chosen since they have perfect autocorrelation properties and can be used for timing synchronization as well [6, 16]. A 6-tap symbol spaced channel model is used. The power delay profile of this channel is given in Table 1. Using (7), the maximum step-size for stability can be calculated for this channel model as $\mu < 0.33$. In our simulations a step size of 0.12 is used. The frequency offset threshold to stop the iteration is set to 0.25 and the maximum number of passes is limited to 10.

Figs. 3 and 4 show the MSE performance of frequency offset and channel estimations respectively at different passes. The error decreases as the number of passes increases as expected. These figures also show the modified Cramér-Rao bounds for joint estimation. These bounds are given in [8] as

$$JMCRB(\epsilon) = \frac{3M \cdot SNR}{4\pi^2(M-1)(M+1)} \quad (13)$$

$$JMCRB(h) = \frac{1}{M \cdot SNR}\left(L + \frac{3(M-1)^2}{2(M^2-1)}\right) \quad (14)$$

The performance of the proposed algorithm is very close to the Cramér-Rao bound as can be seen from these figures. The threshold effect can also be seen in Fig. 3. This effect is caused by the nonlinear nature of frequency offset estimation [13].

Fig. 5 shows the MSE of frequency offset estimation algorithm as a function of normalized frequency offset for 25dB SNR. As can be seen from this figure, the maximum frequency offset that can be estimated is around 80. Note that the range depends on the number of the maximum passes allowed which was 10 in our simulations.

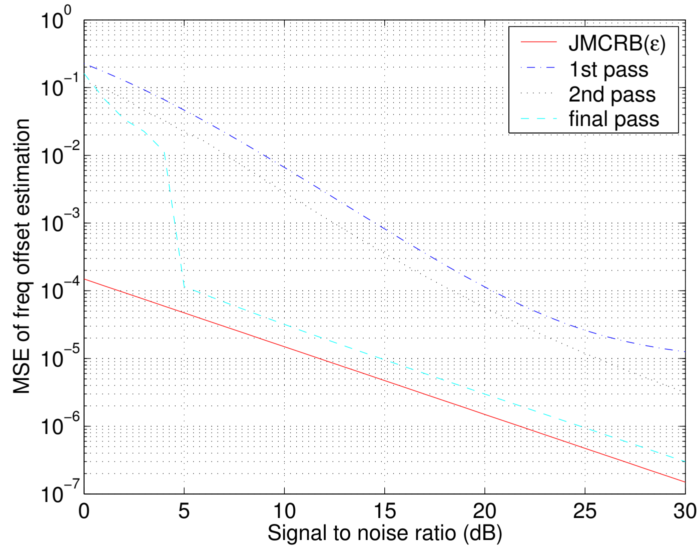

Figure 3: Mean square error performance of frequency offset estimation algorithm for a normalized frequency offset of 1. Joint modified Cramér-Rao bound is also shown.

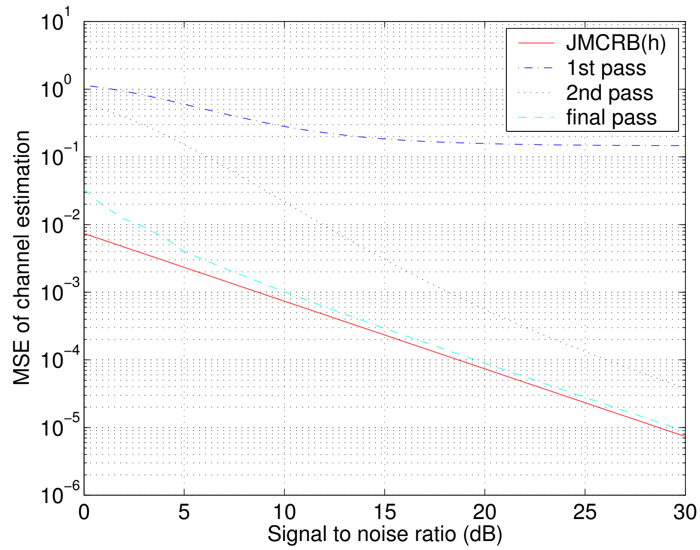

Figure 4: Mean square error performance of channel estimation algorithm for a normalized frequency offset of 1. Joint modified Cramér-Rao bound is also shown.

CONCLUSION AND FUTURE WORK

This paper propose a new joint frequency offset and channel estimation method. Unlike most of the frequency offset estimation algorithms, the proposed method does not require repetitive pattern in the preamble. This provides covertness which is a crucial property for military communication applications. The frequency offset estimation range is not limited to the

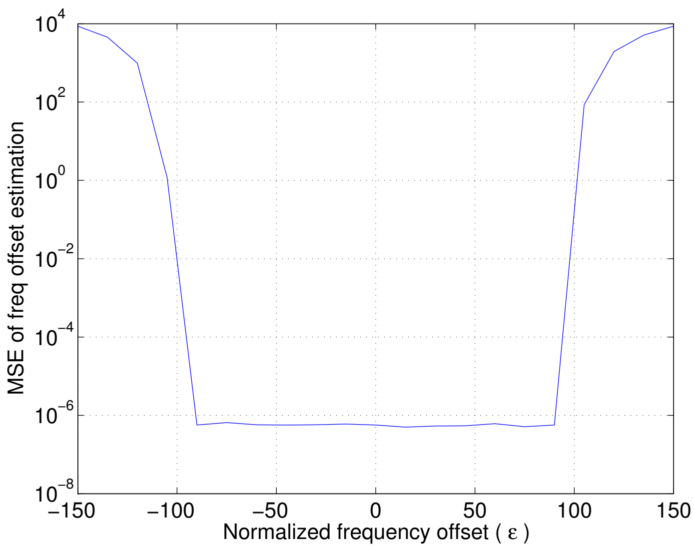

Figure 5: Mean square error of frequency offset estimation as a function of the normalized frequency offset at 25dB SNR.

sub-carrier spacing and the algorithm can correct frequency offsets up to half of the occupied bandwidth. The estimation algorithm is shown to have performance very close to the Cramér-Rao lower bound.

For channel tracking a simple LMS algorithm is used. The performance of the proposed algorithm with other adaptive trackers can be investigated and compared with the LMS algorithms performance. We have assumed that the number of channel taps is known. This assumption can be removed by using order recursive algorithms. This way the algorithm can adapt itself to different environment conditions. Our future research will be focused on these areas.

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