Carrier Frequency Offset Compensation with Successive Cancellation in Uplink OFDMA Systems

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***Abstract*—Similar to OFDM systems, OFDMA systems also suffer from frequency mismatches between the receiver and the transmitter. However, the fact that each uplink user has a different frequency offset makes the compensation more challenging than that of OFDM systems. This letter proposes successive interference cancellation (SIC) for compensating the frequency offset in the uplink OFDMA systems. A decorrelator is used to remove the inter-carrier interference (ICI) within a user's signal and successive cancellation is applied to mitigate the multi access interference (MAI) arising due to the frequency difference among uplink users. The proposed algorithm is shown to eliminate the interference and has a manageable complexity.**

***Index Terms*—OFDMA, frequency offset, successive cancellation.**

I. Introduction

RECENTLY, orthogonal frequency division multiple access (OFDMA) is chosen as a transmission technique for mobile wireless metropolitan area network (WMAN) [1]. In OFDMA, subcarriers are grouped into sets, each of which is assigned to a different user. Interleaved, random, or clustered assignment schemes can be used for this purpose. In the uplink of an OFDMA system, all of the users transmitting in the same symbol should be time and frequency aligned with other users in order to prevent inter-symbol interference (ISI), ICI, and MAI. The focus of this letter is frequency synchronization and perfect timing alignment is assumed. Frequency mismatches among the uplink users as well as between the uplink users and the base station (BS) cause power leakage among subcarriers. This leakage has two effects, namely ICI and MAI. ICI is caused by the leakage or interference between a user's own subcarriers whereas MAI is caused by the power leakage from other users' subcarriers.

Two main approaches, namely *feedback* and *compensation* methods, can be used to mitigate the frequency offset in the uplink of OFDMA systems. In the former, the estimated frequency offset values are fed back to the subscriber stations (SSs) on a control channel so that they can adjust their transmission parameters [2]. However, the obvious disadvantage of this approach is the bandwidth loss due to the need for control channel. In the compensation method, the receiver compensates for the frequency offsets of all users by employing signal processing techniques. Two compensation methods are given in [3]. The effect of frequency offset is represented as a matrix multiplication of transmitted frequency domain signal with the interference matrix[1]. In order to reconstruct the transmitted signal, least squares (LS) and minimum mean-square error (MMSE) algorithms are applied. The LS method requires only the frequency offset knowledge of users while the MMSE algorithm requires the knowledge of signal and noise powers as well. The LS method is also referred as group synchronization scheme in [4]. Both methods require inversion of the large interference matrix which is computationally demanding as the dimensions of this matrix is equal to the number of subcarriers. Cancellation based compensation methods are also used for mitigating the interference due frequency mismatches. The effect of frequency offset can be modeled as a circular convolution in the frequency domain. Using this property, frequency offset is canceled by circularly deconvolving the discrete Fourier transform (DFT) output with the DFT of frequency offset vector in [5]. For reducing complexity, only some elements of the DFT output are used. Moreover, only a specific user's subcarriers are considered. Circular convolution is used in [6] as well to generate the interference in frequency domain after DFT. The generated interference is then removed from the original signal. In [7], SIC is applied to compensate for frequency offset. The MAI due to frequency offset is reduced by reconstructing and removing the interfering signals in the frequency domain. In [8], iterative cancellation is proposed where the ICI is iteratively removed from other subcarriers for each subcarrier. As the subcarriers are not sorted, the required number of iterations might be large. The complexity of this algorithm is also relatively large.

In this letter, we present a SIC method for frequency offset compensation assuming that the frequency offsets of all uplink users are known at the receiver. The letter is organized as follows. In Section II, system model is established for uplink OFDMA systems. Section III presents the proposed frequency offset compensation algorithm followed by the numerical results given in Section IV. Finally, the conclusions are drawn in Section V.

Notation: Bold upper letters denote matrices and bold lower letters denote column vectors; $(\cdot)^T$ denotes transpose; and $\mathbf{I}_K$ is the identity matrix of size K.

II. System Model

We consider an OFDMA system with D simultaneously active users and N subcarriers. The inverse discrete Fourier transform (IDFT) output of ith user can be written as

$$x_m^{(i)}(n) = \sum_{k\in\Gamma_i} X_m^{(i)}(k)e^{-j\frac{2\pi kn}{N}} \qquad -N_G \leq n \leq N-1, \quad (1)$$

[1]The elements of this matrix can be calculated using the knowledge of frequency offsets of each user.

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where m is the symbol index, N_G is the length of cyclic prefix (CP), and $X_m^{(i)}(k)$, $k \in \Gamma_i$, is the value of transmitted symbol on the kth subcarrier. The set of subcarriers assigned to user i is denoted as Γ_i. These sets satisfy $\bigcup_{i=0}^{D} \Gamma_i = \{0, 1, \cdots, N\}$[2] and $\Gamma_i \bigcap \Gamma_j = \emptyset$ if $i \neq j$. The number of users is denoted by D and the number of subcarriers assigned to a user is not necessarily the same and might be changed depending on the bandwidth requirements of the users. Assuming perfect time synchronization, discrete-time model of the received signal at the BS after removal of the CP can be written as

$$y(n) = \sum_{i=1}^{D} y^{(i)}(n) + w(n) \tag{2}$$

$$= \sum_{i=1}^{D} e^{j\frac{2\pi\epsilon_i n}{N}} \sum_{l=0}^{L-1} h^{(i)}(l) \cdot \left[\sum_{m=-\infty}^{\infty} x_m^{(i)}(n - l - m(N + N_G)) \right] + w(n), \tag{3}$$

where $w(n)$ is the complex additive white Gaussian noise (AWGN) sample with variance σ_w^2 and $\epsilon_i = \Delta f_i / f_{sub}$ is the normalized frequency offset *i.e.* the frequency offset Δf_i normalized with the adjacent subcarrier spacing f_{sub}. The channel taps of ith user is denoted by $h^{(i)}(l)$, and L is the total number of channel taps. In the rest of the letter, the symbol index m will be dropped for notational simplicity assuming that the length of the CP, N_G, is larger then the maximum excess delay of the channel.

The receiver applies DFT operation to the received signal $y(n)$ to obtain the frequency domain symbols. The DFT output can be obtained as

$$Y(k) = DFT\{y(n)\} \tag{4}$$

$$= \sum_{i=1}^{D} \sum_{u \in \Gamma_i} X^{(i)}(u) H^{(i)}(u) D(u, k, \epsilon_i) + W(k), \tag{5}$$

where $H^{(i)}$ and W are the DFTs of $h^{(i)}$ and w respectively. $D(u, k, \epsilon_i)$ is the amount of leakage across subcarriers due to frequency offset, and it can be formulated as [9]

$$D(u, k, \epsilon_i) = e^{j\pi(u-k+\epsilon_i)\frac{N-1}{N}} \frac{\sin \pi(u - k + \epsilon_i)}{N \sin \frac{\pi(u-k+\epsilon_i)}{N}}. \tag{6}$$

Assuming $k \in \Gamma_i$, the received signal in kth subcarrier can be written as

$$Y(k) = \underbrace{X^{(i)}(k) H^{(i)}(k)}_{Desired\ signal} D(k, k, \epsilon_i) + \underbrace{\sum_{\substack{u \in \Gamma_i \\ u \neq k}} X^{(i)}(u) H^{(i)}(u) D(u, k, \epsilon_i)}_{ICI} + \underbrace{\sum_{\substack{j=1 \\ j \neq i}}^{D} \underbrace{\sum_{u \in \Gamma_j} H^{(j)}(u) X^{(j)}(u) D(u, k, \epsilon_j)}_{MAI^{(j)}}}_{MAI} + \underbrace{W(k)}_{Noise}, \tag{7}$$

[2]The whole subcarriers are assumed to be used in this letter without loosing generality. DC subcarrier and guard subcarriers might be set to null values in practical implementations.

where the first term is the desired signal with amplitude reduction and phase distortion; second and third terms represent ICI and total MAI as indicated. The MAI from jth user is represented as $MAI^{(j)}$. From (7), it can be observed that MAI to the desired user depends on the frequency offset of other users and their corresponding channel values, *i.e.* power levels.

The system model considered so far was independent of subcarrier allocation. In this letter, we consider clustered subcarrier assignment scheme where available subcarriers are grouped into clusters and each cluster is assigned to a different user [10]. Clustered OFDMA is used in the partially used sub-channeling (PUSC) mode of IEEE 802.16e [1] where every four consecutive subcarriers are grouped into clusters called *tile*. We assume that the size of each cluster is fixed and denoted by K, hence the total number of clusters is N/K.

III. Proposed Compensation Algorithm

In this letter, we assume that the carrier frequency offsets of uplink users are known, or estimated, by the receiver. The effect of imperfect synchronization is investigated in Section IV using computer simulations. Estimation of carrier frequency offset can be achieved by using the properties of the received signal, by transmitting known pilots, or blindly (see [11] and references therein).

The fact that different subcarriers are assigned to different users in OFDMA systems makes the signal separability possible since the subcarriers coming from different users will have independent attenuations. As different users are assigned to neighboring subcarriers, where most of the interference comes from, and their power levels are separable, SIC can be used to remove the interference due to frequency offset. On the other hand, in clustered OFDMA systems, the subcarriers within a cluster will observe similar fading and hence their power levels will be similar. Therefore, successive cancellation will not be efficient for these subcarriers as signal separability is not possible. In order to overcome this problem, we apply decorrelator receiver over subcarriers within each clusters. As the size of each cluster is small (compared to the whole subcarrier range), the decorrelator receiver is possible with manageable complexity. The combination of the decorrelator and successive cancellation is proposed as an efficient method for mitigating the frequency offset in uplink OFDMA systems. Decorrelation is used to remove the ICI effect and successive cancellation among clusters is used to solve the MAI problem. In the proposed method, first the clusters are sorted in descending order of their average powers. Then, starting with the cluster with the largest power, decorrelation is applied and decisions are made. After obtaining the bits transmitted for the current cluster, the MAI to neighboring clusters is reconstructed using the knowledge of the channel and frequency offset values, and subtracted to cancel its MAI. Fig. 1 shows the block diagram of the proposed method. The details of the decorrelator receiver and SIC will be explained in the following sections.

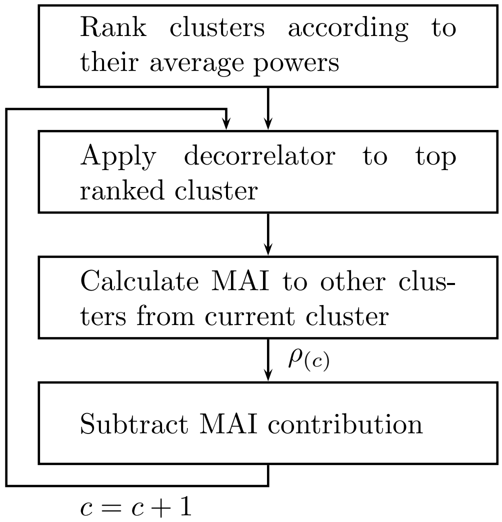

Fig. 1. Block diagram of the proposed compensation algorithm.

A. Decorrelation

The ICI is mostly caused by the power leakage from subcarriers within the same cluster[3]. In the receiver, the DFT output for the cth cluster, which belongs to ith user, can be written in matrix form as

$$\mathbf{y}_c = \mathbf{\Pi}_i \mathbf{h}_c + \mathbf{w}_c, \tag{8}$$

where $\mathbf{y}_c = [Y^{(i)}(k) \dots Y^{(i)}(k+K)]^T$ is the received signal vector, $\mathbf{h}_c = [X^{(i)}(k)H^{(i)}(k) \dots X^{(i)}(k+K)H^{(i)}(k+K)]^T$ is the vector that depends on the transmitted symbols and frequency domain channel, and $\mathbf{w}_c$ is the MAI plus noise vector. The current cluster (cluster c) spans the subcarriers from k to $k+K$, *i.e.* $\Gamma_i^{(c)} = k, \dots, k+K$. The $K \times K$ matrix $\mathbf{\Pi}$ is the ICI matrix whose entries are given as $\mathbf{\Pi}_i(u,k) = D(u,k,\epsilon_i)$, where D is as given in (6).

The leakage among the subcarriers due to frequency offset can be viewed as a matrix diagonalization problem. By multiplying $\mathbf{y}_c$ with $\mathbf{\Pi_i^{-1}}$, the matrix $\mathbf{\Pi_i}$ is diagonalized and hence the power leakage among the subcarriers within the current cluster is removed. Moreover, the phase rotation to the desired subcarriers due to frequency offset $D(k,k,\epsilon_i)$ is also corrected. In order to be able to apply this method, however, the ICI matrix $\mathbf{\Pi_i}$ should be invertible which is proven by the following theorem.

Theorem 1: The $K \times K$ ICI matrix $\mathbf{\Pi}$, whose elements are $\mathbf{\Pi}(u,k) = D(u,k,\epsilon)$, is an invertible matrix for $-1 < \epsilon < 1$.

Proof: See Appendix. ■

The decorrelator output can be calculated using (8) as

$$\hat{\mathbf{h}}_c = \mathbf{\Pi}_i^{-1} \mathbf{y}_c \tag{9}$$
$$= \mathbf{h}_c + \mathbf{\Pi}_i^{-1} \mathbf{w}_c. \tag{10}$$

Using $\hat{\mathbf{h}}_c$ and the channel knowledge, the receiver can detect the transmitted information. This solution is also known as LS method. If the noise power and the autocorrelation of channel is known, a more advanced method such as MMSE might also be used to obtain a better detection performance. A similar approach is applied in [3], [4] to remove ICI and MAI for *all subcarriers*. However, calculating and inverting the $N \times N$ interference matrix is practically impossible for large DFT sizes. The ICI matrix is a Toeplitz matrix and it can be inverted efficiently with $O(K^2)$ computations as compared to $O(K^3)$ for arbitrary matrices [12]. Moreover, the same matrix can be used for all clusters of a user as it is independent of subcarrier positions.

[3]Assuming a user's clusters are separated from each other, the ICI contribution from the user's other clusters will be small. We treat these clusters the same way as MAI and remove their interference by successive cancellation.

B. Successive Cancellation

In this section, the decoded signal in Section III-A is used to reconstruct and remove the MAI of current cluster on other clusters successively. The interference of cth cluster (which belongs to user i) on the uth subcarrier ($u \notin \Gamma_i^{(c)}$) can be reconstructed as

$$\rho_{(c)}(u) = \hat{\mathbf{h}}_c^T \mathbf{d}_{c,u}, \tag{11}$$

where $\mathbf{d}_{c,u} = [D(k,u,\epsilon_i), D(k+1,u,\epsilon_i) \dots D(k+K,u,\epsilon_i)]^T$; $k,\dots,k+K \in \Gamma_i^{(c)}$ gives the power leakage amount. Hence, the uth subcarrier's value after removal of MAI from cth cluster becomes

$$\begin{aligned}\hat{Y}(u) &= Y(u) - \rho_{(c)}(u) \\ &= Y(u) - \hat{\mathbf{h}}_c^T \mathbf{d}_{c,u}.\end{aligned} \tag{12}$$

In the next step, the cluster with the second largest power is decoded by applying decorrelation algorithm given in Section III-A, and its interference to remaining clusters is removed. The MAI free signal $\hat{Y}(u)$ is then used in the next decorrelation process. This successive process is continued until all the clusters are processed (see Fig. 1).

The SIC is prone to error propagation. When errors are made during detection following the demodulation step, these errors will propagate to other clusters as MAI is reconstructed incorrectly in (11). It is especially dominant in low SNR and large frequency offset cases. In order to decrease the amount of error propagation, the detected bits can be decoded and coded again. This will reduce the number erroneous decisions increasing the performance of the proposed method in the expense of larger complexity and delay.

It is well known that the amount of power leakage from a subcarrier to neighboring subcarriers decreases sharply as the separation between the two subcarriers increases. This behaviour is a natural consequence of (6) and it can be used to decrease the complexity of the proposed algorithm as suggested in [5]. The interference from current cluster to only a limited number of neighboring clusters, denoted by N_c, can be calculated and removed in the SIC step. When the frequency offset is large, however, the leakage can not be neglected as it may still be large.

C. Algorithm Complexity

Once the inverse of ICI matrix is calculated, NK multiplications are required in deconvolution. For the successive cancellation with full cancellation, $N^2 - N(N+K)/2$ multiplications are required. Therefore, the total number of multiplications is $N(N+K)/2$ which gives a complexity of $O(N^2)$.

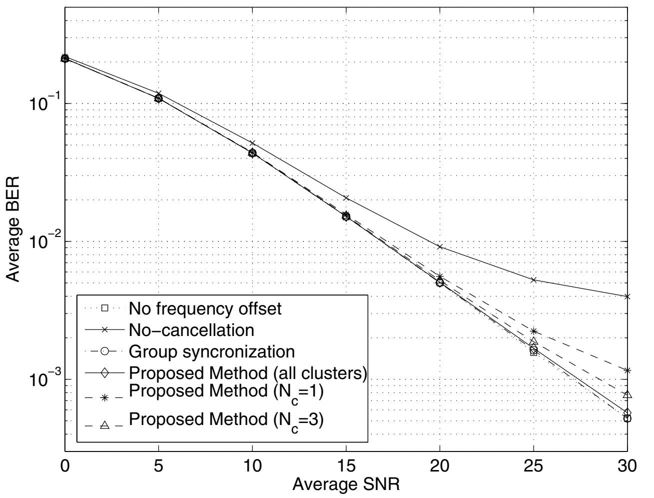

Fig. 2. Uncoded bit error rate (BER) as a function of average SNR. All users are assumed to have the same average power. The normalized frequency offset has a uniform distribution in the range (-0.1, 0.1).

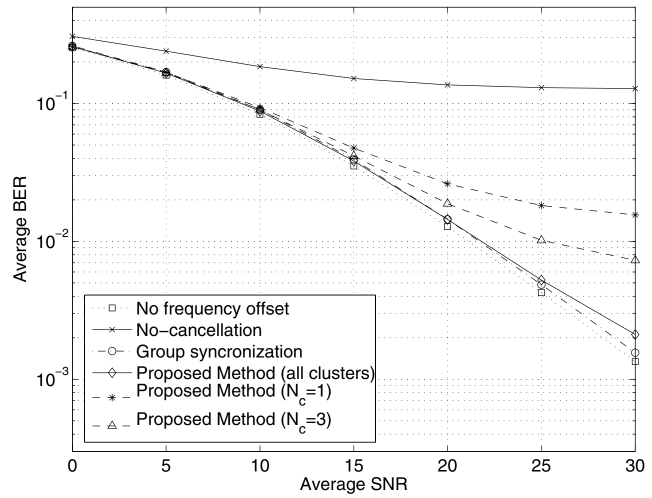

Fig. 4. Uncoded bit error rate (BER) as a function of average SNR. Average powers of users has a difference of 2dB. The normalized frequency offset has a uniform distribution in the range (-0.3, 0.3).

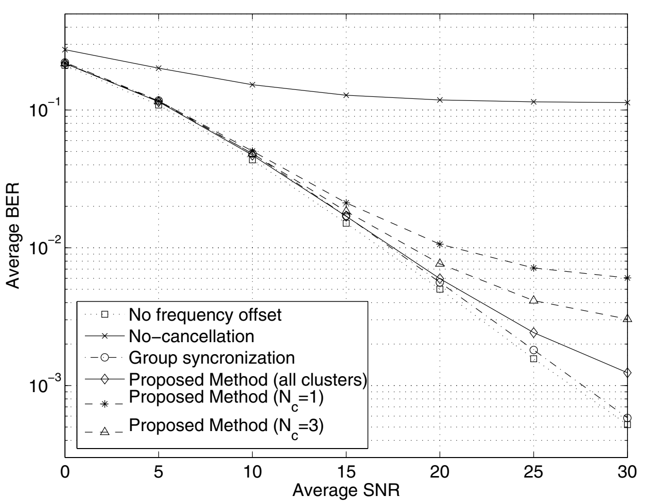

Fig. 3. Uncoded bit error rate (BER) as a function of average SNR. All users are assumed to have the same average power. The normalized frequency offset has a uniform distribution in the range (-0.3, 0.3).

On the other hand, the complexity of group synchronization method proposed in [3] is $O(N^3)$ because of the inversion of $N \times N$ interference matrix. When only N_c neighboring clusters are considered in the proposed method, the number of multiplications in successive cancellation reduces to N_cN, hence, the overall complexity reduces to $O(N)$ from $O(N^2)$. The proposed algorithm also reduces the memory requirements as compared to group synchronization algorithm since it is not necessary to store the large interference matrix.

IV. Numerical Results

The performance of the proposed algorithm is tested with computer simulations. An uplink OFDMA system with 256 subcarriers and 8 users are considered. The size of each cluster is set to 4 ($K = 4$) and the same number of clusters are assigned for each user; hence 8 clusters for each user. The transmission bandwidth is set to 10 MHz. For simulating the wireless channel, the Channel A of ITU-R channel model for vehicular environments with high antenna [13] is used and perfect channel knowledge is assumed for simulations. The users employ QPSK mapping for their data symbols, and subcarrier allocation is assumed to be similar to UL-PUSC of IEEE 802.16 [1]. Independent frequency offsets are assumed for all uplink users. Each frequency offset is a random variable with uniform distribution in (-ϵ_{max}, ϵ_{max}) where ϵ_{max} is the maximum allowed value of users' frequency offsets. Instead of fixing the frequency offsets of each user, they are chosen randomly to simulate a more realistic scenario.

Figs. 2, 3 and 4 show the uncoded bit-error-rates (BERs) versus average SNR of all users. The results with no-frequency offset, no-compensation, group synchronization [4] and the proposed compensation method are presented. Proposed method with cancellation of MAI from all clusters and from only neighboring 1 and 3 clusters are also given in order to show the effect of applying MAI cancellation to only neighboring clusters. Fig. 2 shows the results for $\epsilon_{max} = 0.1$ and Figs. 3 and 4 show the results for $\epsilon_{max} = 0.3$. In Figs. 2 and 3, users are assumed to have independent fading channels with the same average power. By investigating these figures, it can be seen that the proposed algorithm is effective for mitigating the frequency offset for both cases. However, the curves drift from the no-frequency offset case in Fig. 3 because of more error propagation due to larger frequency offset. In Fig. 4, all users are assumed to have different average powers with 2dB steps, *e.g.* 0dB, -2dB, -4dB *etc.* In such a scenario, the proposed algorithm is more effective in removing the error floor (comparing Figs. 3 and 4) as signal separability is more efficient and error propagation is minimal.

When successive cancellation is applied to only neighboring clusters, where the main ICI contribution comes from, the performance of the proposed algorithm decreases. This shows that the leakage to other clusters can not be ignored especially when the frequency offset is large. However, when the frequency offset is small, only adjacent subcarriers might be considered in order to decrease the computational complexity ($O(N^2)$ vs. $O(N)$). Hence, there is a trade-off between the computational complexity and amount of interference that can

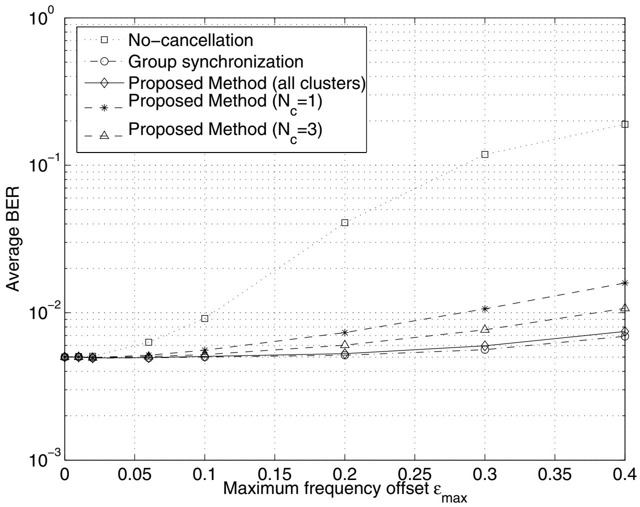

Fig. 5. Uncoded bit error rate (BER) as a function of maximum normalized frequency offset for average SNR of 20dB.

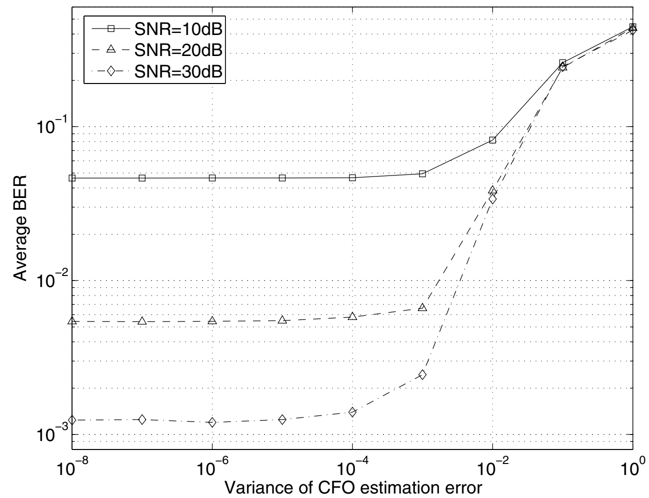

Fig. 6. Uncoded bit error rate (BER) as a function of the variance of frequency offset estimation error. The normalized frequency offset has a uniform distribution in the range (-0.3, 0.3).

be removed. For large frequency offsets, the performance of proposed algorithms drifts from the no-frequency offset case at high SNR values (see Fig. 3). This is caused by the errors in decisions (and hence errors in ICI removal) because of the large frequency offset and high ICI.

Fig 5 presents the uncoded BER as a function of maximum normalized frequency offset ϵ_{max} at 20dB SNR value. It is clear that the proposed algorithm lowers the error floor due to frequency offset mismatches. However, as frequency offset becomes larger, the effectiveness of the algorithm decreases slowly because of the increased amount of error propagation.

Finally, we investigate the effect of frequency offset estimation errors. The estimation errors are assumed to be additive and have a zero-mean Gaussian distribution. Fig. 6 gives the BER results for different SNR values as a function of the estimation error variance. The frequency offsets of users are chosen randomly in the range (-0.3, 0.3), *i.e.* $\epsilon_{max} = 0.3$. It can be seen that the performance starts to degrade as the estimation error variance becomes larger than 10^{-4}. This limit can be satisfied with frequency offset estimation algorithms proposed in literature (See *e.g.* [11]), which shows that the proposed algorithm is robust to frequency offset estimation errors.

V. Conclusions

In this letter, we have proposed a successive cancellation algorithm to mitigate the effects of frequency offset in uplink OFDMA systems where user separation is achieved using the average received power of user's clusters. The proposed method, a combination of decorrelator and successive cancellation, can compensate the different frequency offsets of users with manageable complexity. Hence, it can be used as an alternative to the feedback method. We have shows that the proposed compensation method is robust to frequency offset estimation errors. Moreover, we have proposed to apply SIC to only neighboring subcarriers in order to decrease the computational complexity. The number of clusters over which the SIC is performed can be chosen adaptively by using the knowledge of frequency offset and signal-to-noise ratio (SNR) values for better performance and complexity trade-off.

Appendix A
Proof of Theorem 1

Consider the case $\epsilon = 0$. In this case $\mathbf{\Pi} = \mathbf{I}$ and it is trivial that $\mathbf{\Pi}$ is invertible. Now consider $\epsilon \neq 0$ and $|\epsilon| < 1$. The determinant of $\mathbf{\Pi}$ can be written as

$$\det(\mathbf{\Pi}) = \left[\frac{1}{N} e^{j\frac{\pi\epsilon(N-1)}{N}} \sin \pi\epsilon\right]^K \det(\mathbf{A}) \tag{13}$$

where $\mathbf{A}$ is a $K \times K$ matrix with entries

$$\mathbf{A}(u,k) = \frac{1}{\sin\frac{\pi(u-k+\epsilon)}{N}} \tag{14}$$

$$= \frac{2je^{j\pi(u-k)/N}e^{j\pi\epsilon/N}}{e^{j2\pi(u-k)/N}e^{j2\pi\epsilon/N} - 1} \tag{15}$$

The term in bracket in (13) is not zero for the considered ϵ values. Hence, if the matrix $\mathbf{A}$ is invertible, $\mathbf{\Pi}$ will also be invertible.

Observe that $e^{j2\pi/N}$ is a primitive Nth root of unity and since $K < N$, the powers $e^{j2\pi/N}$ through $e^{j2\pi K/N}$ are distinct. Moreover, denominator of (15) never vanishes.

Suppose $\mathbf{A}\mathbf{v} = 0$ where $\mathbf{v} = [v_1 \ v_2 \ \ldots \ v_K]^T$. Then

$$0 = \sum_{u=1}^{K} v_u A(u,k) = \sum_{u=1}^{K} v_u \frac{2je^{j\pi(u-k)/N}e^{j\pi\epsilon/N}}{e^{j2\pi(u-k)/N}e^{j2\pi\epsilon/N} - 1} \tag{16}$$

$$= 2je^{-j\pi k/N}e^{j\pi\epsilon/N} \cdot \sum_{u=1}^{K} v_u \frac{e^{-j\pi u/N}}{e^{-j2\pi k/N}e^{j2\pi\epsilon/N} - e^{-j2\pi u/N}} \quad 1 \leq k \leq K. \tag{17}$$

Now define $g(X)$ as

$$g(X) = \sum_{u=1}^{K} v_u \frac{e^{-j\pi u/N}}{X - e^{-j2\pi u/N}}. \tag{18}$$

Then $g(X)$ has roots $e^{-j2\pi k/N}e^{j2\pi\epsilon/N}$ for $1 \leq k \leq K$ (see (17)). These K roots are distinct since the $e^{j2\pi k/N}$ terms are distinct. The numerator of $g(X)$ has degree at most $K-1$, so $g(X)$ should be identically zero (as a rational function).

Now lets multiply $g(X)$ with $X - e^{-j2\pi i/N}$ to get

$$g(X)(X - e^{-j2\pi i/N}) = v_i + \sum_{u=1, u\neq i}^{K} v_u \frac{e^{-j\pi u/N}(X - e^{-j2\pi i/N})}{X - e^{-j2\pi u/N}} = 0. \tag{19}$$

Taking the limit of (19) as $X \to e^{-j2\pi i/N}$, we conclude $v_i = 0$ *i.e.* $\mathbf{v} = 0$ as the coefficients $e^{j2\pi u/N}$ are all distinct. Therefore, we show that the system $\mathbf{Av} = 0$ has only the trivial solution $\mathbf{v} = 0$, which is an equivalent condition for the invertibility of the matrix $\mathbf{A}$, and hence for the matrix $\mathbf{\Pi}$.

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