Noise Plus Interference Power Estimation in Adaptive OFDM Systems

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***Abstract*— Noise variance and signal-to-noise ratio (SNR) are important parameters for adaptive orthogonal frequency division multiplexing (OFDM) systems since they serve as a standard measure of signal quality. Conventional algorithms assume that the noise statistics remain constant over the OFDM frequency band, and thereby average the instantaneous noise samples to get a single estimate. In reality, noise is often made up of white Gaussian noise along with correlated colored noise that affects the OFDM spectrum unevenly. This paper proposes an adaptive windowing technique to estimate the noise power that takes into account the variation of the noise statistics across the OFDM sub-carrier index as well as across OFDM symbols. The proposed method provides many local estimates, allowing tracking of the variation of the noise statistics in frequency and time. A mean-squared-error (MSE) expression in order to choose the optimal window dimensions for averaging in time and frequency is derived. Evaluation of the performance with computer simulations show that the proposed method tracks the local statistics of the noise more efficiently than conventional methods.**

I. Introduction

Orthogonal Frequency Division Multiplexing (OFDM) is a multi-carrier modulation scheme in which the wide transmission spectrum is divided into narrower bands and data is transmitted in parallel on these narrow bands. Therefore, symbol period is increased by the number of sub-carriers, decreasing the effect of inter-symbol interference (ISI). The remaining ISI effect is eliminated by cyclically extending the signal. OFDM provides effective solution to high data-rate transmission by its robustness against multi-path fading [1]. Parallel with the possible data rates, the transmission bandwidth of OFDM systems is also large. UWB-OFDM [2] and IEEE 802.16 based wireless metropolitan area network (WMAN) [3] systems are examples of OFDM systems with large bandwidths. Because of these large bandwidths, noise can not be assumed to be white with flat spectrum across subcarriers.

The signal-to-noise ratio (SNR) is broadly defined as the ratio of the desired signal power to the noise power and has been accepted as a standard measure of signal quality. Adaptive system design requires the estimate of SNR in order to modify the transmission parameters to make efficient use of system resources. Poor channel conditions, reflected by low SNR values, require that the transmitter modify transmission parameters such as coding rate, modulation mode *etc.* to compensate channel and to satisfy certain application dependent constraints such as constant bit error rate (BER) and throughput. Dynamic system parameter adaptation requires a real-time noise power estimator for continuous channel quality monitoring and corresponding compensation in order to maximize resource utilization. SNR knowledge also provides information about the channel quality which can be used by handoff algorithms, power control, channel estimation through interpolation, and optimal soft information generation for high performance decoding algorithms.

The SNR can be estimated using regularly transmitted training sequences, pilot data or data symbols (blind estimation). In this paper, we restrict ourself to data aided estimation. A comparison of time-domain SNR estimation techniques can be found in [4]. There are several other SNR measurement techniques which are given in [5] and references listed therein. In literature of OFDM SNR estimation, related work is few. In conventional SNR estimation techniques, the noise is usually assumed to be white and an SNR value is calculated for all subcarriers [4], [6]. In [7], this assumption is removed by calculating SNR values for each subcarrier. However, the correlation of the noise variance across subcarriers is not used since noise variance is calculated for each subcarrier separately.

White noise is rarely the case in practical wireless communication systems where the noise is dominated by interferences which are often colored in nature. This is more pronounced in OFDM systems where the bandwidth is large and the noise power is not the same over all the sub-carriers. Color of the noise is defined as the variation of its power spectral density in frequency domain. This variation of spectral content affects certain sub-carriers more than the others. Therefore, an averaged noise estimate is not the optimal technique to use. In this paper, the assumption of the noise to be white is removed and variation of the noise power across OFDM sub-carriers as well as across OFDM symbols is allowed. The noise variances at each subcarrier is estimated using a two dimensional sliding window whose size is calculated using local statistics of the noise. These estimates are specifically useful for adaptive modulation, and optimal soft value calculation for improving channel decoder performance. Moreover, it can be used to detect and avoid narrowband interference. We investigate a computationally efficient fixed window size algorithm and an adaptive algorithm where the window dimensions are calculated for each subcarrier. The adaptive algorithm is especially suitable for non-stationary interference scenarios. The paper focuses more on estimation of noise power, and assumes that the signal power, and hence SNR, can be estimated from the channel estimates.

This paper is organized as follows. In next section, our system model is described. Section III explains the proposed fixed and adaptive windowing algorithms. Then, numerical results are presented in Section IV and paper is concluded in Section V.

II. System Model

OFDM converts serial data stream into parallel blocks of size N and modulates these blocks using inverse fast Fourier transform (IFFT). Time domain samples of an OFDM symbol can be obtained from frequency domain symbols as

$$\begin{aligned} x_n(m) &= IFFT\{S_{n,k}\} \\ &= \sum_{k=0}^{N-1} S_{n,k} e^{j2\pi mk/N} \qquad 0 \le m \le N-1 \end{aligned} \quad (1)$$

where $S_{n,k}$ is the transmitted data symbol at the kth subcarrier of the nth OFDM symbol and N is the number of subcarriers. After the addition of cyclic prefix and D/A conversion, the signal is passed through the mobile radio channel.

At the receiver, the signal is received along with noise and interference. After synchronization and removal of the cyclic prefix[1], fast Fourier transform (FFT) is applied to the received signal to go back to the frequency-domain. The received signal at the kth subcarrier of nth OFDM symbol can then be written as

$$Y_{n,k} = S_{n,k} H_{n,k} + \underbrace{I_{n,k} + W_{n,k}}_{Z_{n,k}} \quad (2)$$

where $H_{n,k}$ is the value of the channel frequency response (CFR), $I_{n,k}$ is the colored noise (interference), and $W_{n,k}$ is the white Gaussian noise samples. We assume that the impairments due to imperfect synchronization, transceiver non-linearities *etc.* are folded into $W_{n,k}$ and the CFR is not changing within the observation time.

The white Gaussian noise is modeled as $W_{n,k} = \mathcal{N}(0, \sigma_0^2)$ and the interference term as $I_{n,k} = \mathcal{N}(0, \sigma_{n,k}^2)$, where $\sigma_{n,k}$ is the local standard deviation. Note that although the time-domain samples of the interference signal is correlated (colored), the frequency-domain samples ($I_{n,k}$) are not correlated, but their variances are correlated [8]. Assuming that the interference and white noise terms are uncorrelated, the overall noise term $Z_{n,k}$ can be modeled as $Z_{n,k} = \mathcal{N}(0, {\sigma'_{n,k}}^2)$, where ${\sigma'_{n,k}}^2 = \sigma_{n,k}^2 + \sigma_0^2$ is the effective noise variance. The goal of this paper is to estimate ${\sigma'_{n,k}}^2$ which can be used to find SNR. Note that if $\sigma_0 \gg \sigma_{n,k}$, the overall noise can be assumed to be white and it is colored otherwise.

[1]The length of the cyclic prefix is assumed to be larger than the maximum excess delay of the channel.

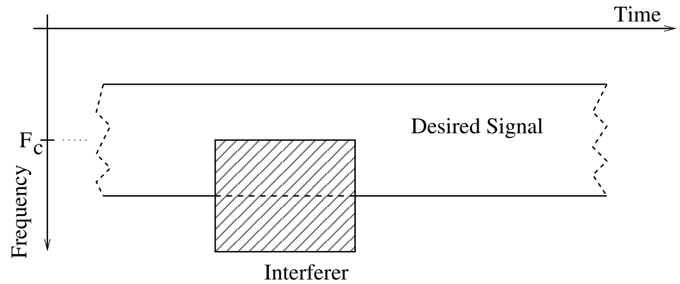

Fig. 1. Illustration of non-stationary interference.

III. Details of the Proposed Algorithm

The commonly used approach for noise power estimation in OFDM systems is based on finding the difference between the noisy received sample in frequency domain and the best hypothesis of the noiseless received sample [6]. It can be formulated as

$$Z_{n,k} = Y_{n,k} - \hat{S}_{n,k} \hat{H}_{n,k} \quad (3)$$

where $\hat{S}_{n,k}$ is the noiseless sample of the received symbol and $\hat{H}_{n,k}$ is the channel estimate for the kth sub-carrier of nth OFDM symbol.

In this paper, three different scenarios for the noise process $Z_{n,k}$ are considered: white noise, stationary colored noise and nonstationary colored noise. The first one is the commonly assumed case, where the frequency spectrum of the noise is uniform. In the second scenario, we assume to have a strong interferer which has larger bandwidth than the desired OFDM signal. A strong co-channel interferer is a good example for this case. In the third one, an interferer whose statistics are not stationary with respect to time and/or frequency is assumed to be present. Adjacent channel interference or a co-channel interference with smaller bandwidth than the desired signal are examples of this type of interference. A scenario where the interference is not stationary both in time and in frequency is illustrated in Fig. 1. Here, the statistics of noise components change as we move along the time or the frequency axis.

We propose to use a two dimensional sliding window for obtaining the noise plus interference power. Windowing will remove the common assumption of having the noise to be white and it will take colored interference (both in time and in frequency) into account. In this case, the estimate of the noise power at kth subcarrier of nth OFDM symbol can be written as

$$\hat{\sigma}_{n,k}^2 = \frac{1}{L_t L_f} \sum_{l=n-L_t/2}^{n+L_t/2+1} \sum_{u=k-L_f/2}^{k+L_f/2+1} |Z_{l,u}|^2 \quad (4)$$

where L_t and L_f are the averaging window lengths in time and frequency respectively.

Sliding window approach given in (4) requires appropriate L_t and L_f values for accurate estimation of noise plus interference power. If the window size is not chosen properly, it degrades the performance of estimation. Estimation error at the kth subcarrier of nth OFDM symbol can be written as

$$E(n,k) = \hat{\sigma}^2_{n,k} - {\sigma'_{n,k}}^2 = \frac{1}{L_t L_f} \sum_{l=n-L_t/2}^{n+L_t/2-1} \sum_{u=k-L_f/2}^{k+L_f/2-1} |Z_{l,u}|^2 - {\sigma'_{n,k}}^2 . \quad (5)$$

Note that the instantaneous errors, (5), will be a function of the window size, how correlated the interference is within the averaging window, average interference power and average noise power. Hence, the optimum values for window sizes will be different for each subcarrier and OFDM symbol, *i.e.* $L_{t,opt} = L_t(n,k)$ and $L_{f,opt} = L_f(n,k)$. In the next section, a suboptimal algorithm that uses the same window sizes for each subcarrier is developed and it is later used to develop the optimum algorithm which calculate the window sizes for each local point.

A. Fixed Window Size

When the interference is stationary (with respect to time or frequency), we propose to use a window with *fixed* dimensions for estimating the total noise power. Although the fixed window size algorithm is sub-optimum, it is computationally simple than the optimum method that will be discussed in the next section.

The window dimensions can be calculated by minimizing the mean-squared-error (MSE), *i.e* by minimizing the expected value of the square of (5). In this case, the MSE can be formulated as

$$\mathbf{MSE} = \mathcal{E}_{n,k}\{E(n,k)\} = \mathcal{E}_{n,k}\left\{\left[\frac{1}{L_t L_f} \sum_{l=n-L_t/2}^{n+L_t/2-1} \sum_{u=k-L_f/2}^{k+L_f/2-1} |Z_{l,u}|^2 - {\sigma'_{n,k}}^2\right]^2\right\} \quad (6)$$

where $\mathcal{E}_{n,k}$ is expectation over subcarriers and OFDM symbols. By further simplification, (6) can be written in terms of the auto-correlation of the variance of the noise component $R_{\sigma'^2}(\tau,\Delta)$ and the window dimensions (L_t and L_f) as shown in (7) at the bottom of the page.

Minimizing (7) achieves a trade-off between large window sizes (for white noise dominated cases) and small window sizes (for colored noise dominated cases). The window size that minimize the MSE should be chosen for averaging, *i.e.*

$$L_{t,fixed} = \arg\min_{L_t} \mathbf{MSE} , \quad L_{f,fixed} = \arg\min_{L_f} \mathbf{MSE} . \quad (8)$$

Note that the window size depends on the statistics of interference and white noise. These statistics can be obtained by averaging since the processes are assumed to be stationary.

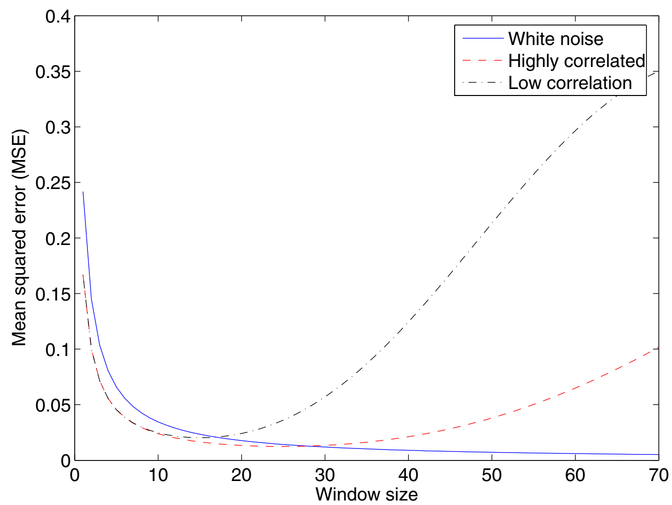

Fig. 2. Mean squared error as a function of window dimension in frequency.

Fig. 2 shows the MSE for different interference scenarios as a function of averaging size. In this figure, only windowing in frequency domain is considered for simplicity although the same concept is true for windowing in time. The best averaging window size becomes infinity for the white noise case and it has different values depending on the auto-correlation of the power of the total noise. As can be seen from this figure, the averaging size that gives the minimum error decreases as the correlation decreases.

By using a fixed sliding window, the common assumption of having the noise to be white is removed and the colored interference (both in time and in frequency) is taken into account. However, the noise statistics are assumed to be constant, *i.e.* $R_{\sigma'^2}(\tau,\Delta)$ is not changing over the estimation period.

B. Adaptive Window Size

In the previous section, a *fixed* window size is used over the whole subcarrier index as well as across OFDM symbols by assuming the noise statistics are constant in frequency and in time. This assumption is not valid when we the interference is not stationary with respect to time (*e.g.* 802.11 interference) or with respect to frequency (*e.g.* narrowband interference [9]) or both. When the dominant interference statistics change over the time or frequency, the algorithm proposed in the previous section will degrade. In this section, we propose to use different window dimensions for each subcarrier. This is achieved by assuming that the interference within the neighborhood of a subcarrier is stationary, *i.e.* the interference is quasi-stationary with respect to time and frequency.

$$\mathbf{MSE} = (1+\frac{1}{L_t L_f})R_{\sigma'^2}(0,0) - \frac{2}{L_t L_f}\sum_{l=-L_t/2}^{L_t/2-1}\sum_{u=-L_f/2}^{L_f/2-1} R_{\sigma'^2}(l,u) + \frac{1}{L_t^2 L_f^2}\sum_{l=-L_t}^{L_t-1}\sum_{u=-L_f}^{L_f-1}(L_t-|l|)(L_f-|u|)R_{\sigma'^2}(l,u) . \quad (7)$$

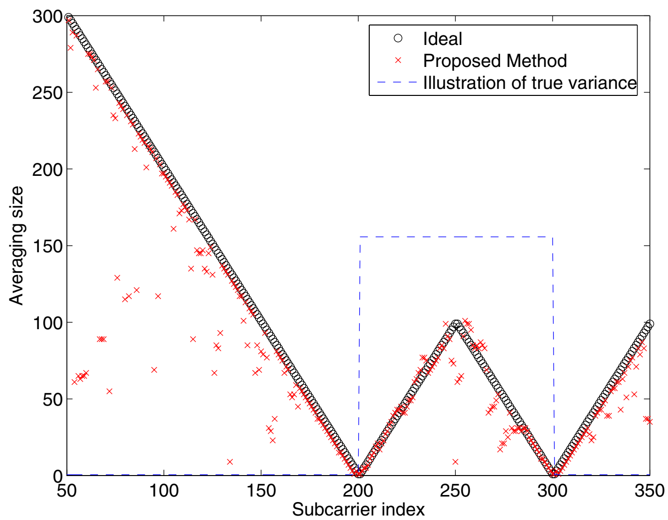

Fig. 3. The averaging size obtained by semi-anlytic method and the proposed adaptive window size algorithms.

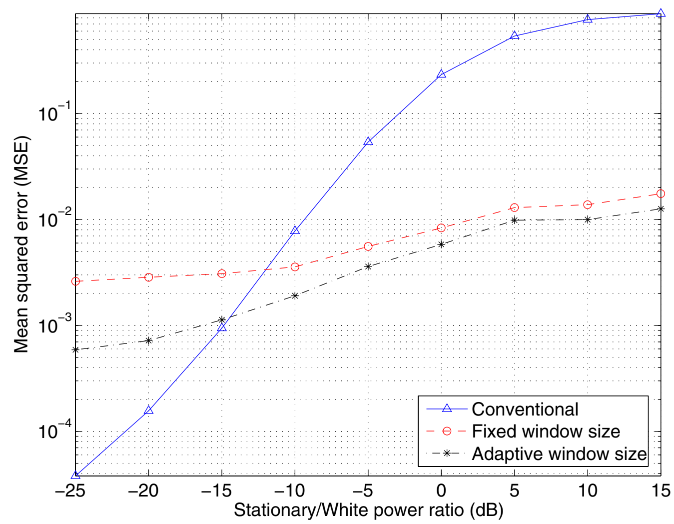

Fig. 4. Mean squared error for different algorithms as a function of the stationary colored noise to white noise power ratios.

In order to be able to find the optimum window dimensions for each local point, we replace the correlation term in (7) with local correlation estimate $\hat{R}_{\sigma'^2}$. $\hat{R}_{\sigma'^2}$ is estimated using only the noise terms $Z_{n,k}$ within the window for which the MSE is calculated.

The optimum window size for each local point is found by minimizing (7). The correlation values are estimated using the noise samples within the hypothesized window. The optimum window size in a subcarrier may be very large if the noise has flat spectral content. In order to decrease the computational complexity, window dimensions are found by assuming the interference is stationary. Then, window sizes less then or equal to this value is tested. If the obtained result is equal to the this maximum value, the maximum window size is increased and the algorithm is repeated.

IV. Numerical Results

An OFDM system with 1024 subcarriers and 20MHz bandwidth is considered. The stationary interference is assumed to be caused by a co-channel user transmitting in the same band with the desired user, and a co-channel signal with 10MHz bandwidth is used to simulate the non-stationary interference. We use averaging over 20 OFDM symbols and consider estimation of L_f only, but the results can be generalized to two dimensional case as well.

Fig. 3 shows the window length in frequency domain for a hypothetical non-stationary interference. Results obtained using the proposed adaptive algorithm and using excessive search are shown. As can be seen, the proposed algorithm finds the correct window dimensions with little error. These error is caused by the absence of enough statistics for obtaining the local correlations. At the edges of the interference the optimum window size goes to zero and it becomes larger where the noise variance is constant.

Figs. 4 and 5 show the MSEs for the conventional, fixed size window and adaptive window size algorithms. Fig. 4 gives the MSEs as a function of the stationary interference to white noise power ratio and Fig. 5 gives the MSE as a function of the non-stationary interference to white noise power ratio. The total noise plus interference power is kept constant for both figures. Note that when the ratio is very small (e.g. -25dB), the total noise can be considered as white noise only, and conventional algorithm performs best because its inherent white noise assumption is true. The estimation error increases as the total noise becomes more colored for all three methods. As noise becomes more colored, the averaging window dimensions become smaller for both fixed and adaptive algorithms increasing the estimation error. For conventional algorithm, the increase in the MSE is expected as variation of the noise power is more.

When the interference is stationary, the performance of the fixed window size algorithm is close to the performance of the adaptive window size algorithm while the performance difference becomes more obvious in the case of non-stationary interference. This is because the stationarity assumption in the derivation of fixed window size algorithm is valid for the stationary interference case (Fig. 4) whereas it is not true for the non-stationary interference (Fig. 5).

V. Conclusion

In this paper, a new noise variance estimation algorithm for OFDM systems, which removes the common assumption of white Gaussian noise and considers colored noise, is proposed. Noise variance, and hence SNR, is calculated by using a two dimensional sliding window in time and frequency. Windows with fixed and adaptive dimensions are considered. The sliding window dimensions in each subcarrier position is calculated adaptively using the local statistics of noise at that subcarrier hence considering the non-stationary interference scenarios. Although, the adaptive window size based algorithm gives the optimum performance, it is computationally complex. Therefore, the fixed window size algorithm may be used in

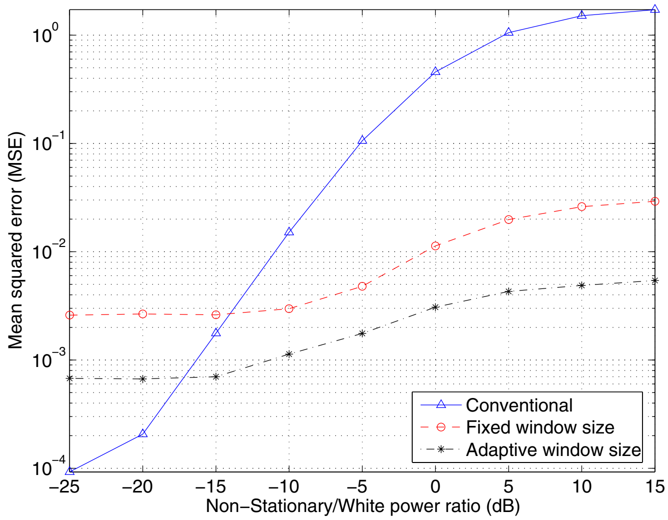

Fig. 5. Mean squared error for different algorithms as a function of the non-stationary colored noise to white noise power ratios.

applications where computational complexity is the limiting factor. Simulation results show that the proposed algorithm out-performs conventional algorithm under colored noise.

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