Carrier Frequency Offset Compensation with Successive Cancellation in Uplink OFDMA Systems

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***Abstract*— Similar to OFDM systems, OFDMA systems also suffer from frequency mismatches between the receiver and the transmitter. However, the fact that each uplink user has a different frequency offset makes the compensation more challenging than that of OFDM systems. This paper proposes successive interference cancellation (SIC) for compensating the frequency offset in the uplink OFDMA systems. A decorrelator is used to remove the inter-carrier interference (ICI) within a user's signal and successive cancellation is applied to mitigate the multi access interference (MAI) arising due to the frequency difference among uplink users. The proposed algorithm is shown to eliminate the interference and has a manageable complexity.**

I. Introduction

Orthogonal Frequency Division Multiplexing (OFDM) is a multi-carrier modulation scheme in which the wide transmission bandwidth is divided into narrower bands and data is transmitted in parallel on these narrow bands. Therefore, symbol period is increased by the number of bands, decreasing the effect of inter-symbol interference (ISI). The remaining ISI effect is eliminated by cyclically extending the signal. Recently, orthogonal frequency division multiple access (OFDMA) is chosen as a transmission technique for mobile wireless metropolitan area networks (WMANs) [1]. In OFDMA, subcarriers are grouped into sets each of which is assigned to a different user. Interleaved, random, or clustered assignment schemes can be used for this purpose. In the uplink of an OFDMA system, all of the users transmitting in the same symbol should be time and frequency aligned with other users in order to prevent ISI, inter-carrier interference (ICI), and multi access interference (MAI). The focus of this paper is frequency synchronization and perfect timing alignment is assumed.

Two main approaches can be used to mitigate the frequency offset in the uplink of OFDMA systems. These approaches are known as *feedback* and *compensation* methods. In the former method, the estimated frequency offset values are fed back to the subscriber stations (SSs) on a control channel so that the SSs can adjust their transmission parameters [2]. Feedback approach is used to compensate for the frequency offset in the uplink channel in [1]. However, the obvious disadvantage of this approach is the bandwidth loss due to the need for control channel. In the compensation method, the receiver compensates for the frequency offsets of all users using signal processing techniques. The simplest method, sometimes termed as single user detection or direct compensation, is compensating each user's frequency offset before fast Fourier transform (FFT), and applying FFT for each user separately. This method is both complex, as it requires different FFT operations for each user, and is not effective as it does not remove MAI and only ICI is prevented.

Two compensation methods are given in [3]. The effect of frequency offsets is represented as a matrix multiplication of transmitted frequency domain signal with the leakage matrix. In order to reconstruct the transmitted signal, least squares (LS) and minimum mean-square error (MMSE) algorithms are applied. The LS method requires only the frequency offset knowledge of users while the MMSE algorithm requires the knowledge of signal and noise powers as well. Both methods require inversion of the large leakage matrix which is computationally demanding as the dimensions of this matrix is equal to the number of subcarriers. In [4], MMSE filtering is applied to extract the desired user's signal from the FFT output. The transmitted data is jointly detected in [5] along with the frequency offset and channel using an iterative algorithm. However, this method is also computationally demanding.

Cancellation based compensation methods are also used for mitigating the interference due frequency mismatches. In [6], the frequency offset is canceled by circularly deconvolving the FFT output with the FFT of frequency offset vector. For reducing complexity, only some elements of the FFT output are used. Moreover, only a specific user's subcarriers are considered. Circular convolution is used in [7] as well to generate the interference in frequency domain after FFT. The generated interference is then removed from the original signal. However, the iterative nature of the algorithm makes the complexity large. The successive interference cancellation (SIC) is applied in [8] to compensate for frequency offset. The MAI due to frequency offset is reduced by reconstructing and removing the interfering signals in the frequency domain. In [9], iterative cancellation is proposed where the ICI is iteratively removed from other subcarriers for each subcarrier. As the subcarriers are not sorted, the number of iterations before stopping might be large. The complexity of this algorithm is also relatively large.

In this paper, we present a SIC method for frequency offset compensation assuming that the frequency offsets of all uplink users are known at the receiver. The paper is organized as follows. In Section II, system model is established for uplink OFDMA systems. Section III presents the proposed frequency offset compensation algorithms followed by the numerical results given in Section IV. Finally, the conclusions are drawn in Section V.

Notation: Bold upper letters denote matrices and bold lower letters denote column vectors; $(\cdot)^T$ denotes transpose; $\mathbf{I}_K$ is the identity matrix of size K; and $\mathscr{E}\{\cdot\}$ denotes expectation.

II. System Model

In this section, the uplink system model of an OFDMA system is introduced. We consider an OFDMA system with D simultaneously active users and N subcarriers. The inverse fast Fourier transform (IFFT) output of ith user can be written as

$$x_m^{(i)}(n) = \sum_{k\in\Gamma_i} X_m^{(i)}(k) e^{-j\frac{2\pi k n}{N}} \quad -N_G \leq n \leq N-1, \quad (1)$$

where m is the symbol index, N_G is the length of cyclic prefix (CP), and $X_m^{(i)}(k)$, $k \in \Gamma_i$, is the value of transmitted symbol on the kth subcarrier. The set of subcarriers assigned to user i is denoted as Γ_i. These sets satisfy $\bigcup_{i=0}^{D} \Gamma_i = \{0, 1, \cdots, N\}$[1] and $\Gamma_i \bigcap \Gamma_j = \emptyset$ if $i \neq j$. The number of users is denoted by D and the number of subcarriers assigned to a user is not necessarily the same and might be changed depending on the bandwidth requirements of the users. The transmitted signal of ith user can now be written as

$$x^{(i)}(n) = \sum_{m=-\infty}^{\infty} x_m^{(i)}(n - m(N+N_G)). \quad (2)$$

The received signal transmitted from ith user arrives to the receiver after passing through a multipath channel. The received baseband signal with frequency mismatch and channel effects can be modeled as

$$y^{(i)}(n) = e^{j\frac{2\pi\epsilon_i n}{N}} \sum_{l=0}^{L-1} x^{(i)}(n-l) h^{(i)}(l). \quad (3)$$

Therein $\epsilon_i = \Delta f_i / f_{sub}$, normalized frequency offset, denotes the frequency offset Δf_i normalized with the adjacent subcarrier spacing f_{sub} *i.e.* the frequency separation between two subcarriers. The channel taps of ith user is denoted by $h^{(i)}(l)$, and L is the total number of channel taps. Moreover, the channel parameters are assumed to be constant within an OFDM symbol in this paper.

In this paper we assume perfect time synchronization, and concentrate on the frequency errors. The symbol index m will be dropped for notational simplicity assuming that the length of the CP, N_G, is larger then the maximum excess delay of the channel. The discrete-time model of total received signal for a symbol at the base station (BS) after removal of the CP can now be written as

$$y(n) = \sum_{i=1}^{D} y^{(i)}(n) + w(n) \quad (4)$$
$$= \sum_{i=1}^{D} e^{j\frac{2\pi\epsilon_i n}{N}} \sum_{l=0}^{L-1} x^{(i)}(n-l) h^{(i)}(l) + w(n), \quad (5)$$

where $w(n)$ is the complex additive white Gaussian noise (AWGN) sample with variance σ_w^2.

The receiver applies FFT operation to the received signal $y(n)$ to obtain the frequency domain symbols. The FFT output can be obtained as

$$Y(k) = FFT\,(y(n)) \quad (6)$$
$$= \sum_{i=1}^{D} \sum_{u\in\Gamma_i} X^{(i)}(u) H^{(i)}(u) D(u,k,\epsilon_i) + W(k), \quad (7)$$

where $W(k)$ is the FFT of $w(n)$ and $D(u,k,\epsilon_i)$ is the amount of leakage across subcarriers due to frequency offset, and it can be formulated as [10]

$$D(u,k,\epsilon_i) = e^{j\pi(u-k+\epsilon_i)\frac{N-1}{N}} \frac{\sin \pi(u-k+\epsilon_i)}{N \sin \frac{\pi(u-k+\epsilon_i)}{N}}. \quad (8)$$

The frequency domain channel $H^{(i)}(u)$ can be obtained using the time domain channel taps as

$$H^{(i)}(u) = \sum_{l=0}^{L-1} h^{(i)}(l) e^{j\frac{2\pi l u}{N}}. \quad (9)$$

Assuming $k \in \Gamma_i$, the received signal in kth subcarrier can be written as

$$Y(k) = \underbrace{X^{(i)}(k)H^{(i)}(k)}_{Desired\ signal} D(k,k,\epsilon_i) + \underbrace{\sum_{\substack{u\in\Gamma_i \\ u\neq k}} X^{(i)}(u)H^{(i)}(u)D(u,k,\epsilon_i)}_{ICI} + \underbrace{\sum_{\substack{j=1 \\ j\neq i}}^{D} \underbrace{\sum_{u\in\Gamma_j} H^{(j)}(u)X^{(j)}(u)D(u,k,\epsilon_j)}_{MAI^{(j)}}}_{MAI} + \underbrace{W(k)}_{Noise}, \quad (10)$$

where the first term is the desired signal with amplitude reduction and phase distortion; second and third terms represent ICI and total MAI as indicated. The MAI from jth user is also shown which is formulated as

$$MAI^{(j)} = \sum_{u\in\Gamma_j} H^{(j)}(u)X^{(j)}(u)D(u,k,\epsilon_j). \quad (11)$$

Hence it can be observed that MAI to the desired user depends on the frequency offset of other users and their corresponding channel values, *i.e.* if a user has a high power level its effect on other users will be large as well.

[1]The whole subcarriers are assumed to be used in this paper without loosing generality. DC subcarrier and guard subcarriers might be set to null values in practical implementations.

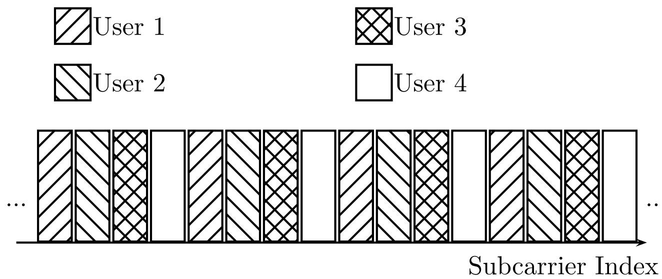

Fig. 1. Illustration of subcarrier allocation in clustered OFDMA.

The system model considered so far was independent of subcarrier allocation. In this paper, we consider the subcarrier allocation shown in Fig. 1. Available subcarriers are grouped into clusters and each cluster is assigned to a different user as shown in Fig. 1. This scheme is termed as *clustered OFDMA* or hybrid subcarrier assignment in the literature [11], and it is used in the partially used sub-channeling (PUSC) mode of IEEE 802.16e [1]. In PUSC allocation, every four consecutive subcarriers are grouped into clusters called *tile*. The combination of 6 tiles (not adjacent to each other) makes a subchannel, one or more of which is assigned to a specific user. We assume that the size of each cluster is fixed and an integer number of clusters are alloted for each user. The size of each cluster is denoted by K, hence the total number of clusters is N/K.

III. Proposed Compensation Algorithm

In this paper, we assume that the carrier frequency offsets of uplink users are known (or estimated) by the receiver. The fact that different subcarriers are assigned to different users in OFDMA systems makes the signal separability possible since the subcarriers coming from different users will have independent attenuations. As different users are assigned to neighboring subcarriers, where most of the interference comes from, and their power levels are separable, SIC can be used to remove the interference due to frequency offset. On the other hand, in clustered OFDMA systems, the subcarriers within a cluster will observe similar fading and hence their power levels will be similar. Therefore, successive cancellation will not be efficient for these subcarriers as signal separability is not possible. In order to overcome this problem, we apply decorrelator receiver over subcarriers within each clusters. As the size of each cluster is small (compared to the whole subcarrier range), the decorrelator receiver is possible with manageable complexity. The combination of the decorrelator and successive cancellation is proposed in this paper as an efficient method for mitigating the frequency offset in uplink OFDMA systems.

In the proposed method, first the clusters are sorted in descending order of their average powers. Then, starting with the cluster with the largest power, decorrelation is applied and decisions are made. After obtaining the bits transmitted for the current cluster, the MAI to neighboring clusters is reconstructed using the knowledge of the channel and frequency offset values, and subtracted to cancel its MAI. Fig. 2 shows the flow chart of the proposed method.

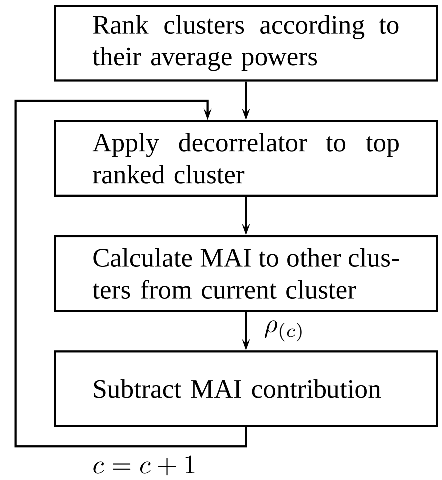

Fig. 2. Flow chart of the proposed compensation algorithm.

A. Decorrelation

The ICI is mostly caused by the power leakage from subcarriers within the same cluster[2]. In the receiver, the FFT output for the cth cluster, which belongs to ith user, can be written in matrix form as

$$\mathbf{y}_c = \mathbf{\Pi}_i \mathbf{h}_c + \mathbf{w}_c, \tag{12}$$

where $\mathbf{y}_c = [Y^{(i)}(k) \dots Y^{(i)}(k+K)]^T$ is the received signal vector, $\mathbf{h}_c = [X^{(i)}(k)H^{(i)}(k) \dots X^{(i)}(k+K)H^{(i)}(k+K)]^T$ is the vector that depends on the transmitted symbols and frequency domain channel, and $\mathbf{w}_c$ is the MAI plus noise vector. The current cluster (cluster c) spans the subcarriers from k to $k+K$, *i.e.* $\Gamma_i^{(c)} = k, \dots, k+K$. The $K \times K$ matrix $\mathbf{\Pi}$ is the ICI matrix whose entries are given as

$$\mathbf{\Pi}_i(u,k) = D(u,k,\epsilon_i), \tag{13}$$

where D is as defined in (8).

The leakage among the subcarriers due to frequency offset can be viewed as a matrix diagonalization problem. By multiplying $\mathbf{y}_c$ with $\mathbf{\Pi_i^{-1}}$, the matrix $\mathbf{\Pi_i}$ is diagonalized and hence the power leakage among the subcarriers within the current cluster is removed. Moreover, the phase rotation to the desired subcarriers due to frequency offset $D(k,k,\epsilon_i)$ is also corrected. In order to be able to apply this method, however, the ICI matrix $\mathbf{\Pi_i}$ should be invertible which can be proven by showing that $\mathbf{\Pi_i v} = \mathbf{0}$ has only trivial solution. We skip the proof in this paper because of the space requirements.

The decorrelator output can be calculated using (12) as

$$\hat{\mathbf{h}}_c = \mathbf{\Pi}_i^{-1} \mathbf{y}_c \tag{14}$$

$$= \mathbf{h}_c + \mathbf{\Pi}_i^{-1} \mathbf{w}_c. \tag{15}$$

[2]Assuming a user's clusters are separated from each other, the ICI contribution from the user's other clusters is not considered and it is removed by successive cancellation along with MAI.

Using $\hat{\mathbf{h}}_c$ and the channel knowledge the receiver can detect the transmitted information. This solution is also known as LS method. If the noise power and the autocorrelation of channel is known, a more advanced method such as MMSE might also be used to obtain a better detection performance. A similar approach is applied in [3] to remove ICI and MAI for *all subcarriers*. However, calculating and inverting the $N \times N$ interference matrix is practically impossible for large FFT sizes.

The interference matrix is a Toeplitz matrix and it can be inverted efficiently with $O(K^2)$ computations as compared to $O(K^3)$ for arbitrary matrices [12]. Moreover, the same matrix can be used for the clusters of each user as it is independent of subcarrier positions.

B. Successive Cancellation

In this section, the decoded signal in Section III-A is used to reconstruct and remove the MAI on other clusters successively. The interference of cth cluster (which belongs to user i) on the kth subcarrier ($k \notin \Gamma_i^{(c)}$) can be reconstructed as

$$\rho_{(c)}(k) = \sum_{u \in \Gamma_i^{(c)}} H^{(i)}(u) X^{(i)}(u) D(u, k, \epsilon_i). \qquad (16)$$

Hence, the kth subcarrier's value after cancellation becomes

$$\begin{aligned} \hat{Y}(k) &= Y(k) - \rho_{(c)}(k) \\ &= Y(k) - \sum_{u \in \Gamma_i^{(c)}} H^{(i)}(u) X^{(i)}(u) D(u, k, \epsilon_i). \qquad (17) \end{aligned}$$

In the next step, the cluster with the second largest power is decoded by applying decorrelation algorithm given in Section III-A and its interference to remaining clusters is removed (see Fig. 2). This process is continued until all the clusters are processed.

It is well known that the power leakage from a subcarrier to neighboring subcarriers decreases as the separation between the two subcarriers increases. This fact can be seen by investigating (8). Hence the interference from current cluster to only a limited number of neighboring clusters might be calculated and removed in order to decrease the computational requirements of the proposed algorithm.

IV. Numerical Results

The performance of the proposed algorithm is tested with computer simulations. An uplink OFDMA system with 256 subcarriers and 8 users are considered. The size of each cluster is set to 4 ($K = 4$) and the same number of clusters are assigned for each user; hence 8 clusters for each user. The transmission bandwidth is set to 10 MHz. Users are assumed to have independent fading channels with the same average power[3], and perfect channel knowledge is assumed for simulations. For simulating the wireless channel, the Channel A of ITU-R channel model [13] for vehicular environments with high antenna is used. The users employ QPSK mapping for their data symbols, and subcarrier allocation is assumed to be similar to UL-PUSC of IEEE 802.16 [1]. Independent frequency offsets are assumed for all uplink users. Each frequency offset is a random variable with uniform distribution in (-ϵ_{max}, ϵ_{max}) where ϵ_{max} is the maximum allowed value of users' frequency offsets. Instead of fixing the frequency offsets of each user, they are chosen randomly to simulate a more realistic scenario.

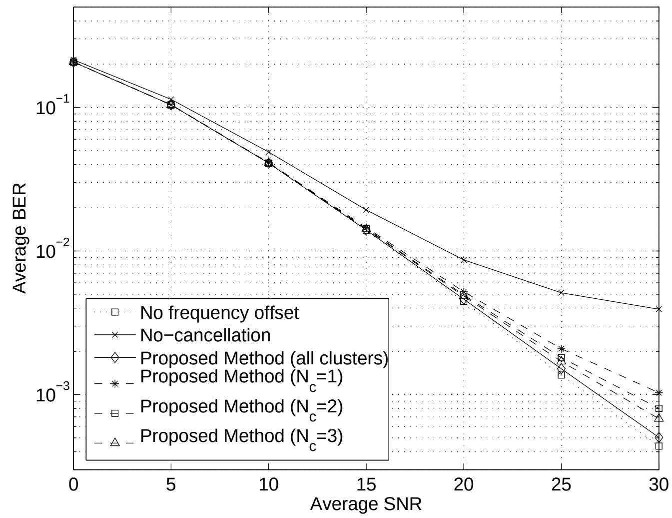

Fig. 3. Uncoded bit error rate (BER) as a function of average SNR. The normalized frequency offset has a uniform distribution in the range (-0.1, 0.1).

Figs. 3 and 4 show the uncoded bit-error-rates (BERs) versus average SNR. The results with no-frequency offset, no-compensation, and the proposed compensation method are presented. Proposed method with cancellation of MAI from all clusters and from only neighboring 1, 2 and 3 clusters are given so that the effect of applying MAI cancellation to only neighboring subcarriers can be seen. Fig. 3 shows the results for $\epsilon_{max} = 0.1$ and Fig. 4 shows the results for $\epsilon_{max} = 0.3$. The effectiveness of the proposed algorithm for mitigating the frequency offset can be seen in these figures. When successive cancellation is applied to only neighboring clusters, where the main ICI contribution comes from, the performance of the proposed algorithm decreases. This shows that the leakage to other clusters can not be ignored especially when the frequency offset is large. However, when the frequency offset is small, only adjacent subcarriers might be considered in order to decrease the computational complexity. Hence, there is a trade-off between the computational complexity and amount of interference that can be removed. For large frequency offsets, the performance of proposed algorithms drifts from the no-frequency offset case at high SNR values (see Fig. 4). This is caused by the errors in decisions (and hence errors in ICI removal) because of the large frequency offset and high ICI. In order to decrease the amount of error propagation, the detected bits can be decoded and coded again. This will reduce the number erroneous decisions increasing the performance of the

[3]This is not a necessary assumption for the proposed algorithm. In fact, when power control is not used in uplink, the proposed algorithm is expected to work better as signal separability is more efficient.

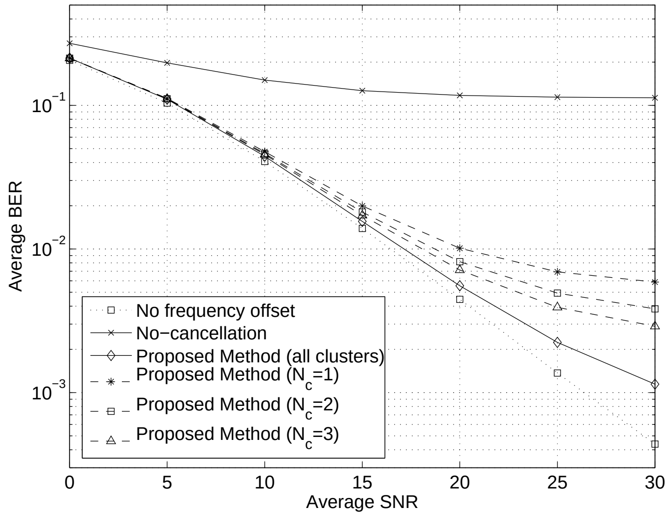

Fig. 4. Uncoded bit error rate (BER) as a function of average SNR. The normalized frequency offset has a uniform distribution in the range (-0.3, 0.3).

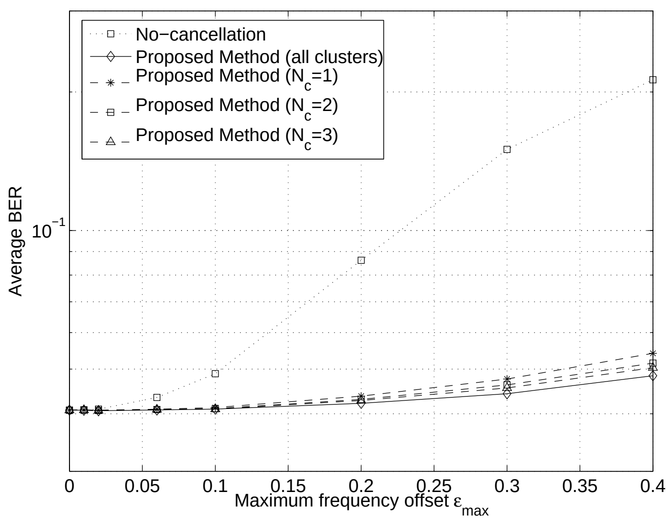

Fig. 5. Uncoded bit error rate (BER) as a function of maximum normalized frequency offset for average SNR of 10dB.

proposed method in the expense of larger complexity.

Figs. 5 and 6 present the uncoded BER as a function of maximum normalized frequency offset ϵ_{max} at 10dB and 30dB SNR values respectively. It is clear that the proposed algorithm lowers the error floor due to frequency offset mismatches. However, as frequency offset becomes larger, the effectiveness of the algorithm decreases because of the error propagation.

V. Conclusions

We have proposed a successive cancellation algorithm to mitigate the effects of frequency offset in uplink OFDMA systems where user separation is achieved using the average received power of user's clusters. The proposed method, a combination of decorrelator and successive cancellation, can compensate the different frequency offsets of users. Hence, it can be used as an alternative to the feedback method.

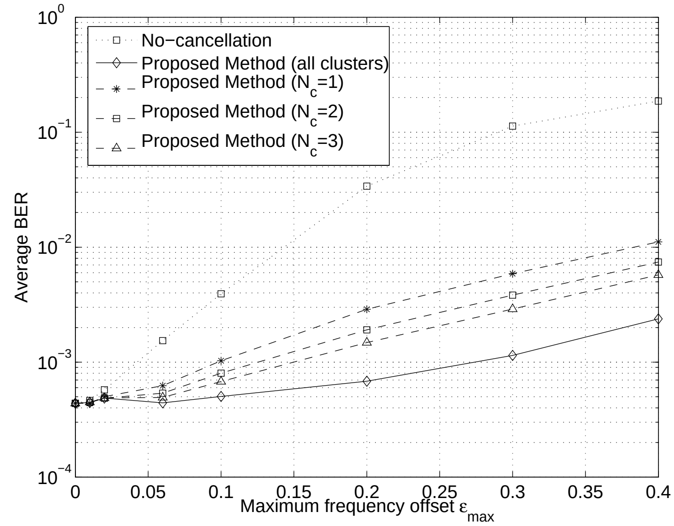

Fig. 6. Uncoded bit error rate (BER) as a function of maximum normalized frequency offset for average SNR of 30dB.

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