

# A Novel Sub-optimum Maximum-Likelihood Modulation Classification Algorithm for Adaptive OFDM Systems

Tevfik Yücek and Hüseyin Arslan

Department of Electrical Engineering, University of South Florida  
4202 E. Fowler Avenue, ENB-118, Tampa, FL, 33620  
Phone : (813) 974 0759 and (813) 974 3940  
E-mail : yucek@eng.usf.edu and arslan@eng.usf.edu

**Abstract**—Adaptive modulation is an effective method to increase the spectral efficiency of OFDM based high-speed wireless data transmission systems in time-dispersive (frequency-selective) channels. Blind modulation classification schemes play an important role in adaptive modulation systems to eliminate the need for transmitting the modulation information, thereby increasing spectral efficiency. In this paper, a maximum-likelihood (ML) modulation classifier which has the optimum performance in the presence of white noise is presented. A sub-optimum classifier, which greatly reduces the complexity, is derived from the optimum ML classifier. The performances of proposed classifiers are tested using Monte-Carlo simulations for ideal and non-ideal cases.

## I. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM), which is a multi-carrier modulation scheme, is a strong candidate for future communication systems to achieve high data rates in multipath fading environments. In OFDM, the wide transmission spectrum is divided into narrower bands and data is transmitted in parallel on these narrow bands. Therefore, symbol period is increased by the number of sub-carriers, decreasing the effect of inter-symbol interference (ISI). The remaining ISI effect is eliminated by cyclically extending the signal. Adaptive modulation is a method to increase the data capacity, throughput, and efficiency of wireless communication systems. In adaptive modulation, the transmitter continually monitors the dynamic channel and adjusts the transmission parameters accordingly to maximize efficiency.

The three steps that entail the process of adaptive modulation are channel estimation, modulation selection, and signaling/blind classification of the modulation used. The focus of this paper is restricted to reliable blind modulation classification schemes. A maximum-likelihood (ML) modulation classifier which gives the optimum performance in the presence of white Gaussian noise is presented. A sub-optimum classifier is derived from the optimum ML classifier to reduce the complexity. The performances of these classifiers are tested using Monte-Carlo simulations for ideal and non-ideal cases.

The rest of the paper is organized as follows. In the next section, adaptive modulation is described. Section III explains the blind modulation classification and gives prior art in this

area. Following the system description, we present the proposed classification algorithms. In Section VI, the simulation results are given, and paper is concluded in Section VII.

## II. ADAPTIVE MODULATION

Adaptive modulation is an effective method to increase the spectral efficiency of OFDM based high-speed wireless data transmission systems in time-dispersive (frequency-selective) channels [1], [2]. The objective of adaptive modulation is to change the modulation used in each sub-carrier depending on the channel strength for that sub-carrier in order to achieve a good trade-off between throughput and overall bit-error-rate (BER). When the channel is highly reliable, the modulation order is increased to take full advantage of the channel condition and to maximize the throughput. As channel fades, the modulation level is decreased to a level so as to provide an acceptable BER. Adaptation is not limited to only varying the modulation schemes alone, but also other parameters such as coding rate, symbol rate *etc.* can be adapted depending on the channel reliability information as mentioned in [3].

Adaptive modulation for OFDM systems is proposed in [1], [2], and [4], where different modulations are used for each sub-carrier. However, this is impractical since adaptation in every sub-carrier would be extremely complicated. Therefore, often, sub-carriers are grouped together, and adaptation is performed on the entire sub-carrier group [5]–[7]. This is termed as *sub-band adaptive modulation* and is illustrated in Fig. 1, which shows the different modulations used at different sub-carrier groups based on the channel conditions. Sub-band adaptation decreases the signaling overhead and/or makes the blind modulation classification possible. In [6] and [7], sub-band adaptive modulation is proposed for OFDM based WLAN systems.

Various modulation selection algorithms have been proposed in literature, which use different criteria to decide between modulation schemes based on the channel conditions. Modulation selection algorithms based on BER constraints, constant throughput, or both, depending on the intended application, are given in [5]. In this paper, modulation selection for each

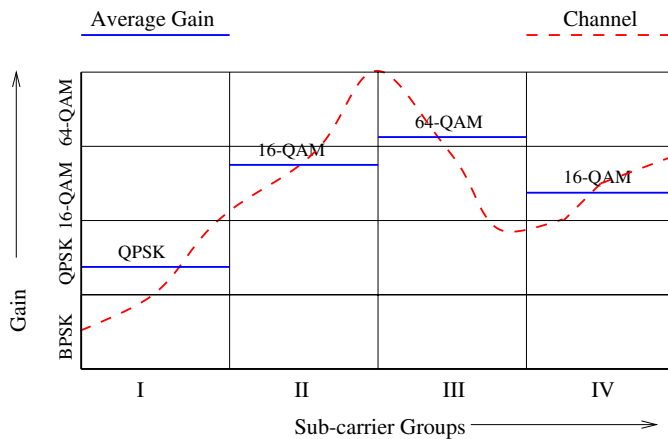


Fig. 1. Modulation mode selection based on perceived channel quality.

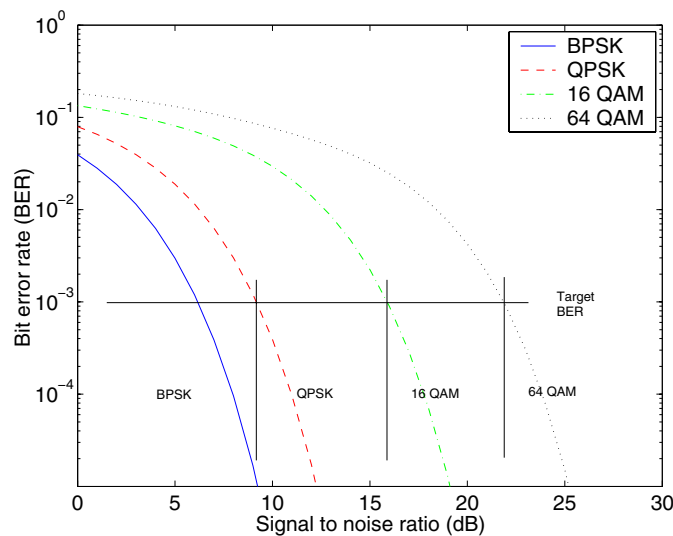


Fig. 2. SNR bounds within which different modulation schemes are used.

sub-band is assumed to be done based on the estimated signal-to-noise ratio (SNR) at the transmitter to provide a desired BER performance while providing highest throughput. Available modulation options are restricted to BPSK, QPSK, 16-QAM and 64-QAM. The SNR bounds within which different schemes are used are shown in Fig. 2 for a target BER of  $10^{-3}$ . The ranges shown in this figure are used in our simulations.

In adaptive modulation, in order to demodulate the transmitted data correctly, the receiver has to know which modulation is being used at the transmitter. Two methods, namely parameter signaling or blind detection, can be employed [5]. Parameter signaling is where the modulation information is embedded within the transmitted data symbols. This has the obvious disadvantage of reducing data throughput due to extra signaling symbols. An alternative would be blind classification of modulation used at the transmitter side which will be explained in next section.

### III. BLIND MODULATION CLASSIFICATION

Blind modulation classification is a method to determine the modulation type of a received signal. It has applications in military communications, interference identification, *etc.*

Modulation classification is a well studied area in wireless communications. The problem has been traditionally approached in two ways, namely pattern recognition [8], [9] and decision theoretic techniques [10]–[12]. In statistical pattern recognition methods (see references in [13]), some statistical representations of the received signal or some of its parameters are extracted and used for determining the modulation employed by the transmitter. In a decision-theoretic framework, modulation classification problem can be viewed as a multi-hypotheses testing problem, *i.e.* choosing from a number of modulations based on the observed waveform [10].

Maximum likelihood methods for modulation classification are studied in [10]–[15]. ML classifier gives the best performance among any other classifiers. In [12], the theoretical limits of the ML classifier are derived.

In adaptive modulation, blind modulation detection schemes can be used to minimize the loss of useful data bandwidth by estimating the modulation scheme used at the transmitter without explicit signaling. This has the advantage of increased spectral efficiency and improved throughput over traditional signaling schemes.

Two methods with reference to blind modulation detection for adaptive OFDM systems are proposed in [16], which are based on the decision theoretic approach. In the first method, the mean Euclidean distance between the received samples and all the closest legitimate constellation points of all possible modulation schemes are calculated. The scheme which minimizes this distance is chosen for demodulation. In this method, however, there is always a bias toward the higher order modulation schemes irrespective of the actual modulation used. In [17], instead of using only the error information, the unique distribution of these errors for each SNR and modulation scheme is taken into account. The received distribution and the known distributions are compared using Kullback-Leibler distance to find the closest match. The second method proposed in [16] makes use of convolutional encoding with Viterbi decoding to estimate the modulation scheme. However, there is a high degree of computational complexity involved and this method entails the use of coding, which is out of the scope of this paper.

### IV. SYSTEM MODEL

OFDM converts serial data stream into parallel blocks of size  $N$ . These blocks are called OFDM symbols and they can be represented by a vector  $\mathbf{X} = [X_1 \ X_2 \ \dots \ X_N]$ . The data symbols  $X_k$  are obtained by using one of the available modulation options (see Section II), and they are not necessarily mapped with the same modulation.

The OFDM symbols are modulated using Inverse Discrete Fourier Transform (IDFT) (of size  $N$ ) to obtain time domain

samples as

$$\begin{aligned} x(n) &= \text{IDFT}\{\mathbf{X}\} \\ &= \sum_{k=0}^{N-1} X_k e^{j2\pi nk/N} \quad 0 \leq n \leq N-1, \end{aligned} \quad (1)$$

where  $X_k$  is the symbol transmitted on the  $k$ th sub-carrier. Time domain signal is cyclically extended to avoid ISI from previous symbol, filtered, passed through D/A conversion block, and transmitted using antenna over a wireless channel.

At the receiver, the signal is received along with noise. After the synchronization, down sampling, and the removal of cyclic prefix, the baseband model of the received frequency domain symbols can be written as

$$Y_k = H_k X_k + n_k, \quad (2)$$

where  $Y_k$  and  $H_k$  are the received symbol and the frequency response of the channel on the  $k$ th sub-carrier respectively, and  $n_k$  is the complex additive white Gaussian noise sample with zero mean and variance of  $\sigma^2$ .

## V. ALGORITHM DESCRIPTION

In this section, ML method for modulation classification is given. Although this method gives the best possible classification performance, it requires calculation of exponentials which increases complexity. A sub-optimum method, which reduces the complexity with a very small degradation in performance, is derived from the optimum method.

### A. Maximum-likelihood Method

Received symbols at each sub-carrier will have the form as in (2). Let  $m_i$  and  $M_i$  ( $i=1, 2, 3, 4$ ) denote each one of the possible modulations and the constellation size for each modulation respectively. In this case, transmitted symbols  $X_k$  are taken from a set of  $M_i$  complex numbers  $\{a_1, a_2, \dots, a_{M_i}\}$  if modulation  $m_i$  is used. This set is known as constellation and it is usually normalized to unity power, *i.e.*,

$$E\{|X_k|^2\} = \frac{1}{M_i} \sum_{j=1}^{M_i} |a_j|^2 = 1. \quad (3)$$

Given a received point  $Y_k$ , the probability that this point belongs to modulation  $m_i$  can be expressed as

$$P(Y_k|m_i) = \sum_{j=1}^{M_i} P(Y_k|a_j^i) P(a_j^i|m_i), \quad (4)$$

where  $P(Y_k|a_j^i)$  is the probability that received point was actually transmitted at  $j$ th constellation point of  $i$ th modulation  $a_j^i$ . Assuming the signal is distorted by white Gaussian noise, this probability can be calculated as

$$P(Y_k|a_j^i) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{|Y_k - \hat{H}_k a_j^i|^2}{2\sigma^2}}, \quad (5)$$

where  $\hat{H}_k$  is the channel frequency response estimate at  $k$ th subcarrier.  $P(a_j^i|m_i)$  is the *a priori* probability of constellation point  $a_j^i$ . Without loss of generality, these *a priori* probabilities

are assumed to be the same for all constellation points which is the case in practical applications. Hence,  $P(a_j^i|m_i) = 1/M_i$ . Using this assumption and (5), (4) can be re-written as

$$P(Y_k|m_i) = \frac{1}{M_i \sqrt{2\pi\sigma^2}} \sum_{j=1}^{M_i} e^{-\frac{|Y_k - \hat{H}_k a_j^i|^2}{2\sigma^2}}. \quad (6)$$

In sub-band adaptive OFDM, the same modulation is used over a group of sub-carriers. Hence, all of those sub-carriers, which are modulated with the same modulation, can be used to predict the modulation type used at the transmitter. As the size of these groups increases, the false classification probability decreases. Let the size of each group be  $L$ , and let us represent each group by  $\mathbf{G} = [Y_1 Y_2 \dots Y_L]$ . Assuming the transmitted symbols are independent of each other, the probability of modulation  $m_i$  can be calculated by

$$\begin{aligned} P(\mathbf{G}|m_i) &= \prod_{n=1}^L P(Y_n|m_i) \\ &= \prod_{n=1}^L \left( \frac{1}{M_i \sqrt{2\pi\sigma^2}} \sum_{j=1}^{M_i} e^{-\frac{|Y_n - \hat{H}_n a_j^i|^2}{2\sigma^2}} \right). \end{aligned} \quad (7)$$

Given these probabilities, Bayes' decision rule can be used to choose the most probable modulation used. In the event of equiprobable modulation types,  $k$ th modulation is chosen among four candidate modulations if  $P(\mathbf{G}|m_i)$  is maximized for  $i = k$ . This method is applicable to any constellation-based modulation types and it is shown that the performance of this classifier can be described by the average SNR [12].

### B. Sub-optimum Method

Although ML method gives the optimum performance, it is computationally intensive due to the need for calculation of the exponential terms in (8). However, the exponential terms in (8) are dominated by one term which corresponds to the distance between received sample and the closest legitimate constellation point. Neglecting the other terms, (8) can be reformulated as

$$P(\mathbf{G}|m_i) \approx \prod_{n=1}^L \frac{1}{M_i \sqrt{2\pi\sigma^2}} e^{-\frac{|Y_n - \hat{H}_n a_n^i|^2}{2\sigma^2}} \quad (9)$$

where  $a_n^i$  is the hypothesis constellation point that gives minimum distance for  $n$ th received sample. Let us define this distance as

$$d_{min}(n, i) = |Y_n - \hat{H}_n a_n^i|. \quad (10)$$

Eqn. 9 can be re-written now as

$$P(\mathbf{G}|m_i) \approx \left( \frac{1}{M_i \sqrt{2\pi\sigma^2}} \right)^L e^{-\sum_{n=1}^L \frac{d_{min}^2(n, i)}{2\sigma^2}}. \quad (11)$$

Equivalently log-likelihood function can be used since logarithm is a monotone one-to-one function. After taking the logarithm of (11) and removing the common term in each modulation, log-likelihood function can be obtained as

$$H(\mathbf{G}|m_i) = -\sum_{n=1}^L d_{min}^2(n, i) + 2\sigma^2 L \log \frac{1}{M_i}. \quad (12)$$

The first term in (12) is equivalent to the metric used in the SNR estimation based modulation classification algorithm given in [16]. As explained earlier, that method has a bias toward higher order modulations. Proposed algorithm removes this bias with the additive term (second term in (12)) resulting in much better performance.

## VI. RESULTS

In order to verify the proposed algorithm, an OFDM system with 64 sub-carriers is simulated. Sub-carriers are grouped into four bands, each with 16 sub-carriers. One of four different modulation schemes, namely BPSK, QPSK, 16-QAM and 64-QAM, are used in each sub-band for transmission based on the observed channel conditions. For simulations, perfect channel knowledge is assumed.

Fig. 3 shows the false classification probability of ML and sub-optimum algorithms when only one type of modulation is used at the transmitter side. In other words, over the whole SNR range, only one modulation is used and four possible modulations are tested. As this figure clearly shows, the performance degradation in sub-optimum algorithm is very small. However, the proposed sub-optimum solution has a bias toward lower order modulations. In sub-optimum solution, only the dominant term in (6) is used and the other terms are ignored which are not the same for each modulation. Actually, the ignored part is larger for higher order modulations which causes a bias toward lower order ones. This bias, however, improves the performance of blind modulation classification in the case of adaptive modulation, since most of the errors occur at lower order modulations. This bias can be removed (if desired) with an SNR depended correction term which is an area of further research.

Fig. 4 shows the classification error rate (CER) when transmitter modulates the input data with one of the available modulations with equal probabilities, regardless of the channel conditions. In this case, the performance of the algorithm is limited by the performance of highest order modulation. These two figures show that the sub-optimum classifier can also be used for modulation classification on applications other than adaptive modulation.

In sub-band adaptive OFDM, certain modulations are used only within a calculated (transmitter) SNR range as explained in Section II. Fig. 5 shows the CER when transmitter has perfect SNR knowledge. Note that the errors are very small, when proposed sub-optimum algorithm is used for classification. The probabilities of transmitting each modulation for a given SNR are shown in Fig. 6(a) in this case.

### A. Effect of using wrong modulation at the transmitter

In Fig. 5, only one modulation is used within a given SNR range in transmitter. However, transmitter might have SNR estimation error leading to use a different modulation than the ideal one. The SNR estimation error at the transmitter side is modeled as shown in Fig. 6(b), where an overlap of 5.5dB is allowed. Fig. 7 shows the corresponding CER. As Figs. 5 and 7 imply, the SNR estimation error at the transmitter increases the

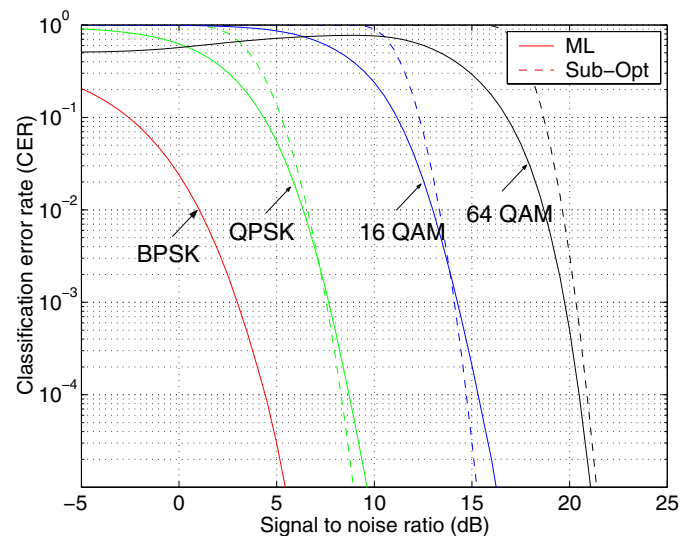


Fig. 3. Classification error rates for each modulation. Solid lines show ML classifier and dashed lines show sub-optimum algorithm.

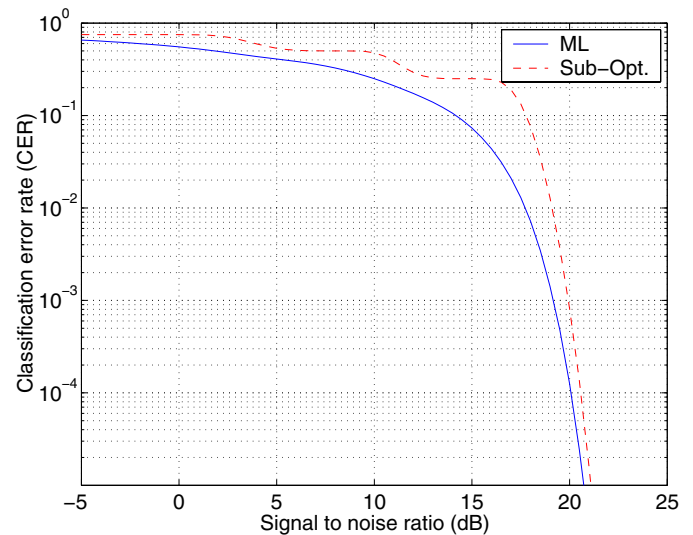


Fig. 4. Classification error rates for equiprobable transmission for each modulation.

false classification probability. Nevertheless, the performance degradation is not significant.

### B. Effect of imperfect noise variance estimation

Optimum and sub-optimum algorithms require noise variance information which can be estimated at the receiver [18]. However, a perfect estimation is not always possible. To test the robustness of the proposed algorithms to imperfect noise variance estimation, a simple noise variance estimator is implemented. Noise variance is estimated after the classification of OFDM symbol is done and this estimate is used in the classification of succeeding OFDM symbols. Estimated noise

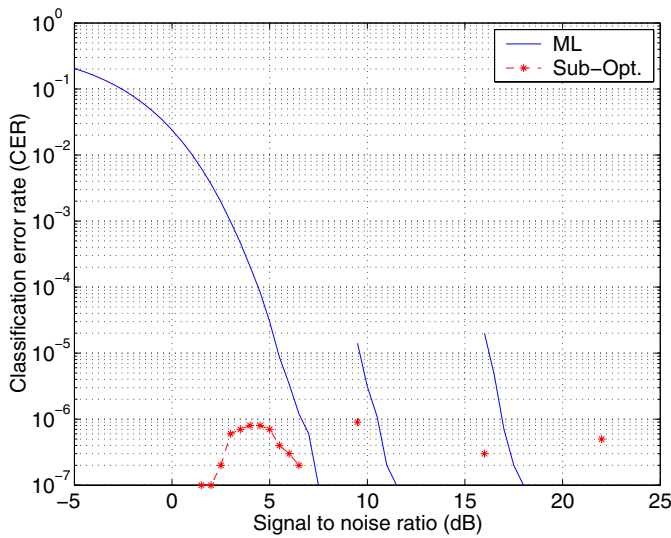


Fig. 5. Classification error rates of ML classifier and sub-optimum classifier in ideal adaptive modulation case. Note that the sub-optimum does not have any errors.

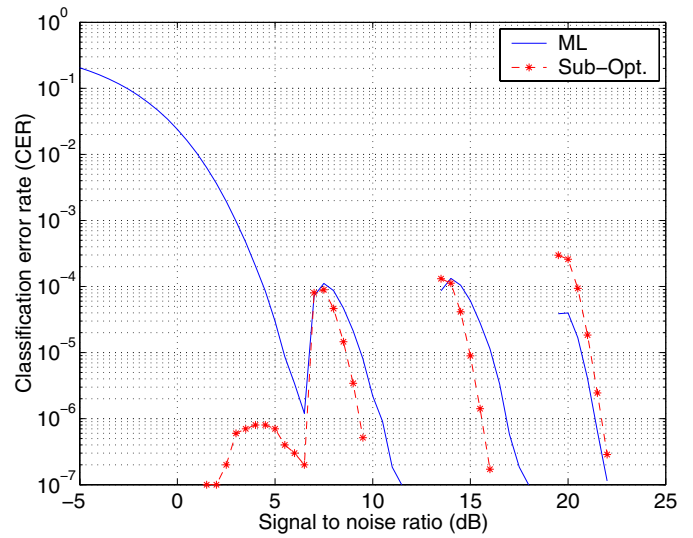
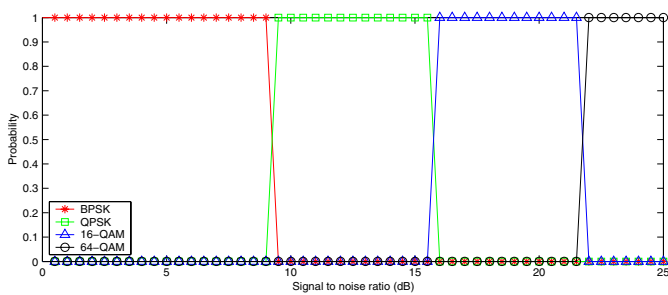
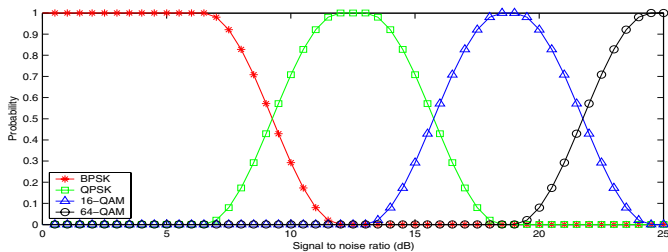


Fig. 7. Classification error rates when transmitter has SNR estimation errors.



(a) Ideal transmitter



(b) Non-ideal transmitter

Fig. 6. Transmit probabilities of each modulation as a function of true SNR for an ideal transmitter and for a transmitter with SNR estimation error.

variance can be formulated as

$$\hat{\sigma}^2 = \frac{1}{K} \sum_{k=1}^K d_{min}^2(n), \quad (13)$$

where  $K$  is the number of symbols over which noise variance is estimated. Noise variances estimated over one sub-band (16 subcarriers) and over an OFDM symbol (64 subcarriers)

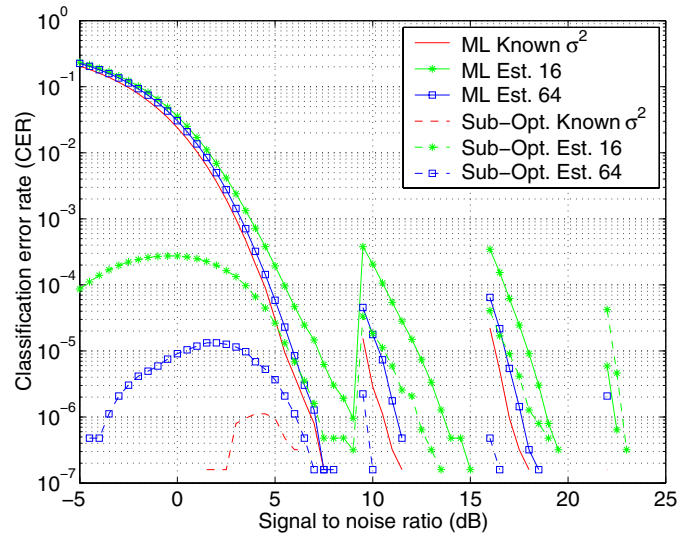


Fig. 8. Classification error rates of ML and sub-optimum algorithms, in the case of perfect and estimated noise variances.

are considered. In practical applications, more complicated estimators can be used. CERs of ML and sub-optimum algorithms, in the case of perfect and estimated noise variances, are shown in Fig. 8. Transmitter is assumed to have perfect SNR information. ML classification algorithm as well as proposed sub-optimum algorithm are quite robust to the noise variance estimation errors as can be seen from Fig. 8. A classification error performance smaller than  $10^{-4}$  is maintained.

## VII. CONCLUSION

In this paper, a ML modulation classifier for adaptive OFDM systems, which gives the optimum performance, is presented. A sub-optimum classifier, which greatly reduces the complexity, is derived from the optimum ML classifier. The performances of these classifiers are tested using Monte-

Carlo simulations for ideal and non-ideal cases. It is found that, the proposed sub-optimum algorithm yields performance close to optimum with much less complexity. Hence, it can be used in practical systems instead of signaling to increase the spectral efficiency. Constant envelope property of some of the employed modulations can also be used to further decrease the computational complexity which will be studied in the future.

#### ACKNOWLEDGMENT

The authors would like to thank Sharath B. Reddy for his helpful comments and suggestions.

#### REFERENCES

- [1] A. Czylik, "Adaptive OFDM for wideband radio channels," in *Proc. IEEE Global Telecommunications Conf.*, vol. 1, London, UK, Nov. 1996, pp. 713–718.
- [2] S. Ye, R. Blum, and J. Cimini, L.J., "Adaptive modulation for variable-rate OFDM systems with imperfect channel information," in *Proc. IEEE Veh. Technol. Conf.*, vol. 2, Birmingham, AL, May 2002, pp. 767–771.
- [3] H. Matsuoka, S. Sampei, N. Morinaga, and Y. Kamio, "Adaptive modulation system with variable coding rate concatenated code for high quality multimedia communication systems," in *Proc. IEEE Veh. Technol. Conf.*, vol. 1, Atlanta, GA, Apr./May 1996, pp. 487–491.
- [4] E. Lawrey, "Multiuser OFDM," in *Proc. Fifth International Symposium on Signal Processing and its Applications*, Brisbane, Australia, Aug. 1999, pp. 761–764.
- [5] T. Keller and L. Hanzo, "Adaptive modulation techniques for duplex OFDM transmission," *IEEE Trans. Veh. Technol.*, vol. 49, no. 5, pp. 1893–1906, Sept. 2000.
- [6] H. R. R. Grünheid, E. Bolinith and K. Aretz, "Adaptive modulation for the HIPERLAN/2 air interface," in *Proc. 5th International OFDM Workshop*, Hamburg, Germany, 2000, pp. 4–1 – 1–4.
- [7] A. Kannu, T. Sivanadyan, and S. Subramaniam, "Performance of block adaptive modulation schemes for OFDM based WLAN systems," in *Proc. IEEE International Conference on Personal Wireless Communications*, Dec. 2002, pp. 6–10.
- [8] B. Mobasseri, "Constellation shape as a robust signature for digital modulation recognition," in *Proc. Mil. Commun. Conf.*, vol. 1, Atlantic City, NJ, Oct./Nov. 1999, pp. 442–446.
- [9] X. Huo and D. Donoho, "A simple and robust modulation classification method via counting," in *Proc. IEEE Int. Conf. on Acoust., Speech, and Signal Processing*, vol. 6, Seattle, WA, May 1998, pp. 3289–3292.
- [10] P. Panagiotou, A. Anastasopoulos, and A. Polydoros, "Likelihood ratio tests for modulation classification," in *Proc. Mil. Commun. Conf.*, vol. 2, Los Angeles, CA, Oct. 2000, pp. 670–674.
- [11] J. Sills, "Maximum-likelihood modulation classification for PSK/QAM," in *Proc. Mil. Commun. Conf.*, vol. 1, Atlantic City, NJ, Oct./Nov. 1999, pp. 217–220.
- [12] W. Wei and J. Mendel, "A new maximum-likelihood method for modulation classification," in *Conference Record of the Twenty-Ninth Asilomar Conference on Signals, Systems and Computers*, vol. 2, Pacific Grove, CA, Oct. 1995, pp. 1132–1136.
- [13] D. Boiteau and C. Le Martret, "A general maximum likelihood framework for modulation classification," in *Proc. IEEE Int. Conf. on Acoust., Speech, and Signal Processing*, vol. 4, Seattle, WA, May 1998, pp. 2165–2168.
- [14] A. Polydoros and K. Kim, "On the detection and classification of quadrature digital modulations in broad-band noise," *IEEE Trans. Commun.*, vol. 38, no. 8, pp. 1199–1211, Aug. 1990.
- [15] C.-Y. Huang and A. Polydoros, "Likelihood methods for MPSK modulation classification," *IEEE Trans. Commun.*, vol. 43, no. 234, pp. 1493–1503, Feb./Mar./Apr. 1995.
- [16] T. Keller and L. Hanzo, "Blind-detection assisted sub-band adaptive turbo-coded OFDM schemes," in *Proc. IEEE Veh. Technol. Conf.*, vol. 1, no. 49, Houston, TX, May 1999, pp. 489–493.
- [17] S. B. Reddy, T. Yücek, and H. Arslan, "An efficient blind modulation detection algorithm for adaptive OFDM systems," in *Proc. IEEE Veh. Technol. Conf.*, Orlando, FL, Oct. 2003.
- [18] H. Arslan and S. B. Reddy, "Noise variance and SNR estimation for OFDM based wireless communication systems," in *Proc. The 3rd Int. Conf. on Wireless and Optical Commun.*, Banff, Alberta, Canada, July 2003.