A Comparative Study of Initial Downlink Channel Estimation Algorithms for Mobile WiMAX

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***Abstract*— Fourth generation (4G) wireless products are expected to be based on worldwide interoperability for microwave access (WiMAX) systems. It is the optimized receiver algorithms that will differentiate competitive products. In this paper, an important receiver algorithm, channel estimation, is studied for mobile WiMAX systems (IEEE 802.16e-2005). Various channel estimation algorithms are studied and compared for initial channel estimation in downlink of IEEE 802.16e-2005. Various methods are analyzed in terms of their computational complexity, mean square error and bit error rate performances under different channel models.**

I. Introduction

Future wireless systems require high data rates since new wireless applications such as video-on-demand (VoD) over internet and internet protocol TV (IPTV) are only feasible when the wireless systems are capable of providing high data rates [1], [2]. While the implementation of new wireless services over the conventional wireless systems is less costly when the infrastructure is considered, these systems are not feasible when it comes to providing very high data rates. Hence, new wireless systems have to be built on technologies that can provide high data rates.

Orthogonal frequency division multiplexing (OFDM) is a multi-carrier modulation scheme in which the wide transmission bandwidth is divided into narrower bands and data is transmitted in parallel on these narrow bands [3]. Therefore, symbol period is increased by the number of bands, decreasing the effect of inter-symbol interference (ISI). The remaining ISI effect is eliminated by cyclically extending the signal. With these properties, OFDM can satisfy the high data requirement of future wireless applications.

The limitation of conventional systems and the re-birth of OFDM with high speed digital signal processors (DSPs) have forced researchers to come up with new wireless systems. Moreover, the motivation to converge existing wireless systems into a newer technology has given birth to a new wireless system for wireless metropolitan area networks (WMANs). Worldwide interoperability for microwave access (WiMAX) is the new hot technology of the wireless community as it enables operators to offer diverse wireless services to fixed and mobile users. For fixed users OFDM and for mobile users orthogonal frequency division multiple access (OFDMA) transmission techniques are employed [4]. One point to note is that the support for mobile users has threatened the cellular systems, since voice communication of mobile users can now be established over IP networks, *i.e.* voice-over-IP (VoIP) [5].

WiMAX promises many advantages to operators as well as wireless users. The allocation of spectrum in licensed/unlicensed bands gives the operators a chance of worldwide service. Moreover, quality of service (QoS) can be implemented and hence users are guaranteed to get certain wireless services at competitive prices [6]. The IP based network connection does not impose a cellular type base station (BS) installation. Hence, WiMAX BSs can easily target local rural areas or places where certain events that may create temporary heavy traffic take place. WiMAX is also attractive to under-developed and developing countries where the cable infrastructure does not exist [7]. Instead of laying cable all around the country, WiMAX BSs can provide the broadband access to users without this need. Hence, WiMAX systems are predicted to be deployed at a fraction of cost compared to wired systems. As can be seen, WiMAX satisfies different needs of operators and users, and hence stands as one of the strongest technology of future wireless applications.

In the realization of advantages promised by WiMAX systems, receiver algorithms play a crucial role. Channel estimation, which is the topic of the current article, is one of the key blocks in WiMAX receivers. It is one of the most important elements of wireless receivers that employs coherent demodulation. For OFDM based systems, channel estimation has been studied extensively [8]. Approaches based on least squares (LS) [9], transform domain [10], [11], and minimum mean-square error (MMSE) [12] estimation are proposed by exploiting the training sequences that are transmitted along with the data. Although there are many channel estimation techniques reported in the literature, for practical WiMAX systems it is important to have an estimation technique that is specifically designed for WiMAX preambles or pilots, and has low computational and hardware complexities. In this paper, different channel estimation techniques are analyzed for downlink OFDMA transmission technique of WiMAX systems. The comparison is made by having different channel models and system parameters. It is observed the use of MMSE estimator with a fixed uniform channel power delay profile (PDP) offers a good trade-off between the performance and complexity.

The rest of the paper is organized as follows. In Section II, system model will be introduced and investigated algorithms will be explained in Section III. Section IV compares the computational and implementation complexities of the studied algorithms. Numerical results will be presented in Section V and the paper will be concluded in Section VI.

II. System Model

OFDM converts serial data stream into parallel blocks of size N and modulates these blocks using inverse fast Fourier transform (IFFT). Time domain samples of an OFDM symbol can be obtained from frequency domain data symbols as

$$\begin{aligned} x(n) &= IFFT\{X(k)\} \\ &= \sum_{k\in\Gamma} X(k)e^{j2\pi nk/N} \qquad 0 \le n \le N-1, \end{aligned} \tag{1}$$

where $X(k)$ is the transmitted data symbol at the kth subcarrier of the OFDM symbol[1], N is the fast Fourier transform (FFT) size, and Γ denotes the set of used subcarriers. After the addition of cyclic prefix (CP) and D/A conversion, the signal is passed through the mobile radio channel. In this paper, the channel is assumed to be constant over an OFDM symbol, but time-varying across OFDM symbols, which is a reasonable assumption for low and medium mobility.

At the receiver, the signal is received along with noise. After synchronization, down-sampling, and removal of the CP, the simplified baseband model of the received samples can be formulated as

$$y(n) = \sum_{l=0}^{L-1} x(n-l)h(l) + w(n), \tag{2}$$

where L is the number of sample-spaced channel taps, $w(n)$ is the additive white Gaussian noise (AWGN) sample with zero mean and variance of σ_w^2, and the time domain channel impulse response (CIR) for the current OFDM symbol, $h(l)$, is given as a time-invariant linear filter. Note that in (2) perfect time and frequency synchronization is assumed[2]. In this case, after taking FFT of the received signal $y(n)$, the samples in frequency domain can be written as

$$\begin{aligned} Y(k) &= FFT\{y(n)\} \\ &= X(k)H(k) + W(k) \qquad k \in \Gamma, \end{aligned} \tag{3}$$

where $\mathbf{H}$ and $\mathbf{W}$ are FFTs of $\mathbf{h}$ and $\mathbf{w}$ respectively.

[1] A particular subcarrier can be assigned to different users depending on the OFDMA allocation.

[2] In IEEE 802.16e, time and frequency synchronization is achieved by using initial and periodic ranging methods [13].

TABLE I
IEEE 802.16e Parameters Used in the Paper.

Parameter	Used Value	Notes
N	1024	FFT size
N_u	840	Number of used subcarriers
N_{CP}	256	CP size
N_p	280	Number of used (pilot) subcarriers in the preamble

The LS estimate of the channel frequency response (CFR) $\mathbf{H}$ can be calculated using the received signal and the knowledge of transmitted symbols as

$$\hat{H}_{LS}(k) = \frac{Y(k)}{X(k)} \tag{4}$$

$$= H(k) + \frac{W(k)}{X(k)}. \tag{5}$$

LS method is the simplest channel estimation method and it is usually used as an initial step for more advanced algorithms as it will be explained in this paper.

A. Preamble Pattern for 802.16e Downlink

The first symbol of each downlink subframe is dedicated as a preamble in OFDMA mode of 802.16e standard. The preamble is generated by modulating each third subcarrier using boosted binary phase shift keying (BPSK) with a specific pseudo noise (PN) sequence [4]. Hence, the time domain preamble consists of three repeating parts. This preamble is used for initial estimation of time-varying channel. In this paper, we analyse the performances and complexities of mobile station (MS) initial channel estimation methods using such a preamble.

Once the receiver transforms the received signal into frequency domain, the LS channel estimates can be calculated as per (5). Yet, more processing is required in order to estimate the complete channel transfer function as channel estimates at only every third subcarrier can be calculated using the preamble. In Section III, we explain and compare four different methods for improving the LS estimates at pilot subcarriers and for calculating the channel values at unused subcarriers.

B. Mobile WiMAX Related Parameters

Based on the IEEE 802.16e standard, we define the parameters given in Table I that will be used throughout the paper. Moreover, we focus on the 1024 FFT size mode of standard. However, our conclusions are similar for other modes of operation. The CP size is chosen to be 256 (the ratio of cyclic extension to “useful” time is 1/4) so as to prevent ISI even in highly multipath environments.

III. Studied Algorithms

In this section, the channel estimation algorithms that are used in this paper are described. We have chosen piecewise linear interpolation, transfer domain estimation, and MMSE algorithms because of their low implementation simplicity and robustness.

A. Piecewise Linear Interpolation

One of the simplest ways of interpolation is the use of piecewise constant [9] or piecewise linear interpolation [14]. In the piecewise constant interpolation, the CFR between pilot subcarriers is assumed to be constant, while in piecewise linear interpolation the channel for non-pilot subcarriers is estimated from a straight line between two closest pilot subcarriers. In the first method, acceptable results can be obtained if the channel frequency response is less frequency selective or the maximum excess delay of multi-path fading channel is very small. Such a constraint makes the CFR at the subcarriers very correlated that CFR at a group of subcarriers can be assumed to be the same. In piecewise linear interpolation some variation is allowed between the pilot subcarriers. Such an approach can result in a lower mean-squared-error (MSE) since noise averaging is performed. In terms of the computational complexity, the first method requires no multiplication. The piecewise linear method requires only one division per estimated subcarrier. In this paper, we only investigate the piecewise linear interpolation method as the complexity increase is negligible compared to piecewise constant method.

B. Transform Domain Estimation

When the CIR length is smaller than the number of pilot subcarriers, an orthogonal transformation applied to the CFR at the pilot subcarriers results in L number of significant values in the transform domain. If the significant values of the transform domain signal are retained, and the non-significant ones are treated as zero, then the noise term will be eliminated significantly especially when $L \ll N_p$ [10]. The CFR can then be obtained by applying the inverse of the orthogonal transformation, since such an operation will also achieve interpolation for non-pilot subcarriers [10], [15]. The transform domain techniques exploit the information about the number of significant values in the transform domain and their location. Moreover, more number of pilot subcarriers are used for the interpolation process. Hence, they perform better than the simple interpolation techniques in general. Although different transform domain techniques are studied for the channel estimation of OFDM based systems [8], in this paper only Fourier Transformation will be considered.

Mathematically, the process of the Fourier Transform technique for the channel estimation of OFDM based systems can be represented as [11],

$$\hat{\mathbf{H}}_{FT} = \sqrt{N/N_p}\mathbf{F}\mathbf{I}_{FT}\mathbf{F}_p^H\hat{\mathbf{H}}_{LS} \tag{6}$$

where $\sqrt{N/N_p}$ is for the normalization, $\mathbf{F}_p$ is the Fourier matrix with the rows corresponding to the subcarrier index of the pilot tones, and $\mathbf{I}_{FT}$ is given by

$$\mathbf{I}_{FT} = \begin{bmatrix} \mathbf{I}_{L\times L} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}. \tag{7}$$

Here, L is the length of the delay profile of the channel. The information about L is important in achieving higher performance in transform domain approaches. A delay profile length taken to be smaller than the actual CIR length will eliminate the significant taps, while a channel taken to be longer will result in less noise suppression. However, the first case is more critical than the second, and hence in practice L is usually taken to be the length of the CP when exact value of L is not available [11]. In this paper, we also use $L = N_{CP}$.

The transform domain techniques suffer from the unused subcarriers or suppressed subcarriers as this corresponds to rectangular windowing in frequency domain and hence $sinc$ convolution in transform domain or equivalent time domain. Practical systems such as WiMAX introduce guard bands in OFDM symbols via the elimination of the use of the subcarriers at the edge of the OFDM symbols. Since the resulting effect is the windowing process, in the equivalent time domain, the taps are leaked to one another due to $sinc$ interpolation, and hence the taps orthogonality is lost. Since the transform domain techniques assume a certain channel length, L, $N-L$ taps are zeroed out during time windowing. When this windowing in time domain is applied for reducing the noise plus interference, it will cause Gibbs phenomenon when the signal in the transform domain is transferred back to the frequency domain. In other words, the channel frequency response will have ripples around the edge carriers. The reason for this is the truncation of the $sinc$ function in time domain. Because of these reasons the transform domain based algorithms perform poorly as will be shown in Section V.

C. MMSE Channel Estimation

MMSE is widely used in the OFDM channel estimation since it is optimum in minimizing the MSE of the channel estimates in the presence of additive white Gaussian noise (AWGN). MMSE uses additional information like the operating signal-to-noise ratio (SNR) and the channel statistics. MMSE is a smoother/interpolater/extrapolater, and hence is very attractive for the channel estimation of OFDM based systems with pilot subcarriers similar to the downlink preamble of IEEE 802.16e. However, the computational complexity of MMSE is very high due to extra information incorporated in the estimation technique [16], [17].

For a given linear system model in the form of

$$\mathbf{y} = \mathbf{A}\mathbf{x} + \mathbf{w}, \tag{8}$$

MMSE of the variable $\mathbf{x}$ is given by,

$$\hat{\mathbf{x}} = \mathbf{R}_{yx}\mathbf{R}_{yy}^{-1}\mathbf{y} \tag{9}$$

where $\mathbf{R}_{xy}$ is the cross-covariance between variables x and y. When the expression (9) is applied to the OFDM channel estimation,

$$\hat{\mathbf{H}}_{MMSE} = \mathbf{R}_{HH_p} \cdot \left(\mathbf{R}_{H_pH_p} + \sigma_w^2(\mathbf{X}\mathbf{X}^H)^{-1}\right)^{-1}\hat{\mathbf{H}}_{LS} \tag{10}$$

can be obtained. Here, H_p is the CFR at the pilot subcarriers, $\mathbf{R}_{HH_p}$ represents the cross-correlation between all the subcarriers and the pilot subcarriers, and $\mathbf{R}_{H_pH_p}$ represents the auto-correlation between the pilot subcarriers. As can be seen in (10), MMSE uses additional information in its estimation process such as the correlation between subcarriers and noise variance. The diagonal matrix $\mathbf{X}$ contains the transmitted symbols, *i.e.* boosted BPSK in our application. Note that $\mathbf{XX}^H$ is the same for all preamble codes used in downlink IEEE 802.16e.

The MMSE estimation of $\mathbf{H}$ in (10) is computationally very heavy. For example, the dependency on the transmitted symbols due to the matrix inversion required at each estimate needs many operations. Moreover, large sized, full matrix multiplication required for a single estimate increases the computational complexity of MMSE as well. The non-trivial matrix inversion required in the MMSE estimation is another factor increasing the computational complexity of MMSE. Therefore, although MMSE is optimal, without reducing its computational complexity, it is difficult to realize its application in practical systems. Moreover, the channel correlation (or PDP) of the channel should be know. In order to reduce the complexity involved in calculating the MMSE coefficient matrix, a single matrix can be calculated and stored into memory. It is shown in [17] that a uniformly distributed PDP assumption offers good results. We investigate this MMSE method as MMSE with fixed coefficients where we set the length of PDP equal to the length of CP.

Subspace methods are investigated for the computational complexity and noise subspace reduction for the MMSE channel estimation. With subspace methods, the number of multiplications required for the channel estimate of a single subcarrier is reduced by exploiting singular value decomposition (SVD) [17], [18]. Subspace methods applied to the MMSE channel estimation reveal the degree of dependency of the subcarriers' auto and cross-correlation matrices. Since the subcarrier correlation is a function of the channel delay spread, it ultimately reveals long-term significant CIR taps or channel PDP. In this paper, subspace methods are not used as complexity reduction was not possible with acceptable performance for the system model considered.

IV. Computational and Hardware Complexities

Required number of complex multiplications are tabulated in Table II for the algorithms given in Section III for a single OFDM symbol. The used system parameters are as given in Table I. The transform domain estimation method requires two FFT operations for which the required number of multiplications is $N/2\log_2 N$. The complexity involved in calculating the MMSE coefficient matrices for optimum MMSE and MMSE with uniform PDP are not included in the table. For optimum MMSE, the coefficient matrix needs to be estimated when the channel statistics change. This operation involves inversion of large matrices (see (10)) and is computationally demanding. As explained in Section III-C, this requirement is removed by assuming a uniform PDP which is the last algorithm in Table II. The MMSE estimator also requires the knowledge of noise variance (or SNR). However, MSs are required to measure the SNR. Hence, in the last algorithm the coefficient matrices for different SNR values can be estimated and stored in ROMs and the channel estimator can use the appropriate matrix based on the estimated SNR value.

TABLE II
Computational complexities and performances of analyzed algorithms.

Estimation Scheme	Complex Multiplications	Performance
Piecewise Linear	$N_u - N_p = 560$	Average
Transform Domain	$N\log_2 N = 10240$	Poor
MMSE	$N_uN_p = 235200$	Good
MMSE with fixed coefficients	same with MMSE	Good

V. Numerical Results

For obtaining the simulation results we have used OFDMA mode of IEEE 802.16e with 1024 subcarriers. Total number of subcarriers is 1024 out of which 841 subcarriers are used for transmitting data information and pilots. The DC carrier is set to zero and the outermost 183 subcarriers (92 on left and 91 on the right of the spectrum) are not used to allow for guard bands. The system bandwidth is 10MHz and the length of the CP is set to 256 samples (the length of the guard band is 25.6μs). For simulating the wireless channel, the Channel A and Channel B of ITU-R channel model [19] for vehicular environments with high antenna is used. The relative delays and average power of each tap for these models are given in Table III.

The MSE performances of the studied algorithms are given in Figs. 1–5 and the uncoded bit-error-rates (BERs) are given in Figs. 2–6. From these curves it can be seen that the performance of the piecewise linear interpolation and transform domain techniques degrades as the time dispersion (*i.e.*

TABLE III
Characteristics of the ITU-R vehicular channel models

Channel A		Channel B	
Delay (ns)	**Power (dB)**	**Delay (ns)**	**Power (dB)**
0	0	0	-2.5
310	-1	300	0
710	-9	8900	-12.8
1090	-10	12900	-10
1730	-15	17100	-25.2
2510	-20	20000	-16

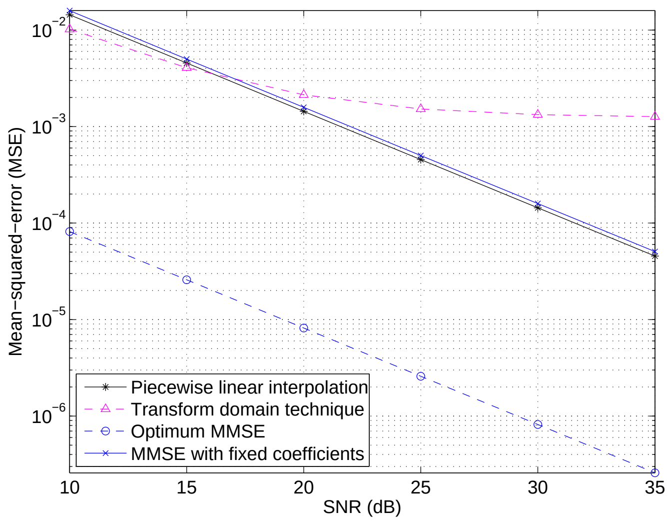

Fig. 1. MSE performance of analysed estimation methods in flat fading scenario.

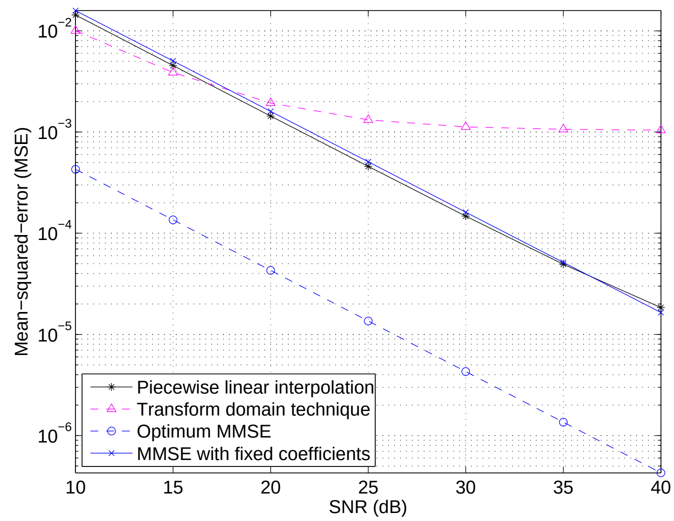

Fig. 3. MSE performance of analysed estimation methods in ITUR-A channel model.

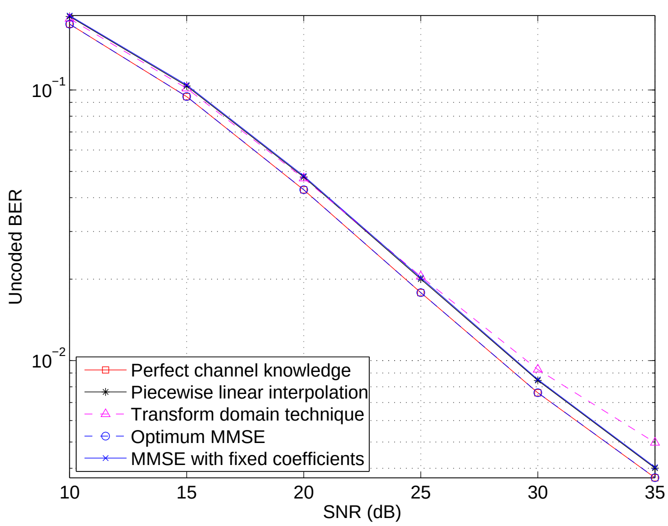

Fig. 2. BER performance of the analysed estimation algorithms for 64-QAM under flat fading.

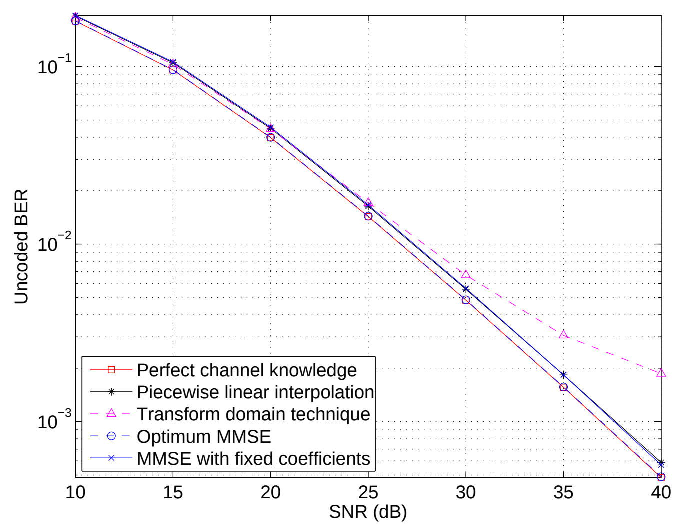

Fig. 4. BER performance of the analysed estimation algorithms for 64-QAM under ITUR-A channel model.

frequency selectivity) of the channel increases. However, the MMSE based algorithms are shown to be robust to various environments.

One interesting observation is that the performance of the piecewise linear interpolation is quite satisfactory (only 0.5dB worse than then best case channel model) at low and medium frequency selectivity. However, at high frequency selectivity the performance degrades as the linearity assumption of the CFR between the pilots is not valid anymore.

MSE of different algorithms is shown in Fig. 7 as a function of subcarrier index for Channel B model. Only the edge of the spectrum is shown in this figure. As this figure shows, the MSE of transform domain technique is large at the edge subcarriers as discussed in Section III-B. The large MSE at these subcarriers degrades the performance of the system and causes error floors. An expected result in this figure is the MSE of linear interpolation. The MSEs at non-pilot subcarriers is larger than the MSEs at pilot subcarriers which is causing the error floor for linear interpolation algorithm. Please note that for MMSE based algorithms, this is not the case.

VI. Conclusion

In this paper, four different channel estimation algorithms for downlink WiMAX are analysed and compared in terms of complexities and performance under different environments. We conclude that the MMSE algorithm with fixed coefficients achieves a good trade off and it can yield acceptable performance with practical hardware and computational complexities. By fixing the parameters of the MMSE coefficients, the complexity of calculating these parameters in real time is

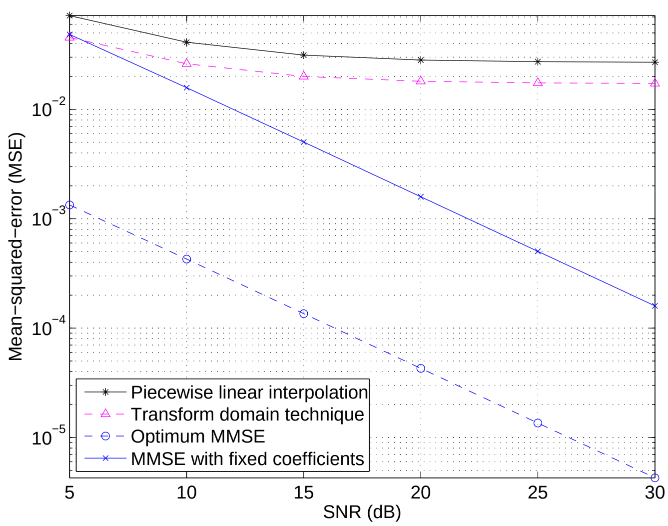

Fig. 5. MSE performance of analysed estimation methods in ITUR-B channel model.

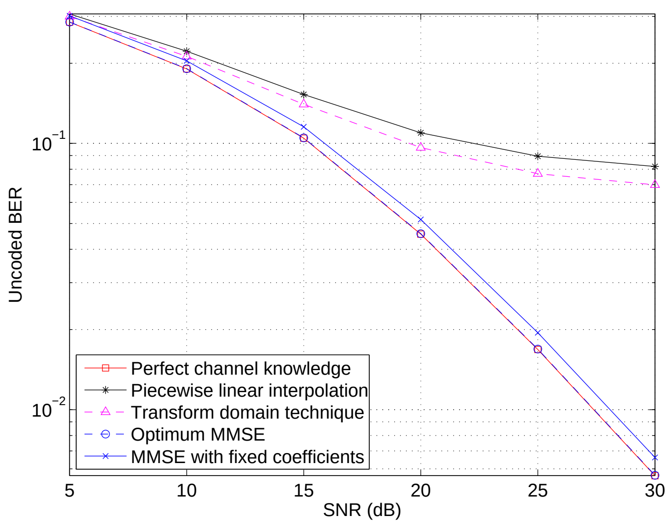

Fig. 6. BER performance of the analysed estimation algorithms for 64-QAM under ITUR-B channel model.

eliminated. On the other hand, simple linear interpolation and transform domain techniques have failed to provide satisfactory performance in highly frequency-dispersive environments.

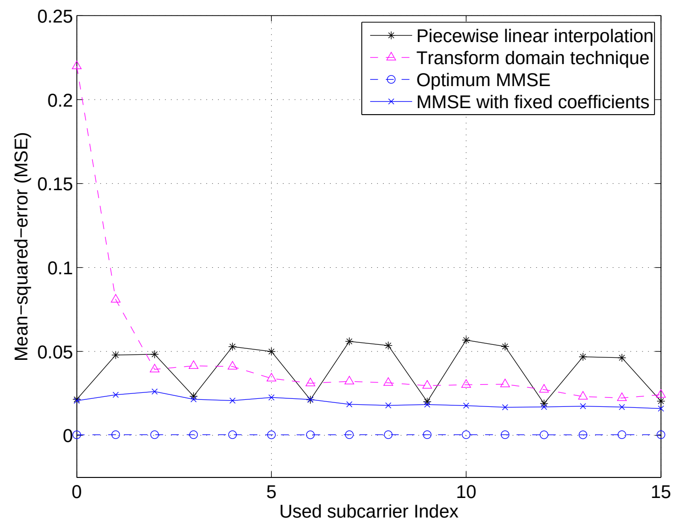

Fig. 7. MSE of channel estimation for ITUR-B channel model as a function of subcarrier index. Only the edge of frequency band is shown.

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