ESTIMATION OF FREQUENCY SELECTIVITY FOR OFDM BASED NEW GENERATION WIRELESS COMMUNICATION SYSTEMS

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Abstract

In this paper, estimation of both global (long term) and local (instantaneous) frequency selectivity and rms delay spread of wireless communication channel are investigated. A practical algorithm for averaged channel frequency correlation estimation, which is used for measuring frequency selectivity, is given. Average rms delay spread of the channel is calculated from the channel frequency correlation estimate. For measuring local frequency selectivity, channel frequency response estimate is exploited. Time domain channel impulse response is obtained by taking IFFT of the sampled channel frequency response. Optimal sampling rate of the estimated channel response is investigated. Performance of the estimates are obtained in noise limited situations. The effect of noise variance and robustness of the estimates against different channel power delay profiles is discussed for global parameter estimation. For local parameter estimation, the effect of channel estimation error is studied.

1 Introduction

In digital wireless communication systems, transmitted information reaches the receiver after passing through a radio channel, which can be represented as an unknown, time-varying filter. Transmitted signals are typically reflected and scattered, arriving at the receiver through multiple paths. When the relative path delays are on the order of a symbol period or more, images of *different symbols* arrive at the same time, causing intersymbol interference (ISI). Traditionally, ISI due to time dispersion is handled with equalization techniques. As the wireless communication systems making transition from voice centric communication to interactive Internet data and multi-media type of applications, the desire for higher data rate transmission is increasing tremendously. The higher data rates, with narrower symbol durations experiences significant dispersion, requiring highly complex equalizers.

Orthogonal Frequency Division Multiplexing (OFDM), which is a multi-carrier modulation technique, handles the ISI problem due to high bit rate communication by splitting the high rate symbol stream into several lower rate streams and transmitting them on different orthogonal carriers. The OFDM symbols with increased duration might still be effected by the previous OFDM symbols due to multipath dispersion. Cyclic prefix extension of the OFDM symbol avoids ISI if the cyclic prefix length is greater than the maximum excess delay of the channel. Since the maximum excess delay depends on the radio environment, the cyclic prefix length needs to be designed for the worst case channel condition which makes cyclic prefix as a significant portion of the transmitted data, reducing spectral efficiency. One way to increase spectral efficiency is to adapt the length of the cyclic prefix depending on the radio environment [1]. The adaptation requires estimation of maximum excess delay of the radio channel, which is also related to the frequency selectivity of the channel.

The information about the frequency selectivity of the channel can be very useful for improving the performance of the wireless radio receivers. For example, in channel estimation using channel interpolators, instead of fixing the interpolation parameters for the worst case expected channel dispersion as commonly done in practice, the parameters can be changed adaptively depending on the dispersion information [2]. Similarly, such dispersion estimation information can be used to control the receiver or the transmitter adaptively for different level of channel dispersion. For example, OFDM symbol duration, or OFDM sub-carrier bandwidth can be changed adaptively. As a result, adaptation through dispersion estimation provides better overall system performance, improved radio coverage, and higher data rates.

Characterization of the frequency selectivity of the radio channel is studied in [3]-[5] using level crossing rate (LCR) of the channel in frequency domain. Frequency domain LCR gives the average number of crossings per Hz at which the measured amplitude crosses a threshold level. Analytical expression between the LCR and the time domain multipath power delay profile (PDP) parameters corresponding to a specific PDP model is given. LCR is very sensitive to noise, which increases the number of level crossing and severely deteriorates the performance of the LCR measurement [5]. Filtering the channel frequency response (CFR) reduces the noise effect, but finding the appropriate filter bandwidth is an issue. If the filter is not chosen properly, one might end up smoothing the actual variation of frequency domain channel response. In [6], instantaneous root-mean-squared (rms) delay spread, which provides information about local (small-scale) channel dispersion, is obtained by estimating the channel impulse response (CIR) in time domain. The detected symbols in frequency domain are used to re-generate the time domain signal through inverse fast Fourier transform (IFFT). Then, this signal is used to correlate the actual received signal to obtain CIR, which is then used for delay spread estimation. Since, the detected symbols are random, they might not have good autocorrelation properties, which can be a problem especially when the number of carriers is low. In addition, the use of detected symbols for correlating the received samples to obtain CIR provides poor results for low signal-to-noise-ratio (SNR) values. In [1], the delay spread is also calculated from the instantaneous time domain CIR, where in this case the CIR is obtained by taking IFFT of the frequency domain channel estimate.

In this paper, both global (long term) and local (instantaneous) frequency selectivity of the CFR will be discussed. First, averaged channel frequency correlation estimate is used to describe the global frequency selectivity of the channel. A novel and practical algorithm for channel frequency correlation estimation will be described. Channel frequency correlation estimates will then be used to get average rms delay spread of the channel. The performance of the estimates will be shown in noise limited situations. The effect of noise variance and robustness of the estimates against different channel PDPs will be discussed. For measuring local frequency selectivity, CFR estimate will be exploited. Unlike [1], CIR is obtained by taking IFFT of the sampled channel frequency response. Sampling the CFR before taking IFFT reduces the computational complexity. The sampling rate of the channel frequency response is chosen based on efficient IFFT implementation, and according to the Nyquist criterion assuming the knowledge of the worst case maximum excess delay value. The effect of channel estimation error will be discussed.

2 System model

An OFDM based system model is used. Time domain samples of an OFDM symbol are obtained from frequency domain symbols as

$$x_m(n) = IDFT\{S_m(k)\} \tag{1}$$

$$x_m(n) = \sum_{k=0}^{N-1} S_m(k) e^{j2\pi nk/N} \qquad 0 \le n \le N-1 \ , \tag{2}$$

where $S_m(k)$ is the k-th subcarrier of the m-th OFDM symbol, and N is the number of subcarriers. After the addition of cyclic prefix and D/A conversion, the signal is passed through the mobile radio channel. Assuming a wide-sense stationary and uncorrelated scattering (WSSUS) channel, the channel $H(f,t)$ can be characterized for all time and all frequencies by the two-dimensional spaced-frequency, spaced-time correlation function

$$\phi(\Delta f, \Delta t) = E\{H^*(f,t)H(f+\Delta f, t+\Delta t)\} \ . \tag{3}$$

In this paper, we assume the channel to be constant over an OFDM symbol, but time-varying across OFDM symbols, which is a reasonable assumption for low and medium mobility.

At the receiver, the signal is received along with noise. After synchronization, down sampling, and removing the cyclic prefix, the simplified received baseband model of the samples can be formulated as

$$y_m(n) = \sum_{l=0}^{L-1} x_m(n-l) h_m(l) + z_m(n) \ , \tag{4}$$

where L is the number of channel taps, $z_m(n)$ is the additive white Gaussian noise (AWGN) component with zero mean and variance of σ_z^2, and the time domain CIR, $h_m(l)$, over an OFDM symbol is given as a time-invariant linear filter. After taking DFT of the OFDM symbol, the received samples in frequency domain can be shown as

$$\begin{aligned} Y_m(k) &= DFT\{y_m(n)\} \\ &= S_m(k)H_m(k) + Z_m(k) \ . \end{aligned} \tag{5}$$

The channel estimates in frequency domain can be obtained using OFDM training symbols, or by transmitting regularly spaced pilot symbols in between the data symbols and by employing frequency domain interpolation. In this paper, we assume transmission of training OFDM symbols. Using the knowledge of the training symbols, CFR can be estimated as

$$\begin{aligned} \hat{H}_m(k) &= \frac{Y_m(k)}{S_m(k)} \\ &= H_m(k) + w_m(k) \ , \end{aligned} \tag{6}$$

where $w_m(k)$ is the channel estimation error which is modeled as AWGN with zero mean and variance of σ_w^2.

3 Delay Spread Estimation

In this section, first a practical method for frequency correlation estimation from CFR is introduced. Then, an analytical expression for the correlation as a function of rms delay spread is derived. Finally, calculation of coherence bandwidth and the corresponding rms delay spread from frequency correlation for a given correlation value is explained.

3.1 Frequency correlation estimation

From the channel estimates, the instantaneous channel frequency correlation values can be calculated as

$$\hat{\phi}_H(\Delta) = E_k\{\hat{H}^*(k)\hat{H}(k+\Delta)\} \ , \tag{7}$$

where E_k is the mean with respect to k (averaging within an OFDM symbol). These instantaneous correlation estimates are noisy and needs to be averaged over several OFDM symbols. An *alpha tracker* is used for averaging the instantaneous values; the nth average can be found as

$$\tilde{\phi}_H^n(\Delta) = \alpha\tilde{\phi}_H^{n-1}(\Delta) + (1-\alpha)\hat{\phi}_H^{n-1}(\Delta) \ . \tag{8}$$

After going through the mathematical details, the averaged correlation estimates can be derived as

$$\tilde{\phi}_H(\Delta) = \begin{cases} \phi_H(\Delta) & \text{if } \Delta \neq 0 \\ \phi_H(0) + \sigma_w^2 & \text{if } \Delta = 0 \ . \end{cases} \tag{9}$$

3.2 Delay spread estimation

The PDP estimate $\tilde{\phi}_h(\tau)$ can be obtained by taking IDFT of the averaged frequency correlation estimates,

$$\begin{aligned} \tilde{\phi}_h(\tau) &= IDFT\{\tilde{\phi}_H(\Delta)\} \\ &= \phi_h(\tau) + \sigma_w^2 \ . \end{aligned} \tag{10}$$

The statistics like rms delay spread and maximum excess delay spread can be calculated from PDP. However, this requires IDFT operation which increases computational complexity. Instead, the desired parameters can be calculated directly from the averaged frequency correlation estimates. In [7], we derived the analytical relation between rms delay spread value and the channel frequency correlation. In order to be able to make this derivation, we assumed an exponential PDP since it has been shown theoretically and with experimental verification as the most accurate model [8]. The simulation results for other PDPs are also given in order to test the robustness of the algorithm.

Absolute value of the channel frequency correlation can be written as

$$|\tilde{\phi}_H(\Delta)| = \sqrt{\frac{1 - 2e^{-\tau_0/\tau_{rms}} + e^{-2\tau_0/\tau_{rms}}}{1 - 2e^{-\tau_0/\tau_{rms}}\cos\frac{2\pi\Delta}{N} + e^{-2\tau_0/\tau_{rms}}}} \ . \tag{11}$$

Details of derivation for the above formula can be found in [7].

3.3 Calculation of rms delay spread and coherence bandwidth

Coherence bandwidth (B_c), which is a statistical measure of the range of frequencies over which the two frequency components have a strong correlation, can be calculated from the averaged frequency correlation estimate. Coherence bandwidth B_c of level K is defined as $|\phi_H(B_c)| < K$ [9]. Popularly used values for K are 0.9 and 0.5. In our simulations we have used $K = 0.9$, since the estimated correlation for small Δ values are more reliable, as more data points are used to obtain these values. Figure 1 shows how to calculate Δ for a given K. Observe that as rms delay spread increases, coherence bandwidth decreases. Three different coherence bandwidth values and rms delay spread estimates for $K = 0.9$ are shown in the figure.

For given K and the corresponding Δ value, rms delay spread can be derived from (11) as

$$\tau_{rms} = \frac{\tau_0}{\log\frac{2-2K^2\cos\frac{2\pi\Delta}{N}+\sqrt{(2K^2\cos\frac{2\pi\Delta}{N}-2)^2-4(1-K^2)^2}}{2(1-K^2)}} \ . \tag{12}$$

Given channel frequency correlation we can set a value for K and calculate Δ. Then, we calculate τ_{rms} using (12) and the value of Δ.

An approximation to (12) is found in [7] in order to reduce the computational complexity. This approximation is in the form $\tau_{rms} = \frac{c}{\Delta}\tau_0$, and gives accurate and less complex results.

3.4 Effect of Additive Noise

Additive noise is one of the most common limiting factors for most algorithms in wireless communications. Effect of noise in the channel frequency correlation is given in (9), where it appears as a DC term whose magnitude depends on the ratio of the power of channel response to estimation error (SER). Therefore, knowing SER, we can remove that DC term easily with simple mathematical operations. A relation between SER and SNR is given in [10].

4 Instantaneous CIR Calculation

Channel frequency response (CFR) for an OFDM system can be calculated using DFT of time domain CIR. Assume that we have an L tap channel, and the value of lth tap for the mth OFDM symbol is represented by $h_m(l)$. Then we can find CFR as

$$H_m(k) = \frac{1}{N}\sum_{l=0}^{N-1} h_m(l)e^{-j2\pi kl/N} \quad 0 \leq k \leq N-1 \quad (13)$$

The above equation implies that if we know CIR, we can calculate CFR using DFT. We can do the reverse operation as well, *i.e.* we can calculate CIR from CFR with IDFT operation.

In almost all communication systems using coherent demodulation, the frequency response of the channel is estimated in order to equalize channel characteristic. We can use estimated CFR of received samples, (6), to calculate time domain CIR. This method is used in [11] to obtain the filter coefficients of channel estimation filter adaptively. However, this method requires IDFT with size equal to the number of subcarriers.

CFR can be sampled to reduce the computational complexity. In this case, we need to sample CFR according to Nyquist theorem in order to prevent aliasing in time domain. We can write this as

$$\tau_{max}\Delta f S_f \leq 1 \ , \quad (14)$$

where τ_{max} is maximum excess delay of the channel, Δf is subcarrier spacing in frequency domain, and S_f is the sampling interval. Note that the right hand side of the above equation is 1 and not 1/2. This is because PDP is nonzero between 0 and τ_{max}. We can represent frequency spacing in terms of OFDM symbol duration ($\Delta f = 1/T_u$), then we can re-write (14) as

$$\tau_{max} \leq \frac{T_u}{S_f} \ . \quad (15)$$

From the above equation by assuming worst case maximum excess delay, sampling rate can easily be calculated.

Using (6) and (13), estimate of CFR can be written as

$$\begin{aligned}\hat{H}_m(k) &= H_m(k) + w_m(k) \\ &= \frac{1}{n}\sum_{l=0}^{L-1} h_m(l)e^{-j2\pi kl/N} + w_m(k) \ , \quad (16)\end{aligned}$$

where $w_m(k)$ are independent identically distributed complex Gaussian noise variables. Note that we have replaced the upper bound of summation with $L-1$ since $h_m(l)$ is zero for $l \geq L$.

The CFR estimate is sampled with a spacing of S_f. Therefore, the sampled version of the estimate can be written as

$$\begin{aligned}\hat{H}'_m(k) &= \frac{1}{N}\sum_{l=0}^{L-1} h_m(l)e^{-j2\pi(S_f k)l/N} \\ &+ \quad w_m(S_f k) \quad 0 \leq k \leq \frac{N}{S_f} - 1 \ . \quad (17)\end{aligned}$$

Without loss of generality, we can assume $\frac{N}{S_f} = L$. Now, we can find the CIR by taking IDFT of the sampled estimate CFR. IDFT size is reduced from N to N/S_f by using sampling. As a result of this reduction, the complexity of the IDFT operation will decrease at least S_f times. For wireless LAN (IEEE 802.11a), for example, the worst case scenario S_f would be 4 (assuming a maximum excess delay equal to guard interval ($0.8\mu s$)), which decreases original complexity at least 75 percent.

Let us apply IDFT of size $N/S_f = L$ to (17) to obtain the estimate of CIR. We have;

$$\begin{aligned}\hat{h}_m(l) &= IDFT\left\{\frac{1}{N}\sum_{n=0}^{L-1} h_m(n)e^{-j2\pi kn/L} + w_m(S_f k)\right\} \\ &= h_m(l) + w'_m(l) \ , \quad (18)\end{aligned}$$

where $w'_m(l)$ is the IDFT of the noise samples.

Equation 18 gives the instantaneous CIR. Having this information, we can calculate the PDP by averaging the magnitude of instantaneous CIR over OFDM symbols.

Channel estimation error will result in additive noise on the estimated CIR. The signal-to-estimation error ratio for CIR will be equal to signal-to-estimation error ratio for CFR since IDFT is a linear operation.

5 Performance Results

Simulation results are obtained in an OFDM based wireless communication system with 64 subcarriers. The delay between multipath components are assumed to be equal to data symbol period. Figure 2 shows the difference between the frequency correlation estimates and ideal correlation values for different rms delay spread values. An exponentially decaying PDP is assumed in this figure. As can be seen, the correlation estimates are very close to the ideal correlation values.

Performance of instantaneous CIR estimation depends on the sampling rate. As sampling rate increases mean square error of the estimates will decrease because of noise averaging. This is shown in Figure 3. Since we have an 8-tap channel, Nyquist sampling period will be $S_f = 8$. This figure shows the normalized mean squared error for different sampling periods. Results are given for unsampled ($S_f = 1$), oversampled ($S_f = 4$), Nyquist rate ($S_f = 8$) and undersampled ($S_f = 10$) cases. Note that if we sample the CFR below Nyquist rate, we get an error floor. This is because of aliasing in time domain due to undersampling.

In order to obtain the PDP from CIR, we need to average the magnitude of CIR. Figure 4 shows ideal and estimated PDPs for different numbers of OFDM frames over which we perform averaging. In this figure, CFR is sampled at Nyquist rate. As number of averages increases estimation error decreases as expected.

Figure 5 shows the performance of the PDP estimation under different signal-to-estimation error ratios (SER). Channel estimation error causes distortion on the channel taps which has low energy. For channel taps with high energy, absolute value function changes sign depending on the signal which makes averaging over noise. Since mean of the noise is zero, effect of noise is very small. However, for taps with low energy absolute value of the noise is averaged causing distortion. We sampled CFR at Nyquist rate to obtain these results. If we sample CFR above Nyquist rate, it reduces the effect of the estimation error.

6 Conclusion

Different methods to estimate the CIR, PDP and rms delay spread of the wireless communication channel are investigated. A practical algorithm for averaged channel frequency correlation estimation has been presented. Average rms delay spread of the channel is calculated from the channel frequency correlation estimate. For measuring local frequency selectivity, CFR estimate is exploited. Time domain CIR is obtained by taking IFFT of the sampled CFR. The optimal sampling rate for sampling the channel response is investigated and simulation results for different sampling rates are given. PDP is obtained by averaging the estimated CIR. The performance of the estimates are obtained in noise limited situations using Monte Carlo simulation. For local parameter estimation, the effect of channel estimation error is studied.

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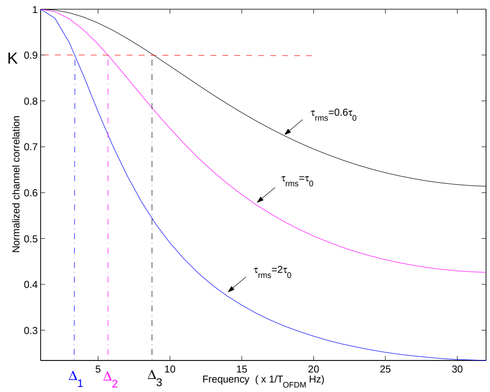

Figure 1: Estimation of coherence bandwidth B_c of level K from absolute correlation estimates, corresponding to different rms delay spread values.

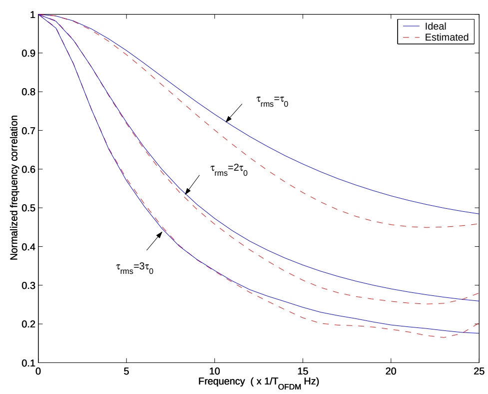

Figure 2: Comparison of the estimated frequency correlation with the ideal correlation for different rms delay spread values.

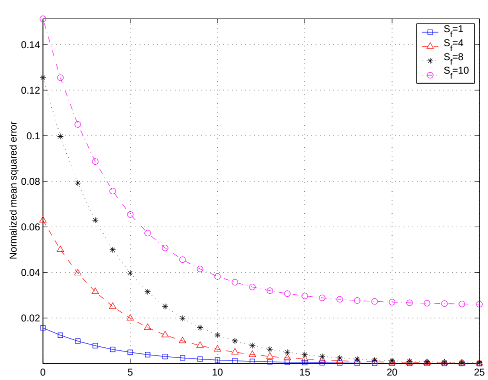

Figure 3: Normalized mean squared error versus signal-to-estimation error ratio for different sampling intervals.

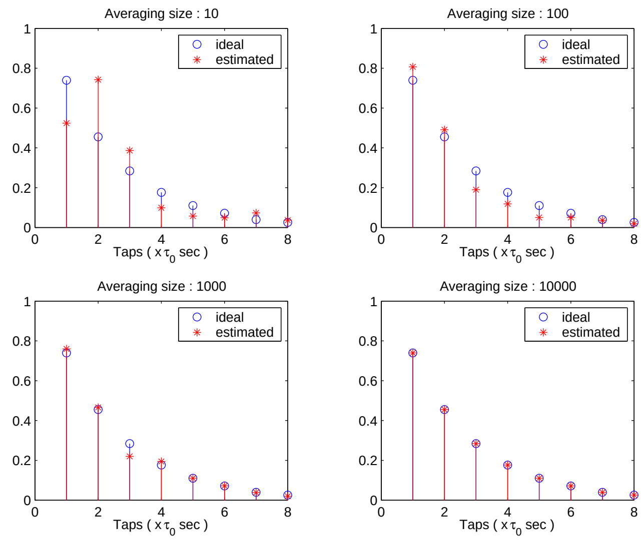

Figure 4: Results for different number of averaging over OFDM frames. CFR is sampled at Nyquist rate and ideal CFR is used.

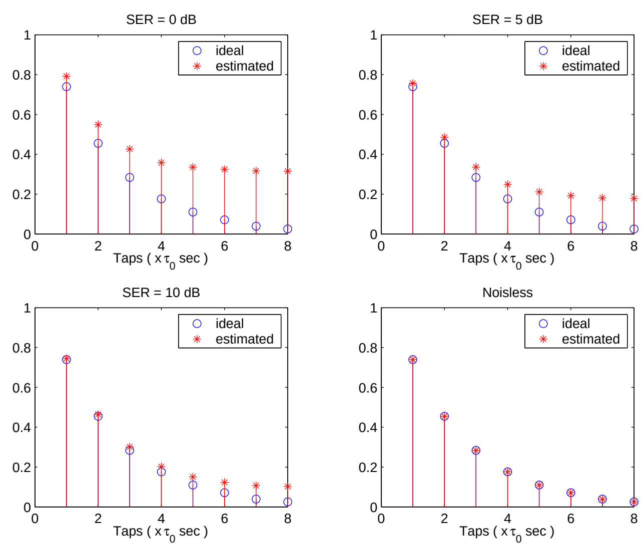

Figure 5: Ideal and estimated PDPs for different signal-to-estimation error ratio values. CFR is sampled at Nyquist rate and 10000 OFDM frames is used for averaging.