1. Decomposition of ozone in water.

2. How long to remove 99% of ozone in a batch reactor?

\[ C_0 = 4 \text{ mg/L} \times \frac{1 \text{ L}}{1000 \text{ mg}} = \frac{1 \text{ mol O}_3}{48 \text{ g O}_3} = 8.3 \times 10^{-5} \text{ mol/L} \]

99% removal \( \Rightarrow C_{\text{final}} = 0.001 C_0 = 8.3 \times 10^{-7} \text{ mol/L} \)

For 2nd-order kinetics in a batch reactor,

\[ \frac{1}{C} = \frac{1}{C_0} + k_2 t \]

\[ \frac{1}{8.3 \times 10^{-7} \text{ mol/L}} = \frac{1}{8.3 \times 10^{-5} \text{ mol/L}} + k_2 t \]

\[ 1,200,000 \text{ L/mol} = 12,000 \text{ L/mol} + k_2 t \]

\[ t = \frac{1,200,000 \text{ L/mol} - 17,300 \text{ L/mol}}{14 \text{ L/mol} \cdot s} = 85,000 \text{ s} = 1414 \text{ min} = 24 \text{ hr} \]

The ozone will be 99% gone in 24 hr.

(b) Ozone in a PFR drops from 4 mg/L to 1 mg/L.

A PFR is like a batch reactor or a conveyor belt.

Batch reactor: \( \frac{1}{C} = \frac{1}{C_0} + k_2 t \)

Therefore PFR: \( \frac{1}{C_E} = \frac{1}{C_2} + k_2 C \)

So, this problem is just like part (a)!

\[ \frac{1}{C_E} = \frac{1}{8.3 \times 10^{-5} \text{ mol/L}} + (14 \text{ L/mol} \cdot s)(80 \text{ min})(60 \frac{1}{1 \text{ min}}) \]

\[ \frac{1}{C_E} = 37,200 \text{ L/mol} \Rightarrow C_E = 2.69 \times 10^{-5} \text{ mol/L} \]

\[ C_E = 2.69 \times 10^{-5} \text{ mol/L} \times \frac{48 \text{ g O}_3}{1 \text{ mol O}_3} \times \frac{1000 \text{ mg}}{1 \text{ g}} = 1.29 \text{ mg/L} \]

We should be ok -- the effluent concentration of ozone is greater than 1 mg/L -- thus disinfection should be effective.
Ozone in a CMFR

\[ \frac{Q}{C_z} \rightarrow c_0 \rightarrow \frac{Q}{C_e} \]

Quantity balanced = mass of ozone
Control volume = reactor
Accumulation = Flow in - Flow out + Sources - Sinks
Steady-state \(\Rightarrow\) accumulation = 0

\[ 0 = Q \cdot C_z - Q \cdot C_e + 0 - V \cdot r \]
Second-order kinetics, \(\Rightarrow\) \(r = k_2 C^2\)

\[ 0 = Q \cdot C_z - Q \cdot C_e - V \cdot k_2 C_e^2 \]
CMFR \(\Rightarrow\) \(C_e = C\) (conc. leaving reactor = conc. in the reactor)

\[ 0 = Q \cdot C_z - Q \cdot C_e - V \cdot k_2 \cdot C_e^2 \]

(b) Solve for exit conc. of ozone

\[ 0 = Q \cdot C_z - Q \cdot C_e - V \cdot k_2 \cdot C_e^2 \]

\[ V \cdot k_2 \cdot C_e^2 + Q \cdot C_e - Q \cdot C_z = 0 \]

Quadratic in \(C_e\)
Use quadratic formula

\[ C_e = \frac{-Q \pm \sqrt{Q^2 + 4V \cdot k_2 \cdot Q \cdot C_z}}{2V \cdot k_2} \]

But only the "+" root will be a positive number and therefore physically meaningful.

So \( C_e = \frac{-Q + \sqrt{Q^2 + 4V \cdot k_2 \cdot Q \cdot C_z}}{2V \cdot k_2} \)
\section{Find effluent concentration}

Take answer from part (d) and divide both numerator, denominator by \( Q \).

Then\[ C_e = \frac{-1 + \sqrt{1 + 4 \frac{2}{k_2 C_I}}}{2 k_2 C_I} \]

\[ C_e = \frac{-1 + \sqrt{1 + (4)(30 \text{ min}) (60 \text{ s/min}) (14 \text{ \ell/min}) (8.3 \times 10^5 \text{ m}^3/\ell)}}{2 (14 \text{ \ell/min}) (30 \text{ min}) (60 \text{ s/min})} \]

\[ C_e = 4.1 \times 10^{-5} \text{ mol/L} = 1.97 \text{ m}^3/\ell \approx 2.0 \text{ m}^3/\ell} \]

\textbf{Note:} By comparing answers from parts (c) and (d), we can see that more ozone is converted in the PFR than in the CMFR, given the same average hydraulic residence time. This is what we would expect. For 1\textsuperscript{st}-order kinetics or higher, the PFR is more efficient than the CMFR because the CMFR has dilution which suppresses the reaction rate.
2. Coagulation and Floculcation

a) Find number concentration of particles entering the lake.

Particles are spheres

Volume of the particle = \( \frac{4}{3} \pi d_p^3 = \frac{4}{3} \pi (2.0 \times 10^{-6} \text{ m})^3 = 4.19 \times 10^{-18} \text{ m}^3 \)

Mass of one particle = \( \rho V_p = (2600 \text{ kg/m}^3)(4.19 \times 10^{-18} \text{ m}^3) = 1.09 \times 10^{-14} \text{ kg} \)

\( (N_0 \text{ in} \frac{\#}{\text{m}^3})(1.09 \times 10^{-14} \text{ kg}) = \left(10 \frac{\text{m}^3}{\text{L}}\right) \left(\frac{1000 \text{ L}}{1 \text{ m}^3}\right) \left(\frac{1 \text{ kg}}{10^6 \text{ m}^3}\right) \)

\( N_0 = 9.2 \times 10^7 \text{ particles/m}^3 \)

b) What transport mechanism dominates flocculation?

Based on the given information, it appears that the particle suspension is monodisperse.

Therefore, differential settling will be negligible -- \( \beta_m = 0 \).

That leaves microscale flocculation or macroscale flocculation.

Microscale:

\( \beta_m = \frac{8}{3} \frac{k T}{\mu} \)

\( = \frac{8}{3} \frac{(1.38 \times 10^{-23} \text{ J/K})(18 + 273 \text{ K})}{1.06 \times 10^{-3} \text{ Pa.s}} = 1.0 \times 10^{-17} \text{ m}^3/\text{s} \)

Macroscale:

\( \beta_m = \frac{4}{3} \frac{G d_p^3}{\sqrt{p/\rho}} \)

\( G = \sqrt{\frac{p}{\rho \mu}} = \sqrt{\frac{p/\mu}{1.06 \times 10^{-3} \text{ Pa.s}}} = 9.71 \text{ s}^{-1} \)

\( \beta_m = \frac{4}{3} (9.71 \text{ s}^{-1})(2.0 \times 10^{-6} \text{ m})^3 = 1.0 \times 10^{-16} \text{ m}^3/\text{s} \)

Macroscale flocculation dominates -- it is 10 times more important than microscale flocculation.
What is concentration of particles exiting the lake?

For macro scale flocculation of monodisperse suspension,
\[
\Gamma = \frac{4}{\pi} \Omega C \alpha \eta N
\]

First order kinetics

If lake is well mixed, treat as a CMFR

\[
\frac{N_E}{N_i} = \frac{1}{1 + \frac{4}{\pi} \Omega C \alpha \eta \tau}
\]

... assume \( \alpha = 1.0 \) because "completely destabilized"

What is \( \Omega \)?

\[
\Omega = N_v V_p = (9.18 \times 10^9 \text{ particles/m}^3)(4.19 \times 10^{-14} \text{ m}^3/\text{particle}) = 3.85 \times 10^{-6}
\]

\[
\frac{N_E}{N_i} = \frac{1}{1 + \left(\frac{4}{\pi}\right)(3.85 \times 10^{-6})(9.71 \times 10^{-5})(1.0)(30 \text{ d})(24 \text{ hr/d})(60 \text{ min/hr})(60 \text{ sec/min})}
\]

\[
\frac{N_E}{N_i} = 0.008 \quad \text{-- we get 99.2% reduction in number concentration -- good}
\]

\[
N_E = (0.008)(9.18 \times 10^9 \text{ particles/m}^3) = 7.4 \times 10^8 \text{ particles/m}^3
\]

Why might it be a bad idea to add a coagulant?

I can think of a couple problems. One is that alum and/or ferric chloride coagulants consume alkalinity because they form strong acids when they hydrolyze. Hence you might acidify the lake if you add a coagulant. Also, these coagulants produce a lot of sludge in the form of amorphous Fe(OH)₃ or Al(OH)₃ solids. This sludge might settle to the bottom of the lake and you'd start building up a lot of sludge over time. Also, although the coagulants aid in flocculation and eventually in improved settling, the Al(OH)₃ or Fe(OH)₃ flocs create a lot of suspended solids which would, in the short term, decrease water quality -- by blocking light penetration, for instance. So overall I'd leave the coagulants to engineered systems at a treatment plant, but not a lake.
3. Sedimentation

(4) What floc density did Stu Domt use in his calculations? Stu predicted 90% removal at \( V_{OF} = 25 \text{ gal/min} \).

So Stu must have \( \frac{V_s}{V_{OF}} = 0.9 \Rightarrow V_s = 0.9 \times V_{OF} \).

Let's put \( V_{OF} \) in useful units:

\[
25 \frac{\text{gal}}{\text{min}} \times \left( \frac{1 \text{ ft}^3}{0.5048 \text{ m}^3} \right)^2 \times \frac{3.785 \text{ L}}{1 \text{ m}^3} \times \frac{1 \text{ m}^3}{1000 \text{ L}} = 1.02 \text{ m/min} \]

\[
= 0.017 \text{ m/s}
\]

Stu has \( V_s = 0.9 \times V_{OF} = 0.9(0.017 \text{ m/s}) = 0.0153 \text{ m/s} \)

\[
\frac{g (\rho_f - \rho_L) d_p^2}{18 \mu} = 0.0153 \text{ m/s} \text{ according to Stu}
\]

\[
\frac{(4.81 \text{ m/s}^2)(\rho_f - 998.5 \text{ kg/m}^3)(4 \times 10^{-4} \text{ m})^2}{(18)(1.06 \times 10^3 \text{ Pa.s})} = 0.0153 \text{ m/s}
\]

\[
\rho_p = 998.5 \text{ kg/m}^3 + 185.7 \text{ kg/m}^3 = 1184 \text{ kg/m}^3
\]

(6) Demonstrate that Stokes' law is not valid.

\[
Re = \frac{\rho_v V_s d_p}{\mu} = \frac{(998.5 \text{ kg/m}^3)(0.0153 \text{ m/s})(4 \times 10^{-4} \text{ m})}{1.06 \times 10^3 \text{ Pa.s}} = 5.76
\]

\( Re > 1 \) and \( Re > 2 \) and \( Re > 5 \) ... Stokes' law not valid.

It's not way out of range ... not like \( Re = 100 \) or so ... but even at \( Re = 5.8 \) I'd say Stokes' law is not valid.
Will Stu's design perform better or worse than he predicted?

Will $v_s$ be higher or lower than Stu predicted?

If $v_{s_{\text{est}}}/v_s > 1$, then the system will perform even better than expected.

But if $v_{s_{\text{est}}}/v_s < 1$, then the system will perform worse than predicted.

Can we get a better estimate of $v_s$?

Various ways to do this... one is to say $C_D \approx 18.5 (Re)^{-0.4}$ in the transition region.

Stu used Stokes' law, $C_D = \frac{24}{Re} = \frac{24}{5.76} = 4.16$

But a better estimate is $C_D = 18.5 (Re)^{-0.4} = 0.47$

So the actual drag coefficient might be about 1.56 times higher than Stu's.

And $v_s \propto \frac{1}{\sqrt{C_D}}$ so the actual $v_s$ might be about $\frac{1}{1.56}$ times Stu's value.

$v_s$ might be $\approx 0.0123 \text{ m/s}$ not $0.0153 \text{ m/s}$

Thus actual removal might be around $\frac{0.0123 \text{ m/s}}{0.017 \text{ m/s}} = 0.72 = 72\%$

(You could get an even better estimate of $v_s$ by iterating or by using the formula derived on homework #4... but this is pretty good.)

I'd expect removal closer to 70% than 90% for the flux of interest.