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2.1) (a) Let \( z_1 = \frac{100 + j100}{50} = 2 + j2 \)

\( z_1 \) is shown in the Smith chart

(b) \( y_1 = \frac{1}{z_1} = 0.25 - j0.25 \)

\( y_1 \) is shown in the Smith chart

(c) In the \( z \)y Smith chart the location of \( z_1 \) is the same as in (2). The value of \( y_1 \) is read from the green circles.

(d) \( z_2 = \frac{50 - j100}{50} = 1 - j2 \)
\( y_2 = \frac{1}{z_2} = 0.2 + j0.4 \)

\( z_3 = \frac{25 - j25}{50} = 0.5 - j0.5 \)
\( y_3 = \frac{1}{z_3} = 1 + j \)

\( z_4 = \frac{j50}{50} = j \)
\( y_4 = \frac{1}{z_4} = -j \)
\( y_5 = \infty \)

\( z_2, z_3, z_4, \) and \( z_5 \) are shown in the Smith chart

2.2) (a) \( z = -r + jx \), \( \Gamma = \frac{z - 1}{z + 1} = \frac{-r + jx}{1 - r + jx} \)  \hspace{1cm} (1)

\[ |\Gamma| = \frac{\sqrt{r^2 + x^2}}{\sqrt{(1-r)^2 + x^2}} = \left[ \frac{r^2 + 2y + 1 + x^2}{(1-y)^2 + x^2} \right]^{1/2} \]  \hspace{1cm} (2)

In (1) the numerator is greater than the denominator, since \( 2r > -2r \). Hence, \(|\Gamma| > 1\)

(b) From (1): \[ \frac{1}{\Gamma} = \frac{(1-r) - jx}{(1-r) - jx} = \frac{(y-1) + jx}{(y+1) - jx} \]  \hspace{1cm} (3)

Eq. (3) is identical to the transformation in (2.2.2) where \( z = y + jx \) with \( y > 0 \). Hence, negative resistances can be handled in the Smith chart by plotting \( \frac{1}{\Gamma} \) and interpreting the resistance circles as being negative, and the reactance circles as marked.
(c) \[ Z_1 = \frac{Z_0}{50} = \frac{-20 + j16}{50} = -0.4 + j0.32 \]

From the Smith chart we read:
\[ \frac{1}{\Gamma^*} = 0.47 \angle 139^\circ \]

Hence \( \Gamma = 2.1 \angle 139^\circ \)

For:
\[ Z_2 = \frac{-200 + j25}{50} = -4 + j0.5 \]
\[ \frac{1}{\Gamma^*} = 0.61 \angle 3.75^\circ \]

Hence \( \Gamma = 1.6 \angle 3.75^\circ \)

(d) See the compressed Smith chart. For \( Z_1 = -0.4 + j0.32 \) we read: \( |\Gamma| = 2.1, \Gamma = 139^\circ \).

Hence \( \Gamma = 2.1 \angle 139^\circ \).

Similarly, for \( Z_2 \) we obtain \( \Gamma = 1.6 \angle 3.75^\circ \)

(e) \( |\Gamma| \approx 3.2 \)

2.3) \[ Z(d) = Z_0 \frac{Z_0 + jZ_0 \tan \beta d}{Z_0 + j \frac{Z_0 \tan \beta d}{Z_0 + jZ_0 \tan (\beta d + \eta d / 2)}} \]

\[ Z(d + \eta d / 2) = Z_0 \frac{Z_0 + j Z_0 \tan (\beta d + \eta d / 2)}{Z_0 + j \frac{Z_0 \tan (\beta d + \eta d / 2)}{Z_0 + jZ_0 \tan (\beta d + \eta d / 2)}} \]

\[ \tan (\beta d + \eta d) = \tan \beta d \]

Hence, \[ Z(d + \eta d / 2) = Z(d) \]
2.4) **From (2.2.6) and (2.2.7):** Let \( \Gamma_0 = |\Gamma_0| e^{j\phi} \) then
\[
\Gamma = \Gamma_0 e^{-j2\beta d} = (|\Gamma_0| e^{j(\phi_0 - 2\beta d)}) = |\Gamma| e^{j\phi} \quad \text{where} \quad |\Gamma| = |\Gamma_0| \\
\phi = \phi_0 - 2\beta d
\]

\[
Z(d) = Z_0 \left[ \frac{1 + |\Gamma| e^{j\phi}}{1 - |\Gamma| e^{j\phi}} \right] = Z_0 \left[ \frac{1 + |\Gamma| \cos \phi + j|\Gamma| \sin \phi}{1 - |\Gamma| \cos \phi - j|\Gamma| \sin \phi} \right] \]

or
\[
Z(d) = Z_0 \left[ \frac{(1 - |\Gamma|^2 \cos^2 \phi) + j2|\Gamma| \sin \phi}{(1 - |\Gamma|^2 \cos^2 \phi - (|\Gamma| \sin \phi)^2)} \right] = R(d) + jX(d)
\]

**Hence:**
\[
R(d) = Z_0 \frac{1 - |\Gamma|^2}{1 - 2|\Gamma| \cos \phi + |\Gamma|^2} \quad X(d) = Z_0 \frac{2|\Gamma| \sin \phi}{1 - 2|\Gamma| \cos \phi + |\Gamma|^2}
\]

Also, from (1)
\[
12(d) = Z_0 \left[ \frac{(1 + |\Gamma| \cos \phi)^2 + (|\Gamma| \sin \phi)^2}{(1 - |\Gamma| \cos \phi)^2 + (|\Gamma| \sin \phi)^2} \right] = Z_0 \left[ \frac{1 + 2|\Gamma| \cos \phi + |\Gamma|^2}{1 - 2|\Gamma| \cos \phi + |\Gamma|^2} \right]^{1/2}
\]

\[
\theta_d = \tan^{-1} \frac{X(d)}{R(d)} = \tan^{-1} \left[ \frac{2|\Gamma| \sin \phi}{1 - |\Gamma|^2} \right]
\]

2.5)

```
\[
\begin{aligned}
\text{at } &0.25\lambda \\
\text{ZO}=50 &\quad Z_L=50+j100 \\
\text{at } &0.187\lambda \\
Z_{IN}= &50+j100 = 1+j2
\end{aligned}
\]
```

**Locate** \( Z_L \) **in the Smith chart.**

**At** \( Z_L \) **read 0.187\lambda.** **Draw a constant** \( |\Gamma| \) **circle.** \( Z_{IN} \) **is at** 0.25\lambda **from** \( Z_L \) **(or 0.187\lambda + 0.25\lambda = 0.437\lambda).**

**Hence:***
\[
\begin{aligned}
Z_{IN}= &0.21-j0.41 \\
Z_{IN}= &50 \times j100 = 10.5-j20.5 \Omega \\
\Gamma_0= &0.707 \quad 45^\circ \\
VSWR = &1+0.707 = 5.83
\end{aligned}
\]

2.21
2.6) (a) \( |V(d)|_{\text{max}} = |A_i| \left(1 + |\Gamma_0|\right) \) and \( |I(d)|_{\text{min}} = \frac{|A_i|}{Z_0} \left(1 - |\Gamma_0|\right) \)

Hence: \( R(d)_{\text{max}} = \frac{|V(d)|_{\text{max}}}{|I(d)|_{\text{min}}} = \frac{Z_0 \left(1 + |\Gamma_0|\right)}{1 - |\Gamma_0|} \)

\( r(d)_{\text{max}} = \frac{R(d)_{\text{max}}}{Z_0} = \frac{1}{VSWR} \)

(b) \( |V(d)|_{\text{min}} = |A_i| \left(1 - |\Gamma_0|\right) \) and \( |I(d)|_{\text{max}} = \frac{|A_i|}{Z_0} \left(1 + |\Gamma_0|\right) \)

Hence: \( R(d)_{\text{min}} = \frac{|V(d)|_{\text{min}}}{|I(d)|_{\text{max}}} = \frac{Z_0 \left(1 - |\Gamma_0|\right)}{1 + |\Gamma_0|} \)

\( r(d)_{\text{min}} = \frac{R(d)_{\text{min}}}{Z_0} = \frac{1}{VSWR} \)

2.7) (a) \( Z_{\text{IN}} = -\frac{j 25}{50} = -j 0.5 \)

From the Z Smith chart: \( \lambda = 0.426 \lambda \)

(b) \( Y_{\text{IN}} = j 2 \)

From the Y Smith chart: \( \lambda = 0.176 \lambda \)
2.8) The frequency response follows a constant conductance circle with \( g = 1 \). The equivalent circuit is (see Fig. 2.3.2b)

\[
Y = \frac{1}{R} - \frac{j}{WL}
\]

\[
y = \frac{Y}{Y_0} = \frac{Z_0}{R} - \frac{j}{WL}
\]

\[
g = 1 = \frac{Z_0}{R} \Rightarrow R = 50 \Omega
\]

At \( f_b = 1 \text{GHz} \):

\[-j \frac{50}{0.5(2\pi \times 10^9)} = -j0.5 \]

\[
L = \frac{50}{0.5(2\pi \times 10^9)} = 15.9 \text{nH}
\]

Observe that at \( f_a = 500 \text{MHz} \):

\[-j \frac{50}{\omega_a L} = -j1\]

2.9) (a) \( y = 0.4 \), \( R = 50 \Omega = 50(0.4) = 20 \Omega \)

At \( f_a \), the reactance is \( -j0.6 \); and at \( f_b \) is \( -j0.32 \).

\[
Z = 20 - \frac{j}{\omega C}
\]

\[
\therefore \frac{1}{\omega C} = 0.6(50) = 30 \quad (C = 50 \text{ pF})
\]

Or \( f_a = \frac{1}{2\pi(30) \times 50 \times 10^{-12}} = 106.1 \text{ MHz} \)

And \( \frac{1}{\omega_b C} = 0.32(50) = 16 \Rightarrow f_b = \frac{1}{2\pi(16) \times 50 \times 10^{-12}} = 198.9 \text{ MHz} \)

(b) The equivalent circuit is a series \( R, L, C \) circuit.

At \( f_a = 500 \text{ MHz} \), \( Z = 20 + j10 \)

\[
\therefore 20 + j10 = R + j(\omega_a L - \frac{1}{\omega_a C}) \quad (1)
\]

At \( f_b = 1 \text{ GHz} \), \( Z = 20 + j30 \)

\[
\therefore 20 + j30 = R + j(\omega_b L - \frac{1}{\omega_b C}) \quad (2)
\]

From (1) and (2) it follows that \( R = 20 \Omega \) and

\[
10 = 2\pi(f_a L - \frac{1}{f_a C}) \quad \text{and} \quad 30 = 2\pi(f_b L - \frac{1}{f_b C})
\]

The simultaneous solution of these equations is:

\[
L = 5.31 \text{nH}
\]

\[
C = 47.64 \text{ pF}
\]
SHUNT C: \( y_c = -j0.46 - (-j1.19) = j0.73 \)
\( Z_c = \frac{50}{jy_c} = -j68.5 \, \Omega \)

SERIES C: \( z_c = 0 - j1.6 = -j1.6 \)
\( Z_c = 50 z_c = -j80 \, \Omega \)

\( Z_{IN} = 50 \, \Omega \)

SHUNT C: \( y_c = j0.45 - (-j1.19) = j1.64 \)
\( Z_c = \frac{50}{jy_c} = -j30.5 \, \Omega \)

SERIES L: \( z_l = 0 - (-j1.6) = j1.6 \)
\( Z_l = 50 z_l = j80 \, \Omega \)

\( Z_{IN} = 50 \, \Omega \)

SERIES C: \( z_c = j0.4 - 0.8 = j0.4 \)
\( Z_c = 50 z_c = -j20 \, \Omega \)

SHUNT C: \( y_c = 0 - (-j2) = j2 \)
\( Z_c = \frac{50}{j2} = -j25 \, \Omega \)

\( Z_{IN} = 50 \, \Omega \)
\[ y_L = \frac{Y_L}{Y_0} = Z_0 Y_L = 50(4 - j4) \times 10^{-3} = 0.2 - j0.2 \]

\[ y_L = -j0.2 \]

\[ y_L = -j0.2 - j0 = -j0.2 \]

\[ Z_L = 50 \frac{y_L}{y_0} = 83.3 \Omega \]

\[ Z_L = j \omega L = 83.3 \]

\[ L = \frac{83.3}{2 \pi 700 \times 10^6} = 18.9 \ \text{nH} \]

\[ Z_C = -jL = -j100 \]

\[ C = \frac{1}{100(2 \pi 700 \times 10^6)} = 2.27 \ \text{pF} \]
2.12) ONLY CIRCUIT (b) CAN MATCH $Y_L = 8-j12 \ \text{mS}$ TO $Z_{in} = 50 \Omega$.

**Series C:**
- $Z_C = -j0.011 - j1.15 = -j1.56$
- $Z_C = 50 \Omega$

**Capacitor C:**
- $C = \frac{1}{\omega (18) 2 \pi 10^9 (18)} = 2.04 \mu F$

**Shunt L:**
- $\frac{Z_L}{Y_L} = 0 - j0.52 = -j0.52$
- $Z_L = 50 \Omega$

**Inductor L:**
- $L = \frac{96.15}{\omega} = \frac{96.15}{2 \pi 10^9} = 15.3 \ \text{H}$

$$ Y_L = \frac{1}{j \omega L} = \frac{1}{j 10^9 50 \times 10^9} = -j100 \ \text{or} \ \frac{3}{2} \frac{1}{\frac{50}{50}} = \frac{3}{2} \ \text{or} \ \frac{3}{2} \frac{1}{\frac{100}{50}} = \frac{3}{2} $$

$$ Z_{in} = 50(1+j2) = 50 + j100 \ \Omega $$

$$ Z_{in} = 1 + j2 $$

$$ Z_{in} = 50 Z_{in} = 50 + j100 \ \Omega $$

2.13) 

$$ Z_L = j \omega L = j 10^9 50 \times 10^9 = j50 \ \text{or} \ \frac{Z_L}{Y_L} = \frac{1}{j 10^9 \times 10^9} = -j100 \ \text{or} \ \frac{3}{2} \frac{1}{\frac{50}{50}} = \frac{3}{2} \ \text{or} \ \frac{3}{2} \frac{1}{\frac{100}{50}} = \frac{3}{2} $$
2.14) \text{LET } Z_0 = 100 \Omega.
\text{Then } Z_L = \frac{Z_L}{100} = 1 - j
\text{AND } Z_{IN} = \frac{Z_{IN}}{100} = 0.25 + j0.25
\text{SHUNT C: } \gamma_c = \sqrt{131} - j0.5 = j1.263
Z_c = \frac{100}{\gamma_c} = -j1.263 \\Omega
\text{SERIES L: } Z_L = j0.25 - (j0.67) = j0.92
Z_L = 100Z_L = j92 \Omega

\begin{align*}
Z_{IN} &= 25 + j25 \\Omega \\
&= (0.25 + j0.25)
\end{align*}

2.15) (a) \text{Z_L = } \frac{50}{50} = 1
\text{Z_{IN} = } \frac{20 + j20}{50} = 0.4 + j0.4
\text{Draw the Q=5 circles (see Fig. 2.4.16)}
\text{The motion from A to B -- SERIES \text{L}_1:}
\text{At B: } Z_B = 1 + j3
Z_L = -j3 \text{ or } Z_L = j3(50) = j150 \Omega
\text{The motion from B to C -- SHUNT C:}
\text{At B: } \gamma_B = 0.1 - j0.3
\text{At C: } \gamma_c = 0.1 + j0.5
\gamma_c = j0.5 - (j0.3) = j0.8
Z_c = \frac{50}{j0.8} = -j62.5 \Omega
\text{The motion from C to D -- SERIES \text{L}_2:}
\text{At C: } Z_C = 0.4 - j1.9
\text{At D: } Z_D = 0.4 + j0.4
Z_L = j0.4 - (j1.9) = j2.3
Z_L = 50(j2.3) = 115 \Omega

\text{Z_{IN} = 20 + j20 \Omega}

2-27
(b) $Z_L = 1$ and $Z_{in} = 0.5$

At B: $g_B = 1 - j2.6$, $Z_B = 0.13 + j0.335$
At C: $g_C = 2 - j3.4$, $Z_C = 0.13 + j0.215$
At D: $g_D = 2$, $Z_D = Z_{in} = 0.5$

Shunt L: $Z_L = -j2.6$
$Z_L = \frac{50}{-j2.6} = j9.2\Omega$

Series C: $Z_C = j0.215 - j0.335 = -j0.12$
$Z_C = 50(-j0.12) = -j6\Omega$

Shunt C: $Z_C = 0 - (j3.4) = j3.4$
$Z_C = \frac{50}{j3.4} = -j14.7\Omega$

$Z_{in} = 25\Omega$

2.16) (a) From Fig. 2.5.2 with $Z_0 = 50\Omega$ and $\varepsilon_r = 2.23$ we obtain:

$$\frac{W}{h} \approx 3.1 \quad \text{or} \quad W = 3.1(0.7874) = 2.44 \ \text{mm}$$

From Fig. 2.5.3 with $\frac{W}{h} = 3.1$ and $\varepsilon_r = 2.23$ we obtain:

$$\frac{\lambda}{\lambda_{TEM}} = 1.08 \quad \text{or} \quad \frac{\lambda}{\lambda_{TEM}} = 1.08 \frac{\lambda_0}{\sqrt{2.23}} = 0.723 \lambda_0$$

Since $\lambda = \frac{\lambda_0}{\sqrt{\varepsilon_{eff}}}$, then $\frac{1}{\sqrt{\varepsilon_{eff}}} = 0.723$ or $\varepsilon_{eff} = 1.91$

(b) From (2.5.11): $\frac{W}{h} = \frac{2}{\pi} \left[ B - 1 - \frac{1}{2}(B-1)^2 + \frac{2.23-1}{2(2.23)} \left[ \frac{\lambda_0}{(B-1) + 0.39 + 0.61}{2.23} \right] \right] \quad (1)$

Where $B = \frac{377\pi}{2(50)(2.23)} = 7.931 \quad (2)$

Substitute (2) into (1) to obtain: $\frac{W}{h} = 3.073$

From (2.5.8): $\frac{\lambda_0}{\sqrt{2.23}} \left[ \frac{2.23}{1 + 0.63(2.23-1)(3.073)0.1285} \right] = 0.724 \lambda_0$

$\frac{1}{\sqrt{\varepsilon_{eff}}} = 0.724$ or $\varepsilon_{eff} = 1.91$

2 - 28
2.17) **Approximate Values can be obtained from Figs. 2.5.2 and 2.5.3.**

**Exact values are given in Fig. 2.5.4.**

\[
\begin{align*}
\frac{W}{h} &= 1.5 \quad \text{or} \quad W = 1.5(25) = 37.5 \text{ mils} \\
\end{align*}
\]

From Fig. 2.5.3: \( \lambda = 1.18 \quad \text{or} \quad \lambda = 1.18 \frac{\lambda_{TEM}}{f} = 1.18 \frac{310^6}{V6 \cdot 10^9} = 14.45 \text{ cm} \)

\[
\begin{align*}
\frac{Z}{4} &= 7.0 \\
\frac{Z}{50} &= 62.8 = \frac{1.256}{4} \\
\text{Hence:} \quad \lambda &= 0.393 \lambda \\
\lambda &= 0.393(14.45) = 5.68 \text{ cm} \\
\text{or} \quad \lambda &= 5.68 \left( \frac{1000}{25.4} \right) = 2236.2 \text{ mils}
\end{align*}
\]

2.18) **At** \( f = 1 \text{ GHz} \), \( \lambda_0 = 30 \text{ cm} \). **Microstrip:** \( \varepsilon_r = 2.23, h = 0.7874 \text{ mm} \)

**Line with** \( Z_0 = 29.9 \Omega \) and \( \lambda = \frac{\lambda_0}{4} \); \( B = \frac{377 \pi}{2(29.9)2.23} = 13.26 \)

\[
\begin{align*}
\frac{W}{h} &= \frac{2}{\pi} \left[ 13.26 - 1 - \ln(2(12.26) - 1) + 2.23 \left( \frac{1}{2(2.23)} \left[ \ln(12.26) + 0.39 - 0.66 \right] \right) \right] = 6.203 \\
\text{or} \quad W = 6.203, h = 6.203(0.7874) = 4.884 \text{ mm} \\
\lambda &= \frac{\lambda_0}{\sqrt{2.23}} \left[ \frac{2.23}{1 + 0.63(2.23 - 1)(0.7874)0.7874} \right]^{1/2} = 0.7122 \lambda_0
\end{align*}
\]

**Note:** Since \( \lambda = \frac{\lambda_0}{4} \), then \( \frac{V_{eff}}{f_{eff}} = 0.712 \) or \( \varepsilon_{eff} = 1.974 \)

\[
\lambda = \frac{\lambda_0}{4} = 0.25(0.712) = 0.25(0.712)(30) = 5.34 \text{ cm}
\]

Similarly, we obtained:

\[
\begin{align*}
\frac{Z}{50} &= 52.4 \Omega, \quad \lambda = \frac{3\lambda_0}{8} \\
\frac{W}{h} &= 2.89 \quad \text{or} \quad W = 2.27 \text{ mm} \\
\lambda &= 0.726 \lambda_0, \quad \varepsilon_{eff} = 1.89
\end{align*}
\]

\[
\begin{align*}
\frac{Z}{50} &= 95.2 \Omega, \quad \lambda = \frac{3\lambda_0}{8} \\
\frac{W}{h} &= 0.99, \quad W = 0.78 \text{ mm} \\
\lambda &= 0.742 \lambda_0, \quad \varepsilon_{eff} = 1.79
\end{align*}
\]

\[
\begin{align*}
\frac{Z}{50} &= 79.1 \Omega, \quad \lambda = \frac{\sqrt{2}}{4} \\
\frac{W}{h} &= 1.44, \quad W = 1.34 \text{ mm} \\
\lambda &= 0.74 \lambda_0, \quad \varepsilon_{eff} = 1.925
\end{align*}
\]

\[
\begin{align*}
\lambda &= \frac{\lambda_0}{4} = 5.55 \text{ cm}
\end{align*}
\]

---

2 - 29
2.19) (a) \( Z_{\text{IN}} = \frac{Z_{\text{IN}}}{50} = 2 + j2 \)
\( y_{\text{IN}} = \frac{1}{3_{\text{IN}}} = 0.25 + j0.25 \)
\( l_1 = 0.339 \lambda - 0.25 \lambda = 0.089 \lambda \)
\( l_2 = 0.042 + (0.5 \lambda - 0.32 \lambda) = 0.219 \lambda \)

(b) If \( l_1 \) is an open-circuited stub, then
\( l_1 = 0.25 \lambda + 0.089 \lambda = 0.339 \lambda \)

(c) For \( Z_{\text{IN}} = \frac{Z_{\text{IN}}}{50} = 2 + j2 \), one answer is:

\[ Z = 100 + j100 \lambda \]

2.20) (a) \( Z_L = \frac{15 + j25}{50} = 0.3 + j0.5 \)
\( y_L = \frac{1}{3_{L}} = 0.882 - j1.47 \)

Rotate along a constant \(|y|\) circle, from \( y_L \) until the unit conductance circle is reached at \( y_x \).
\( y_x = 1 + j1.55 \)
\( l_1 = 0.178 \lambda + (0.5 \lambda - 0.32 \lambda) = 0.349 \lambda \)
\[ y_{IN} = y_{s.c} + y_x \]
\[ 1 = y_{s.c} + (1 + j1.55) \]
**Hence,**
\[ y_{s.c} = -j1.55 \]

The length \( l_2 \) of the short-circuited stub must have
\[ y_{s.c} = -j1.55 \]

From the Y Smith Chart:
\[ l_2 = 0.342\lambda - 0.25\lambda = 0.092\lambda \]

(b) **If the characteristic impedance of the stub is** \( Z_0' = 100\Omega \).

\[ Z_{s.c} = \frac{50}{-j1.55} = j32.26\Omega \]

Then:
\[ j32.26 = j\frac{Z_o'tan\phi d}{100tan\phi d} \Rightarrow \phi d = 0.312 \]
\[ d = \frac{l_2}{2\pi /\lambda} = 0.05\lambda \]

**Another Way:** Normalize \( Z_{s.c} \) with \( Z_0' \), \( Z_{s.c} = \frac{j32.26}{100} = j0.323 \)

Then:
\[ y_{s.c} = \frac{1}{j3.09} \text{. Locate } y_{s.c} = -j1.09 \text{ in the Y Smith Chart, and read } l_2 = 0.302\lambda - 0.25\lambda = 0.052\lambda \]

2.21) (a) \[ Y_{IN} = G_{IN} + jB_{IN} = 50 + j40 \text{ mS} \]
\[ R_{IN} = \frac{1}{G_{IN}} = \frac{1}{50} = 20 \text{ m} \]
\[ Z_{o1} = \sqrt{Z_LR_{IN}} = \sqrt{50(20)} = 31.62 \Omega \]

In a short-circuited \( 3\lambda \) stub: \( Y_{s.c} = jY_{o2} \). Hence, \( jY_{o2} = jB_{IN} = j40 \text{ mS} \)

Or \( Y_{o2} = 40 \text{ mS} \), \( Z_{o2} = \frac{1}{Y_{o2}} = 25 \Omega \)

(b) \[ Y_{IN} = G_{IN} - jB_{IN} = 50 - j40 \text{ mS} \]
\[ R_{IN} = \frac{1}{G_{IN}} = \frac{1}{50} = 20 \Omega \]

Then, \( Z_{o1} = \sqrt{50(20)} = 31.62 \Omega \). In a short-circuited \( 3\lambda \) stub:

\[ Y_{s.c} = -jY_{o2} \]. Hence, \( Y_{o2} = 40 \text{ mS} \) or \( Z_{o2} = \frac{1}{40} = 25 \Omega \).

(c) \[ Y_{IN} = G_{IN} + jB_{IN} = 10 + j20 \text{ mS} \]
\[ R_{IN} = \frac{1}{G_{IN}} = \frac{1}{10} = 100 \Omega \]

Then, \( Z_{o1} = \sqrt{50(100)} = 70.7 \Omega \). In an open-circuited \( 3\lambda \) stub:

\[ Y_{o.c} = -jY_{o2} \]. Hence, \( Y_{o2} = 20 \text{ mS} \) or \( Z_{o2} = \frac{1}{20} = 50 \Omega \).

(d) \[ Y_{IN} = 10 - j20 \text{ mS} \]. Hence: \( Z_{o1} = \sqrt{50(100)} = 70.7 \Omega \).

In an open-circuited \( 3\lambda \) stub: \( Y_{o.c} = -jY_{o2} \). Hence, \( Z_{o2} = \frac{1}{10} = 10 \Omega \)
2.22) (a) $\lambda = 0.5 \text{ m}, \theta = 0.6 + 0.8$

\[ y_n = \frac{1}{\lambda} = 0.6 - j0.8 \]

From the Smith Chart:

\[ l_1 = 0.136 \lambda, \quad l_2 = 0.375 \lambda - 0.166 \lambda = 0.209 \lambda \]

In Fig. 2.22(b):

\[ Y = \frac{0.6 - j0.8}{0.8 - j1.6} \text{ mS} \]

\[ Z_0 = \sqrt{\frac{50}{100(\frac{1}{12.10})}} = 64.5 \Omega \]

Using a $\frac{3\lambda}{2}$ open-circuited stub:

\[ \gamma = -j16 \text{ mS}, \text{ or } Y_0 = 16 \text{ mS} \]

\[ Z_0 = \frac{1}{16.10} = 62.5 \Omega \]

(b) Balanced Form of the Stubs.

For Fig. 2.22(a): $y_{bal} = \frac{1.15}{2} = j0.575 \Rightarrow l_{bal} = 0.083 \lambda$

For Fig. 2.22(b): $Z_{02(bal)} = 2(62.5) = 125 \Omega$

2.23) The $\frac{3\lambda}{8}$ stub with $Z_0 = 26.32 \Omega$ has an admittance of $-j0.0385$.

In a $Z_0 = 26.32 \Omega$ system:

\[ y_{oc} = (26.32)(-j0.038) = -j1 \]

Then:

\[ y_{oc(bal)} = \frac{-j1}{2} = -j0.5 \Rightarrow l_{bal} = 0.426 \lambda \]

\[ y_{oc(bal)} = \begin{array}{c}
\frac{26.32}{8} \\
\frac{26.32}{8}
\end{array} \]

\[ l_{bal} = 0.426 \lambda \]

The $\frac{3\lambda}{8}$ stub with $Z_0 = 47.6 \Omega$ has an admittance of $-j0.021 \Omega$.

\[ y_{oc} = 47.6(-j0.021) = -j1 \]

Then:

\[ y_{oc(bal)} = \frac{-j1}{2} = -j0.5 \Rightarrow l_{bal} = 0.426 \lambda \]

2 - 32
2.24) (a) \( \Gamma_l = 0.4 \angle 120^\circ \), \( Z_l = 0.538 - j0.444 \), \( Y_l = \frac{1}{Z_l} = 1.105 + j0.912 \)

\[ Y_l = \frac{Y_l}{50} = 22 + j18 \, \text{mS} \]

Hence: \( Z_{01} = \sqrt{50 \left( \frac{1}{22 \times 10^3} \right)} = 47.67 \, \Omega \) and

\[ j \, Y_{02} = j18 \, \text{mS} \text{ or } Z_{02} = \frac{1}{Y_{02}} = \frac{1}{18 \times 10^3} = 55.56 \, \Omega \]

\[ \Gamma_l = 0.4 \angle 120^\circ \]

(6) Each side of the balance stubs has an admittance of \( j9 \, \text{mS} \). If its characteristic impedance is \( Z_0 = 111.11 \, \Omega \), then \( Y_{(111.11)} = j9 \times 10^{-3} (55.56) = j0.5 \). Hence: \( l = 0.323 \lambda \).

2.25) \( \Gamma_l = 0.8 \angle 160^\circ \), \( Y_l = \frac{1}{\Gamma_l} = 2.64 - j4 \)

\( \Gamma_l = 0.7 \angle 20^\circ \), \( Y_l = \frac{1}{\Gamma_l} = 0.182 - j0.171 \)

The design of the matching circuits is shown in the Y Smith charts.
MATCHING TO $\Gamma_2$:

OPEN STUB LENGTH: $l_1 = 0.192\lambda$
SERIES TRANS. LINE: $l_2 = 0.098\lambda$

\[
\Gamma_1 = 0.81160^\circ
\]

BALANCED SHORT STUB: $y_{(bn)} = \frac{j\pi}{2} z_{1.3}$
LENGTH OF EACH SIDE: $l'_1 = 0.146\lambda$

MATCHING TO $\Gamma_L$:

SHORT-CIRCUITED STUB LENGTH: $l_1 = 0.076\lambda$
SERIES TRANS. LINE: $l_2 = 0.159\lambda$

\[
\Gamma_L = 0.71280^\circ
\]

BALANCED SHORT STUB: $y_{(bn)} = \frac{j\pi}{2} z_{1.995}$
LENGTH OF EACH SIDE: $l'_1 = 0.127\lambda$

AMPLIFIER DIAGRAM
The admittance of the 0.125 λ stub is:

\[ y_{oc} = j \]

Hence:

\[ y_x = 1 + j + j = 1 + 2j \]

Locate \( y_x \) in the Y-chart, and rotate 0.1 λ to find \( y_2 \). Hence

\[ y_2 = 2 - j2.7 \quad \text{AND} \quad \Gamma_2 = 0.71152° \]

(b)

Normalizing with \( z_o = 30 \lambda \)

\[ z = \frac{30}{30} = 1.66 \]

Hence, the impedance \( z_x \) at 0.5 λ from the load end is:

\[ z_x = 1.66 \quad \text{or} \quad z_x = 30(1.66) = 50 \lambda \]

And \( y_x = \frac{1}{z_x} = 20 \text{ mS} \)

The 75 λ, 0.46 λ balance stub has:

\[ y_{oc} = -j0.26 \quad \text{or} \quad y_{oc, total} = 2(-j0.26) = -j0.52 \]

\[ y_{oc, total} = -j0.52 = -j7 \text{ mS} \]

\[ Y_x = Y_x + Y_{oc, total} \]

\[ Y_2 = 20 - j7 \text{ mS} \]

In a 50-λ system:

\[ y_2 = 50y_2 = 1 - j0.35 \]

\[ z_2 = \frac{1}{y_2} = 0.891 + j0.312 \]

And \( \Gamma_2 = 0.172199.9° \)

2 - 35
2.27) (a) \( \lambda = \frac{3 \times 10^8}{6 \times 10^9} = 5 \text{ cm} \), \( \lambda' = 1.25 \text{ cm} = 1.25 \times \frac{1}{5} = \frac{1}{4} \)

\( \lambda_2 = 1.87 \text{ cm} = 1.87 \times \frac{1}{5} = 0.375 \lambda = \frac{3}{8} \)

For the \( \frac{1}{4} \) line:

\[
Z_x = \frac{Z_0}{\lambda} = \frac{30^2}{30} = 18 \Omega
\]

\[
Y_x = \frac{1}{Z_x} = 56 \text{ mS}
\]

For the \( \frac{3}{8} \) stub:

\[
Y_{oc} = \frac{1}{\lambda} \quad Y_{oc} = \frac{1}{27} \quad -j37 \text{ mS}
\]

Hence: \( Y_L = Y_x + Y_{oc} = 5.6 - j37 \text{ mS} \)

In a 50 \( \Omega \) system: \( y_L = 50 \cdot Y_L = 2.8 - j1.85 \) and \( \Gamma = 0.61 \angle 80.2^\circ \)

(b) For balance stubs use \( Z_0 = 2(27) = 54 \Omega \) with lengths of \( \frac{3}{8} \).

2.28) \( \Gamma_{in} = 0.5 \angle 100^\circ \), \( Z_{in} = 0.527 + j0.692 \)

The 50 \( \Omega \) 0.15 \( \lambda \) transforms \( Z_{in} \) to the impedance \( Z_x = 2.94 - j0.74 \),

or \( y_x = \frac{1}{Z_x} = 0.32 + j0.08 \)

The admittance of the \( \frac{1}{8} \) stub is: \( y_{oc} = \frac{1}{\lambda} \). Hence, for the two stubs:

\[
y_{oc, \text{total}} = \frac{1}{\lambda} + \frac{1}{\lambda} = 2 \frac{1}{\lambda}
\]

Then,

\[
y_L = y_x + y_{oc, \text{total}} = 0.32 + j0.08 + j2
\]

\[
= 0.32 + j2.08
\]

\[
Z_A = 50 \cdot \frac{Z_A}{y_A} = (0.072 - j0.47)50 = 3.6 - j23.5 \Omega
\]

2.29) \( Z_L = \frac{Z}{Z_0} = 1 - j \)

\( Z_{in} = \frac{Z_{in}}{Z_0} = 0.5 + j0.5 \)

\( y_L = \frac{1}{Z_L} = 0.5 + j0.5 \)

\( y_{in} = \frac{1}{Z_{in}} = 1 - j \)

\[2 - 36\]
\[ y_L \text{ and } y_{in} \text{ are on the same constant } |\Gamma| \text{ circle. Hence, a series transmission line of length:} \]
\[ l = 0.338\lambda - 0.088\lambda = 0.25\lambda \]

will change \( y_L \) to \( y_{in} \).

\[ Z_{in} = 25 + j25.\lambda \]
\[ (y_{in} = 1 - j) \]

2.30) \[ z_L = \frac{50 + j50}{50} = 1 + j \]

The 50\,\Omega, 0.125\,\lambda changes \( z_L = 1 + j \) to \( z_x = 2 - j \) (or \( y_x = 0.4 + j0.2 \)).

The 50\,\Omega, 0.125\,\lambda stub has: \( y_{oc} = j \).

Hence: \( y_L = y_x + y_{oc} = 0.4 + j0.2 + j = 0.4 + j1.2 \) and \( \Gamma_L = 0.7281104^\circ \)

2.31) Locate \( \Gamma_a = 0.571116^\circ \) in the Smith chart and read:
\[ z_x = 0.27 \text{ or } Z_x = 50 \angle 13.5^\circ \]

The transformation of 50\,\Omega to \( Z_x = 13.5\,\Omega \) can be done with a \( 4 \) \text{ line}
with: \( Z_0 = \sqrt{50(13.5)} = 26 \,\Omega \).

Then, a series transmission line of length: \( l = 0.088\lambda \) transforms
\[ Z_x = 13.5\,\Omega \] to \( Z_a = 19.5 + j28.\lambda \) or \( \Gamma_a = 0.571116^\circ \)

\[ Z_a = 50(0.37 + j0.562) = 18.5 + j28.\lambda \]

2-37
2.32) (a) \( z_L = \frac{100 + j100}{50} = 2 + j2 \)

The transmission line produces a motion along a constant IM circle, and the inductor L produces a motion along a constant conductance circle.

Using a ZY Smith chart:
\( \lambda_1 = 0.429\lambda - 0.209\lambda = 0.22\lambda \)

At \( y_x = 1 - j1.6 \). Then, \( y_L = 0 - (j1.6) = -j1.6 \)

or \( z_L = 50Z_L = \frac{50}{-j1.6} = j31.3\Omega \)

(b) In this circuit the inductor produces a motion along a constant resistance circle.

\( \lambda_1 = 0.322\lambda - 0.209\lambda = 0.113\lambda \)

At \( z_x = 1 - j1.6 \)

\( z_L = 0 - (j1.6) = j1.6 \)

\( z_L = 50Z_L = 50(j1.6) = 780\Omega \)

\( Z_{1N} = 50\Omega \)

\( Z'_{1N} = 50\Omega \) (\( \omega_{1N} = 1 \))
2.33) (a) \( Z_{IN} = \frac{25-j25}{50} = 0.5-j0.5 \)

\[ Z_0 = 30.8 \Omega \]

\[ Z_x = 19.38 \Omega \]

\[ Z_{IN} = 25-j25 \]

\[ Z_x \text{ and } Z_{IN} \text{ must be on the same constant } |\pi| \text{ circle. One solution is shown on the Smith chart.} \]

\[ Z_x = 50 \beta_x = 50(0.38) = 19.38 \Omega \]

Then:

\[ Z_0 = \sqrt{Z_x Z_{IN}} = \sqrt{50(19)} = 30.8 \Omega \]

AND \( \lambda_1 = 0.412 \lambda \)

(b) \( Z_{IN} = 0.5-j0.5 \), \( Y_{IN} = \frac{1}{Z_{IN}} = 1 + j \)

Letting \( Z_0 = 50 \Omega \) AND \( \lambda_1 = \text{any length, the admittance } Y_x = 1 \)

THEN, THE SHUNT STUB MUST PROVIDE: \( Y_{SC} = j \). Hence: \( \lambda_2 = 3 \lambda / 8 \)

(c) \( Z_{IN} = \frac{50+j50}{50} = 1+j \), \( Y_{IN} = 0.5-j0.5 \), \( Y = 10-j10 \text{ in } S \)

\[ Z_0 = \sqrt{Z_x Z_{IN}} = \sqrt{50(10j)} = 70.7 \Omega \]. THE STUB ADMITTANCE MUST BE: \( Y_{SC} = -j0.5 \). Hence: \( \lambda_2 = 0.426 \lambda \)

\[ Y_{IN} = Y_x + Y_{SC} \]

\[ = 1 + j \]

\[ 0.5 \]

\[ 50 \]

\[ 50 \]

\[ 0 \]

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\[ 70.7 \]

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2.34) STARTING WITH (2.6.13):

\[
G_T = \frac{(1-\Gamma_{IN}^2) |S_{21}|^2 (1-\Gamma_{L}^2)}{|(1-S_{11} \Gamma_{L}) (1-S_{22} \Gamma_{L}) - S_{12} S_{21} \Gamma_{L} |^2}
\]

(1)

THE DENOMINATOR CAN BE EXPRESSED AS

\[
|1-S_{22} \Gamma_{L}|^2 |1-S_{11} \Gamma_{L} - S_{12} S_{21} \Gamma_{L} \frac{\Gamma_{L}}{1-S_{22} \Gamma_{L}}| = |1-S_{22} \Gamma_{L}|^2 |1-\Gamma_{L} \Gamma_{IN}|^2
\]

HENCE, (1) CAN BE EXPRESSED IN THE FORM

\[
G_T = \frac{(1-\Gamma_{IN}^2) |S_{21}|^2 (1-\Gamma_{L}^2)}{|1-\Gamma_{L} \Gamma_{IN}|^2} \frac{\Gamma_{OUT}}{|1-S_{22} \Gamma_{L}|^2}
\]

(2.6.14)

ANOTHER WAY OF WRITING THE DENOMINATOR IS:

\[
|1-S_{11} \Gamma_{L}|^2 |1-S_{22} \Gamma_{L} - S_{12} S_{21} \Gamma_{L} \frac{\Gamma_{L}}{1-S_{11} \Gamma_{L}}| = |1-S_{11} \Gamma_{L}|^2 |1-\Gamma_{L} \Gamma_{OUT}|^2 \frac{\Gamma_{OUT}}{|1-S_{11} \Gamma_{L}|^2}
\]

HENCE, (1) CAN ALSO BE EXPRESSED AS:

\[
G_T = \frac{(1-\Gamma_{IN}^2) |S_{21}|^2 (1-\Gamma_{L}^2)}{|1-S_{11} \Gamma_{L}|^2} \frac{\Gamma_{OUT}}{|1-\Gamma_{L} \Gamma_{OUT}|^2}
\]

(2.6.15)

2.35) \[
\begin{align*}
&b_1 = \frac{S_{11}(1-\Gamma_{L} S_{22}) + S_{21} \Gamma_{L} S_{12}}{D}, \quad b_2 = \frac{S_{21}}{D} \\
&a_1 = \frac{1-S_{22} \Gamma_{L}}{D}, \quad a_2 = \frac{S_{21} \Gamma_{L}}{D}, \\
&D = 1 - (S_{11} \Gamma_{L} + S_{22} \Gamma_{L} + S_{21} \Gamma_{L} S_{12} \Gamma_{IN}) + S_{11} \Gamma_{L} S_{22} \Gamma_{L}
\end{align*}
\]

HENCE:

\[
A_T = \frac{a_1 b_1 + b_2 b_2}{a_1 b_2 + b_1 b_2} = \frac{S_{21} \Gamma_{L} + S_{21}}{S_{11}(1-\Gamma_{L} S_{22}) + S_{21} \Gamma_{L} S_{12} + 1-S_{22} \Gamma_{L}}
\]

or

\[
A_T = \frac{S_{21}(\Gamma_{L} + 1)}{1-S_{22} \Gamma_{L} + S_{11}(1-\Gamma_{L} S_{22}) + S_{21} \Gamma_{L} S_{12}}
\]

2 - 40
2.36) (a) From (2.8.6), with $\Gamma_L = \Gamma_{out}^* = 0.682^{\degree} 97.9^\circ$, we obtain $|\Gamma_b| = 0$. Then, using (2.8.4), $(\text{VSWR})_{out} = 1$.

(b) When $\Gamma_L = \Gamma_{out}^*$ we have $|\Gamma_b| = 0$ or $\Gamma_b = 0$. Hence,

$Z_b = Z_0 = 50 \, \Omega$.

(c) $|\Gamma_b| = \left| \frac{\Gamma_{out} - \Gamma_{out}^*}{1 - \Gamma_{out} \Gamma_L} \right| = \left| \frac{0.51^{\degree} 60^\circ - 0.682^{\degree} 97.9^\circ}{1 - 0.51^{\degree} 60^\circ(0.682^{\degree} 97.9^\circ)} \right| = 0.546$

$(\text{VSWR})_{out} = \frac{1 + 0.546}{1 - 0.546} = 3.41$

2.37) (a) From (2.8.3), with $\Gamma_a = \Gamma_{in}^* = 0.545^{\degree} 177.7^\circ$, we obtain $|\Gamma_a| = 0$.

Then, using (2.8.1), $(\text{VSWR})_{in} = 1$.

(b) When $\Gamma_a = \Gamma_{in}^*$ we have $|\Gamma_a| = 0$ or $\Gamma_a = 0$. Hence,

$Z_a = Z_0 = 50 \, \Omega$.

(c) $|\Gamma_a| = \left| \frac{\Gamma_{in} - \Gamma_{in}^*}{1 - \Gamma_{in} \Gamma_a} \right| = \left| \frac{0.41^{\degree} 45^\circ - 0.545^{\degree} 177.7^\circ}{1 - 0.41^{\degree} 45^\circ(0.545^{\degree} 177.7^\circ)} \right| = 0.735$

$(\text{VSWR})_{in} = \frac{1 + 0.735}{1 - 0.735} = 6.54$