Homework: Condition Number of a Square Matrix

Follow the same format as given at http://www.eng.usf.edu/~kaw/class/EML3035/homework/sample_homework.htm

Look under Sample HW for assignments HW#5 and beyond

What to Submit:
The sequence of items attached needs to be as follows
1. Affidavit sheet
   http://numericalmethods.eng.usf.edu/EML3035/Independent_affidavit_sheet.pdf
2. Pseudo-code for the function (Skip this part for this HW)
3. Flow chart (needs to be done with a word processor)
4. mfile for the function
5. Published mfiles (one for each of the three inputs)

Title
Finding the condition number of the matrix.

Background
The condition number, $\kappa$, of matrix $A$ is defined as follows:

$$\kappa = \| A \| \| A^{-1} \|$$

where

$\| A \|$ = The norm of matrix $A$

$\| A^{-1} \|$ = The norm of the inverse of matrix $A$

One of the ways to define the norm of a rectangular matrix $[B]_{m \times n}$ is called the column sum norm

$$\| B \| = \max_{1 \leq j \leq n} \sum_{i=1}^{m} |b_{ij}|$$

The condition number of the coefficient matrix in a set of simultaneous linear equations is a measure of the accuracy of the solution.

Example
\[ A = \begin{bmatrix} 1 & 3 & 4 \\ 4 & 5 & 6 \\ -15 & 6 & 9 \end{bmatrix} \]

\[ A^{-1} = \begin{bmatrix} 0.3333 & -0.1111 & -0.0741 \\ -4.6667 & 2.5556 & 0.3704 \\ 3.6667 & -1.8889 & -0.2593 \end{bmatrix} \]

\[ \| A \| = 20 \]

\[ \| A^{-1} \| = 8.6667 \]

\[ \kappa = \| A \| \| A^{-1} \| = 20 \times 8.6667 = 173.3340 \]

**Specifications**

1. You are going to write your own MATLAB function called `condmatrix` with the form
   
   \[ \text{function } [\text{condval}] = \text{condmatrix}(A) \]

to find the condition number of the matrix. Your program should work with square matrix of any size. The input parameter is \( A = \text{square matrix of order } n \times n \). You need to use loop(s) and conditional statement(s) (**you are not allowed to use the MATLAB `max` and `sum` or equivalent commands**) in the procedure to find the norm of the matrices.

You can use MATLAB `inv` function to find the inverse of a square matrix.

Return one variable – `condval`, that is, the value of the condition number of the matrix.

2. Now write another mfile called `test_condmatrix.m` that uses the MATLAB function `condmatrix` to find the condition number of the matrix.

3. Test the program with the following matrix and two other matrices of your choice of various sizes (greater than \( 4 \times 4 \)).

\[ \begin{bmatrix} 2 & 1 & 0 & 0 & 0 \\ 3 & 3 & 12 & 0 & 0 \\ 0 & 4 & -33 & 21 & 0 \\ 0 & 0 & 12 & 0 & 23 \\ 5 & 0 & 0 & 14 & 67 \end{bmatrix} \]