2.1.1. A capillary tube containing water has a radius 0.1-mm. If the hydraulic gradient is 0.01, what is the average velocity and how far would water move during a one-year period?

Answer. The average velocity is given by Eq. (2.1.8). For the capillary of radius 0.1-mm the average velocity is \( \nu = \frac{1 \text{g/cm}^3 \times 981 \text{cm/s}^2 \times (0.01 \text{cm})^2 \times 0.01}{8 \times 0.01 \text{g/cm} \cdot \text{s}} = 0.0123 \text{ cm/s} \). During a period of one-year, water would move a distance \( L = \nu \cdot 1 \text{ yr} = 0.0123 \text{ cm/s} \times 0.01 \text{ m/cm} \times 86400 \text{ s/d} \times 365 \text{ d/yr} = 3.870 \text{ m/yr} \).

2.1.2. For flow between parallel plates, Equation 2.1.6 reduces to \( -\frac{d^2u}{dy^2} = \frac{\rho g l}{\mu} \) for the function \( u(y) \). The boundary conditions are \( u(0) = 0 \) and \( u(H) = 0 \). Verify that Equation 2.1.9 gives the average velocity within the flow space.

Answer. Integrating the ODE twice gives \( u(y) = \frac{-\rho g l}{2\mu} y^2 + Ay + B \), where \( A \) and \( B \) are integration constants. Boundary condition \( u(0) = 0 \) gives \( B = 0 \), while the boundary condition \( u(H) = 0 \) then gives \( A = \frac{\rho g l}{2\mu} H \). The equation for the resulting velocity distribution is \( u(y) = \frac{\rho g l}{2\mu} y (H - y) \). The average velocity is found from \( \nu = \frac{1}{H} \int_0^H u(y) \, dy = \frac{\rho g l}{2\mu} H \left( \frac{2}{3} \right) = \frac{\rho g l H^2}{12\mu} \).

2.1.3. The sand in a filter that is 1.2 m thick with a cross-section area of 40 m² has a mean grain size of 0.4 mm and a porosity of 0.35. When water is ponded to a depth of 0.5 m on the filter and the base is free draining (the pressure is atmospheric), what is the discharge across the filter? Use the Kozeny-Carman equation to estimate the hydraulic conductivity.

Answer. From the Kozeny-Carman equation, the hydraulic conductivity is

\[
K = \frac{\rho g}{\mu} \left( \frac{n^3 d^2}{180 (1 - n)^2} \right) = \frac{1 \text{g/cm}^3 \times 981 \text{cm/s}^2 \times (0.35^3 \times (0.04 \text{cm})^2)}{0.01 \text{g/cm} \cdot \text{s} \times 180 \times (1 - 0.35)^2} = 0.088 \text{ cm/s} = 76 \text{ m/d}.
\]

2.2.1. The manometers shown in Figure 2.2.1 measure a water head difference of 3 ft across the sand column. The cross-section area is 0.8 ft² and the length of the sand column is 3 ft. If the measured discharge is 8 ft³/d, what is the hydraulic conductivity in ft/d?

Answer. From Equation 2.2.1, \( K = \frac{Q L}{A (z_1 - z_2)} = \frac{8 \times 3 \text{ ft}}{0.8 \text{ ft}^2 \times 3 \text{ ft}} = 10 \text{ ft/d} \).

2.2.2. A semiconfined aquifer is recharged from an overlying unconfined aquifer through an aquitard. From water balance studies it is estimated that the recharge rate is 2.9 cm/year. If the average piezometric surface of the confined aquifer is 10 m below the water table in the unconfined aquifer and the aquitard is 1 m thick, what is \( K_\theta \) of the aquitard in m/d?

Answer. Assuming that there is no head loss associated with vertical flow through the aquifer, Darcy's law gives \( K_\theta = q_v \Delta h = 0.029 \text{ m/yr} \times 1/365 \text{ yr/d} \times 1 \text{ m} / 10 \text{ m} = 8 \times 10^{-6} \text{ m/d} \).
2.2.3. The experiment of problem 2.2.1 is repeated with the same set-up and soil but with oil with a dynamic viscosity of 2 cp (centipoise) and specific gravity of 0.8. What is the daily discharge through the column for an oil head drop of 3 ft?

**Answer.** From Equation 2.1.11, \( K_{oi} = K_{water} \frac{\rho_{oi}}{\rho_{water}} \frac{H_{oi}}{H_{water}} = 10.8 \times 0.8 \times \frac{1 \text{ cp}}{2 \text{ cp}} = 4 \text{ ft/d} \). Equation 2.2.1 then gives \( Q = K_{A} \frac{Z_1 - Z_2}{L} = 4.5 \times 0.8 \times 2 \times \frac{3 \text{ ft}}{3 \text{ ft}} = 3.2 \text{ ft}^3/\text{d} \).

2.2.4. Three piezometers monitor water levels (piezometric heads) in a confined aquifer. Piezometer A is located 3000 ft due south of piezometer B. Piezometer C is located 2000 ft due west of piezometer B. The surface elevations of A, B, and C are 480, 610, and 545 ft, respectively. The depth to water in A is 40 ft, in B is 140 ft, and in C is 85 ft. Determine the direction of groundwater flow through the triangle ABC and calculate the hydraulic gradient.

**Answer.** The water levels in wells A, B, and C are at elevations (480 - 40 = 440 ft), (610 - 140 = 470 ft), and (545 - 85 = 460 ft), respectively, above the datum. Through the three elevations one may fit the equation for a plane: \( h = a + b x + c y \). Evaluating this at well A where \( h = 440 \text{ ft}, x = y = 0 \), gives \( a = 440 \text{ ft} \). Evaluating this next at well B where \( h = 470 \text{ ft}, x = 0, y = 3000 \text{ ft} \) gives \( c = (470 - 440) / 3000 = 0.01 \). Finally, evaluating this at well C where \( h = 460 \text{ ft}, x = -2000 \text{ ft}, y = 3000 \text{ ft} \) gives \( b = (460 - 440 - 0.01 \times 3000)/(-2000) = 0.005 \). Thus the head field satisfies \( h (ft) = 440 + 0.005 x (ft) + 0.01 y (ft) \). The hydraulic gradient is given by \( \frac{\partial h}{\partial x} + \frac{\partial h}{\partial y} j \), or \( \frac{\partial h}{\partial y} = -0.005 i - 0.01 j \). The magnitude of the gradient is \( \| \frac{\partial h}{\partial y} \| = \sqrt{0.005^2 + 0.01^2} = 0.0112 \).

The direction of flow is given by \( \theta = \tan^{-1} \left( \frac{0.005}{0.01} \right) = 34.4^\circ \). The gradient shows that the direction of flow is towards the third quadrant, to the angle from due east is \( 180^\circ + 34.4^\circ = 214.4^\circ \), towards the south, southwest.

2.2.5. Two piezometers are situated side-by-side and penetrate an unconfined aquifer. The first piezometer is screened at a depth of 35 ft while the second is screened at a depth of 60 ft. Downhole pressure transducers which are located adjacent to the screened intervals record pressures of 12.88 and 23.98 psi in piezometers one and two, respectively. Determine whether this is a recharge or discharge zone and estimate the depth to the water table in feet below the land surface. If the vertical conductivity is estimated to be 0.1 ft/d, calculate the yearly infiltration or evaporation in inches per year.

**Answer.** Choose the screen elevation in piezometer 2 for the datum. The pressure heads are \( p_{i}/\gamma = 12.88 \text{ lb/ft}^3 x 144 \text{ in}^2/\text{ft}^2 / 62.4 \text{ lb/ft}^3 = 29.723 \text{ ft}, p_{2}/\gamma = 23.98 x 144/62.4 = 55.338 \text{ ft} \). Thus \( h_1 = 25 + 29.723 = 54.723 \text{ ft} \) and \( h_2 = 55.338 \text{ ft} \), which implies upward flow. The upward gradient \( I = \frac{h_1 - h_2}{\Delta z} = 0.0246 \). The upward flow rate is calculated from \( q = K I = 0.1 \text{ ft/d} \times 0.0246 = 0.00246 \text{ ft/d} = 10.8 \text{ in/yr} \). To find the elevation of the water table apply the same gradient from well #2 to the water table where its \( h_{w1} = z_{w1} \), with the elevation datum at the well #2 screen. This gives \( z_{w1} = \frac{h_2}{1 + I} = 54.009 \text{ ft} \). The depth below the ground surface is \( D = 60 - 54.009 = 5.991 \text{ ft} \), or approximately \( D = 6.0 \text{ ft} \).

2.2.6. A piezometer is screened at a depth of 20 m below land surface and records a pressure of 120 kPa on a pressure transducer. An immediately adjacent piezometer is screened at a depth of 10 m below land surface, and the depth to water in this piezometer is 7 m below land surface. Is the vertical component of
flow upward or downward at this location, and what is the depth of the water table below land surface?

Assume the specific weight of water is 9810 N/m³.

*Answer.* Choose the depth of the first piezometer as the elevation datum. Then the head in the first piezometer is \( h_1 = p_1/\gamma = 120,000 \text{ Pa} / 9810 \text{ N/m}^3 = 12.323 \text{ m} \). The head in the second piezometer is \( h_2 = 10 \text{ m} + (10 \text{ m} - 7 \text{ m}) = 13 \text{ m} \). Because \( h_2 > h_1 \), the gradient and flow are downward. The magnitude of the gradient is 
\[
I_z = \frac{h_2 - h_1}{z_2 - z_1} = \frac{13 \text{ m} - 12.323 \text{ m}}{10 \text{ m} - 0 \text{ m}} = -0.0768 \text{.}
\]
Using the same gradient between piezometer 1 and the water table gives 
\[
I_z = \frac{z_{wt} - h_1}{z_{wt} - z_1} = \frac{h_1 + z_1 I_z}{1 + I_z},
\]
so with the calculated values this gives 
\[
z_{wt} = \frac{12.232 \text{ m} + 0 \text{ m} \times (-0.0768)}{1 - (-0.0768)} = 13.250 \text{ m}.
\]
The depth below the land surface is \( D = 20 \text{ m} - 13.250 \text{ m} = 6.750 \text{ m} \).

2.2.7. At a location \((x_1, y_1, z_1) = (1000 \text{ m}, 800 \text{ m}, 120 \text{ m})\) the pressure head is 130 m and the hydraulic gradient is 
\[
\bar{I} = 0.001 \hat{i} - 0.008 \hat{j} + 0.05 \hat{k}. \]
What are the hydraulic head and pressure (in Pa) at location \((x_2, y_2, z_2) = (2000 \text{ m}, -200 \text{ m}, 220 \text{ m})\) if the head gradient is assumed to be uniform. The specific weight of water is 9,810 N/m³.

*Answer.* For a uniform hydraulic gradient the following relationship holds: 
\[
\bar{I} = \frac{h_2 - h_1}{x_2 - x_1}. \] 
This gives 
\[
h_2 = h_1 + I_1 (x_2 - x_1) = (120 \text{ m} + 130 \text{ m}) - (0.001) \times (2000 \text{ m} - 1000 \text{ m}) - (0.008) \times (-200 \text{ m} - 800 \text{ m}) - 
(0.05) \times (220 \text{ m} - 120 \text{ m}) = 250 \text{ m} - 1 \text{ m} - 8 \text{ m} - 5 \text{ m} = 236 \text{ m}. \]
The pressure head is \( p_2/\gamma = h_2 - z_2 = 236 \text{ m} - 220 \text{ m} = 16 \text{ m}. \) The corresponding pressure is \( p_2 = 16 \text{ m} \times 9810 \text{ N/m}^3 = 15,696 \text{ Pa}. \)

2.2.8. A uniform granular media has a mean grain diameter of 0.3 mm and is classified as medium sand. This sand will be packed to a porosity of about 0.4 to make a 2 m thick sand filter. If the maximum allowable depth of water ponding on top of the filter is 0.75 m, what radius of the circular filter is required to pass a discharge of 200 L/s if the water temperature is 25 degrees C and the filter is freely draining at its base.

How would the required radius change if the water temperature was 5 degrees C?

*Answer.* At 25°C, Table B.2 gives the density and viscosity of water as 0.9970 kg/L and 0.00089 Pa-s (0.0089 g/cm-s), respectively. According to the Kozeny-Carman equation (Equation 2.1.13) the hydraulic conductivity is 
\[
K = \frac{0.997 \text{ g/cm}^3 \times 981 \text{ cm/s}^2}{0.0089 \text{ g/cm-s}} \left( \frac{0.4 \times (0.03 \text{ cm})^2}{180 \times (1 - 0.4)} \right) = 0.0977 \text{ cm/s} = 84.4 \text{ m/d}. \]

With this hydraulic conductivity Equation 2.2.1 gives 
\[
A = \frac{Q L}{K \Delta z} = (0.2 \text{ m}^3/\text{s} \times 86400 \text{ s/d}) \times (2 \text{ m})/(84.4 \text{ m/d}) = 2.75 \text{ m}^2. \] 
The corresponding radius is 
\[
R = \sqrt{\frac{A}{\pi}} = \sqrt{\frac{149 \text{ m}^2}{\pi}} = 6.89 \text{ m}. \]

At 5°C the density and viscosity of water are 1.000 g/cm³ and 0.01518 g/cm-s. The hydraulic conductivity is 
\[
K(5^\circ \text{C}) = K(25^\circ \text{C}) \times \mu_{5}/\mu_{25} \times \rho_{25}/\rho_{5} = 84.4 \text{ m/d} \times (1.000/0.9977) \times (0.00089/0.01518) = 49.6 \text{ m/d}. \]
The required radius is 
\[
R = \sqrt{\frac{84.4 \text{ m}}{49.6}} = 8.98 \text{ m}. \]

2.2.9. Four horizontal, homogeneous, isotropic geologic strata overlie one another. In descending order, they have thickness and hydraulic conductivity values of 15 ft and 12 ft/d; 6 ft and 0.1 ft/d; 5 ft and 6 ft/d; and 15 ft and 0.3 ft/d. Calculate the horizontal and vertical components of hydraulic conductivity for the